A Joint Factor Model for Bonds, Stocks, and Options *

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This version: September 12, 2023

Abstract

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Keywords: factor model, IPCA, corporate bond, option returns

JEL classification: G10, G11, G12

*We thank participants of the 2023 Esade spring workshop, the University of Hannover, Fidelity Investments, and Robeco for valuable feedback and discussions.

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Abstract

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1 Introduction

We propose a joint factor model for stocks, bonds, and options, motivated by the theories developed in Du, Elkamhi, and Ericsson (2019), Geske (1979), and Merton (1974). In Merton's (1974) framework, stocks and bonds issued by the same firm represent claims on the same underlying assets of the firm. Specifically, equity (debt) securities can be viewed as a long (short) position in call (put) option on the firm's assets. Du, Elkamhi, and Ericsson (2019) augment the Merton (1974) model with time-varying and priced asset volatility, and show that this can explain both the level and the dynamics of credit spreads and equity volatilities. Geske (1979) shows that an option contract written on corporate securities, such as an equity option, can be viewed as an option on an option, or a compound option. Consequently, as long as the three markets are partially integrated, they share a common factor structure. Our paper is devoted to characterizing this factor structure.

There is an intensive discussion in the literature on the degree of integration between the bond and stock markets (see, for example, Choi and Kim, 2018, Chordia, Goyal, Nozawa, Subrahmanyam, and Tong, 2017, and Sandulescu, 2023). The literature has also documented that option trading activity can influence the prices of individual stocks and bonds issued by the firm (Easley, O'Hara, and Srinivas, 1998). Similarly, informed option trading demand pressure can have an impact on option prices (Gârleanu, Pedersen, and Poteshman, 2008). There is also substantial evidence of information flow between individual equity option and stock markets (An, Ang, Bali, and Cakici, 2014), as well as individual equity option and bond markets (Cao, Goyal, Xiao, and Zhan, 2023).[1] More specifically, variables constructed with option market information predict future returns of individual stocks (Neuhierl, Tang, Varneskov, and Zhou (2023)), and stock characteristics are an important determinant of future option returns (Bali, Beckmeyer, Moerke, and Weigert, 2023).

As a result, we argue that option traders' expectations and their actual trades in the option markets have a significant impact on the future prices of individual stocks and bonds, which eventually influence the firms' asset returns. To be able to capture these complicated dynamics in the bond, option, and stock markets and their impact on future firm values, we extend the instrumented principal component analysis (IPCA) of Kelly, Pruitt, and Su

[1] An, Ang, Bali, and Cakici (2014), Bali and Hovakimian (2009), Cremers and Weinbaum (2010), and Xing, Zhang, and Zhao (2010) find a significantly positive cross-sectional relation between call-minus-put option implied volatility spreads and future returns of optionable stocks. Johnson and So (2012) find a positive relation between the ratio of trading volume in the stock to option trading volume and future stock returns. The findings of these studies suggest a link between investors' demand for options and future returns of the underlying stock. Cao, Goyal, Xiao, and Zhan (2023) propose a similar argument for the connection between equity options and future returns of corporate bonds.

(2019) to allow for heterogeneity in how firm characteristics inform the pricing of bonds, options, and stocks. Our IPCA-based joint factor model produces a more realistic expected return benchmark for the firm.

We note that our objective is not to propose a new structural model. Instead, motivated by Geske (1979) and an extended version of the Merton (1974) model by Du, Elkamhi, and Ericsson (2019), we propose a reduced-form factor model that jointly prices stocks, bonds, and options. The joint factor model can be justified within a compound option pricing model with stochastic asset price and asset volatility dynamics (Doshi, Ericsson, Fournier, and Seo, 2022). Since it is difficult to accurately characterize asset value and asset volatility dynamics for individual firms, we rely on joint IPCA with a large set of bond, option, and stock characteristics to back out a joint risk factor model from the time-series and cross-section of bond, option, and stock returns.

Main Findings: We extend the IPCA framework of Kelly, Pruitt, and Su (2019) by accommodating asset class-level differences in how characteristics instrument the variation in factor sensitivities, while maintaining a joint factor structure for firms' bonds, options, and stocks. In our joint IPCA methodology, individual asset returns for all three asset classes are driven by the same K latent factors through time-varying factor loadings, which we parameterize as a linear function of observable firm characteristics. We allow this linear function to vary for each asset class and use a large number of firm-level characteristics, which incorporate information from the firms' bonds, options, and stocks.

Specifically, we use the bond-level characteristics of Bali, Goyal, Huang, Jiang, and Wen (2022), the option-level characteristics from Bali, Beckmeyer, Moerke, and Weigert (2023), as well as the stock-level characteristics from Jensen, Kelly, and Pedersen (2021). From this exhaustive list of characteristics, we use 107 firm characteristics that result in significant Sharpe ratios (SRs) for at least one asset class. We form characteristic managed portfolios (CMPs) from these characteristics and find that 82 generate a significant SR for one asset class, 22 for two, and only two (the stock-to-option volume and weighted put-call spread of Cremers and Weinbaum (2010)) generate a significant SR for all three bond, option, and stock portfolios. This preliminary analysis already shows that asset classes are partially integrated, with the same conditioning information producing valuable trading strategies across the three asset classes.

Our estimated joint IPCA describes returns of bonds, options, and stocks well. A six-factor model explains 31% of the total variation of overall asset returns; 22% for options, 32% for stocks, and 37% for bonds. We use a simple aggregation scheme for the returns of a hypothetical investor looking to hold an equal investment in a firm's bond, option, and stock. Based on this aggregation scheme, we find that adjusting aggregate returns for risk using our joint IPCA model leaves only 7.6% of alphas significant, with an average unconditional R^2 of 28.6%.

Because the joint IPCA estimates a single factor structure for the three asset classes, we can back out the implied mean-variance efficient (MVE) tangency portfolio. Cochrane (2009) shows that shocks to the MVE portfolio are directly proportional to shocks to the stochastic discount factor. Our six-factor joint IPCA model generates an annualized SR of 6.9 in-sample (IS) and 6.7 out-of-sample (OOS). We also show that $t+1$ returns to the MVE portfolio are related to the VIX in t (Martin, 2017), and that the joint factor structure allows the model to benefit from substantial diversification benefits, by investing in multiple assets of a firm. The MVE's subportfolios of bonds, options, and stocks are uncorrelated or even negatively correlated. Finally, we show that the net-of-fees SR remains large and above 2.0 for realistic-to-high levels of relative transaction costs (Frazzini, Israel, and Moskowitz, 2018).

The common factor structure of joint IPCA is essential for explaining returns across bonds, options, and stocks. We assess how well the model captures unconditional alphas for the 107×3 CMPs of bonds, options, and stocks. Unconditionally, we find that 169 CMPs generate full-sample average returns that are statistically significant at the 5% level. After adjusting for risk using our six-factor joint IPCA model, only 7 alphas remain significant; 5 for bonds, 2 for options, and none for stocks.

Next, we investigate the commonality in return predictability in three different ways. We first analyze whether there is commonality in explanatory power for bonds, options, and stocks. We sort bonds, options, or stocks into decile portfolios by the model's respective R^2. When sorting by one asset class, we also document the resulting R^2 for each decile for the remaining two asset classes. If the markets are partially integrated, we expect that, for stocks that are well priced by the joint IPCA, bonds of the same firms and options on the same stock are well-priced too. Empirically, we find that in deciles sorted by bond's R^2, the R^2 spread of sorting options and stocks is 13% and 11%, respectively. For a sort on option R^2s, the bond and stock spread is 16% and 18%, respectively; and for a sort on stock R^2, the decile R^2 spread for bonds and options is 13% and 19%, respectively.

Second, we study which factors are most important for explaining returns of each asset class. In particular, are all six factors required for each asset class? To answer this question, we iteratively "turn off" the influence of each of the six factors and document the resulting drop in R^2 across asset classes. We find that three factors are responsible for roughly 90%

of the model's ability to explain bond returns, two factors for roughly 84% of the model's ability to explain option returns, and three factors for 91% of the model's explanatory power for stock returns. The six common factors benefit the explanatory power for each asset class.

A third way of understanding the usefulness of the joint IPCA is to compare its performance with IPCA models estimated separately for bonds, options, and stocks. We have mentioned earlier that our joint IPCA leaves only 7 statistically significant alphas from a total of 169 CMPs. We find that each of the three single IPCA models, estimated separately for one asset class, perform worse. The six-factor bond-based IPCA fails to explain 28 alphas, the option-based IPCA 75, and the stock-based IPCA 153. We also consider a combined six-factor model, which uses two factors estimated separately for each asset class, matching the number of latent factors estimated in the joint IPCA. This combined model fails to explain 63 alphas. Furthermore, we find that the joint IPCA's tangency portfolio significantly outperforms the tangency portfolio implied by each of the single asset class IPCA models.

Finally, we also compare our joint IPCA with existing benchmark bond-, option-, and stock-level factor models put forth by the literature to explain returns within one of the three asset classes. We find that the five-factor bond model by Kelly, Palhares, and Pruitt (2023) fails to explain the returns of 33 CMPs, the two-factor straddle model of Coval and Shumway (2001) a total of 126, and the five-factor model of Fama and French (2015) augmented with momentum leaves 156 CMP returns unexplained. Even a combination of the 13 factors (5 bond, 2 option, and 6 stock factors) fails to explain 29 alphas.

Since IPCA factors are latent in nature, it is useful to investigate the dynamics governing the joint six-factor structure. We first show that the six factors capture significant variations in macroeconomic fundamentals. As an example, we show that the first latent factor is related to the business cycle, with a positive correlation to innovations of the Chicago Fed National Activity Index, a negative correlation to the macroeconomic uncertainty measure of Jurado, Ludvigson, and Ng (2015), and a positive correlation to the intermediary capital ratio of He, Kelly, and Manela (2017). Our second IPCA factor hedges macroeconomic uncertainty and intermediary risks, and the fifth factor captures the spread between overall macroeconomic risks and the risks of the intermediary sector. We furthermore show that the six latent factors capture important information from the three asset classes that cannot be replicated by the three macroeconomic indicators or the 13 benchmark factor models. For this, we propose a novel method to interpret latent factors, which replaces each factor's realizations by the fitted values from regressing it on a set of macroeconomic indicators and benchmark factors.

Related Literature: Our paper contributes to the literature investigating the integration of the bond and stock markets. If the two markets are (partially) integrated, risk premia should show up in both markets. Koijen, Lustig, and Van Nieuwerburgh (2017) show that some bond factors are priced in the cross-section of stock returns, whereas Chordia, Goyal, Nozawa, Subrahmanyam, and Tong (2017) argue that equity and corporate bond returns require a different set of risk factors, and Choi and Kim (2018) find that the risk premia of stock factors differ when using bond returns, accounting for the implied hedge ratio. On the modeling front, Du, Elkamhi, and Ericsson (2019) extend the Merton (1974) structural credit risk model with priced asset variance risk and show that this resolves the credit risk puzzle.

Doshi, Ericsson, Fournier, and Seo (2022) extend the compound option pricing model of Geske (1979) by allowing for idiosyncratic volatility and asset value jumps in the stochastic volatility model of Du, Elkamhi, and Ericsson (2019). While Collin-Dufresne, Junge, and Trolle (2022) argue that CDX and SPX options are disintegrated, the model setup of Doshi, Ericsson, Fournier, and Seo (2022) allows the authors to jointly explain the level and time variation of both SPX and CDX options. Culp, Nozawa, and Veronesi (2018) derive the notion of "pseudo firms" from the insights of the Merton (1974) model using traded SPX option prices and show that the credit risk puzzle can be explained by a risk premium for tail and idiosyncratic asset risks. There is, however, little research on the joint integration of bond, option, and stock markets. Our paper adds additional insights into the risk and return trade-off of bonds, options, and stocks from a reduced form factor model with a shared factor structure.

There is a vast literature on factor models for equity returns (see, for example Fama and French, 2015 and Hou, Xue, and Zhang, 2015), corporate bonds (Kelly, Palhares, and Pruitt, 2023), currency (Lustig, Roussanov, and Verdelhan, 2011), commodity futures (Szymanowska, de Roon, Nijman, and van den Goorbergh, 2014), and cryptocurrencies (Liu, Tsyvinski, and Wu, 2022). Despite the proliferation of factor models for stocks, bonds, and other asset classes, the literature on factor models for option returns is relatively sparse, with a few recent advances in Bali, Cao, Chabi-Yo, Song, and Zhan (2022), Goyal and Saretto (2022), and Horenstein, Vasquez, and Xiao (2022). Kozak, Nagel, and Santosh (2020) advocate for a low-dimensional factor structure. We achieve parsimony by estimating a model of common latent risk factors across asset classes, which exploits the markets' partial integration. Just six factors are sufficient to accurately and jointly express the risk and return trade-off of bonds, options, and stocks.

2 Theoretical Motivation

Following Bates (1996), Du, Elkamhi, and Ericsson (2019), Heston (1993), and Leland (1994), we assume that the dynamics of a firm's asset value, V_t, and asset variance, σ_t^2, are governed by the following stochastic volatility-jump diffusion model under the physical probability measure $\mathbb{P}$:

$$\begin{aligned} dV_t/V_t &= (\mu_V^{\mathbb{P}} - q)dt + \sigma_t \cdot \left(\rho dW_{2,t} + \sqrt{1-\rho} dW_{1,t}\right) + \phi dZ_t \\ d\sigma_t^2 &= \kappa(\theta - \sigma_t^2)dt + \gamma\sigma_t dW_{2,t} \\ \text{Prob}(dZ_t = 1) &= \lambda dt, \qquad \ln(1+\phi) \sim N\left[\ln(1+\bar{\psi}) - \delta^2/2,\ \delta^2\right], \end{aligned} \tag{1}$$

where $\mu_V^{\mathbb{P}}$ is the expected return on the firm's assets, q is the proportional payout rate, ρ is the instantaneous correlation between the Brownian motion $W = \rho W_2 + \sqrt{1-\rho^2} W_1$ that drives asset value uncertainty and W_2 that drives uncertainty with respect to asset variance. The parameter κ is the speed of mean reversion, θ is the long-run average of variance, and γ is the scale parameter for the diffusion process of the asset variance. The parameter λ is the annual frequency of jumps, ϕ is the random percentage jump conditional on the occurrence of a jump at time t, δ^2 is the variance of jumps, and Z_t is a Poisson process with constant intensity λ.

The specification in Eq. (1) involves both diffusive and jump risks as well as volatility risk (or uncertainty about future asset variance). The risk-adjusted dynamics for V_t and σ_t^2, under the risk-neutral probability measure $\mathbb{Q}$, are given by:

$$\begin{aligned} dV_t/V_t &= (r - q - \lambda^* \bar{\phi}^*)dt + \sigma_t \cdot \left(\rho dW_{2,t}^{\mathbb{Q}} + \sqrt{1-\rho} dW_{1,t}^{\mathbb{Q}}\right) + \phi^* dZ_t^{\mathbb{Q}} \\ d\sigma_t^2 &= \kappa^*(\theta^* - \sigma_t^2)dt + \gamma\sigma_t dW_{2,t}^{\mathbb{Q}} \\ \text{Prob}(dZ_t^{\mathbb{Q}} = 1) &= \lambda^* dt, \qquad \phi^* \sim N\left[\bar{\phi}^*,\ Var(\phi^*)\right], \end{aligned} \tag{2}$$

where r is the risk-free rate and starred variables in Eq. (2) represent the risk-adjusted versions of the true variables, taking into account the pricing of jump risk and volatility risk. Following Du, Elkamhi, and Ericsson (2019), we specify the market prices of diffusive risk and variance risk in such a way that the dynamics of the two Brownian shocks under the risk-neutral measure ($W_{1,t}^{\mathbb{Q}}$, $W_{2,t}^{\mathbb{Q}}$) follow:

$$\begin{aligned} dW_{1,t} &= dW_{1,t}^{\mathbb{Q}} - \Psi_D \cdot \sigma_t dt \\ dW_{2,t} &= dW_{2,t}^{\mathbb{Q}} - \Psi_V \cdot \sigma_t dt \end{aligned} \tag{3}$$

Under this specification, the asset risk premium is defined as,

$$\mu_V^{\mathbb{P}} - r = \left(\sqrt{1-\rho}\Psi_D + \rho\Psi_V\right) \cdot \sigma_t^2, \tag{4}$$

and compensates for diffusive risk via Ψ_D and variance risk via Ψ_V.[2] Although Ψ_D and Ψ_V are assumed to be constant in the aforementioned theoretical models, in practice the market prices of diffusive and volatility risks are known to exhibit significant time-series variation and nonlinearity. Thus, it is very difficult to provide an accurate characterization of firm value dynamics in a theoretical setting.

Following Du, Elkamhi, and Ericsson (2019) and Leland (1994), we assume that the firm issues consol bonds so that the equity value can be written as the difference between the levered firm value (F) and the debt value (D), that is, $E(V) = F(V) - D(V)$. Assuming stochastic volatility without jumps, Du, Elkamhi, and Ericsson (2019) show that the levered firm's value is given by:

$$F(V_t) = V_t + \zeta c/r \left(1 - p_D(V_t, \sigma_t^2)\right) - \alpha V_D p_D(V_t, \sigma_t^2) \tag{5}$$

where V, ζ, c, α, V_D, p_D, respectively, denote the initial unlevered asset value, the tax rate, the coupon rate, the liquidation cost, the default boundary, and the present value of \$1 at default.

The debt value is the sum of the present value of the coupon payments before default and the recovered value of the firm at default, and is given by:

$$D(V_t) = \frac{c}{r} + \left[(1-\alpha)V_D - \frac{c}{r}\right] \cdot p_D(V_t, \sigma_t^2). \tag{6}$$

The equity value is therefore given by:

$$E(V_t) = V_t - \frac{(1-\zeta)c}{r} + \left[\frac{(1-\zeta)c}{r} - V_D\right] \cdot p_D(V_t, \sigma_t^2). \tag{7}$$

Applying Ito's lemma, Du, Elkamhi, and Ericsson (2019) obtain the stochastic process for the equity value as follows:

$$\frac{dE_t}{E_t} = \mu_{E,t}dt + \frac{V_t}{E_t}\frac{\partial E_t}{\partial V_t}\sigma_t\left(\rho dW_{2,t} + \sqrt{1-\rho}dW_{1,t}\right) + \gamma\frac{1}{E_t}\frac{\partial E_t}{\partial V_t}\sigma_t dW_{2,t}, \tag{8}$$

[2]To generate a positive risk premium associated with the first Brownian motion (W_1), Ψ_D should be positive. Given that asset return and volatility are known to be negatively correlated ($\rho < 0$), Ψ_V should be negative to generate a positive risk premium associated with the second Brownian motion (W_2).

where $\mu_{E,t}$ is the instantaneous equity return. Under the risk-adjusted measure $\mathbb{Q}$, $p_D = \mathbb{E}_t^{\mathbb{Q}}[e^{-r(\Gamma-t)}]$, where $\Gamma = \inf\{s \geq t | V_s = V_D\}$ denotes the default time. To solve for p_D, Du, Elkamhi, and Ericsson (2019) show that one needs to compute the probability density function of the stopping time Γ under the risk-adjusted probability measure $\mathbb{Q}$. To do so, Du, Elkamhi, and Ericsson (2019) rely on Fortet (1943)'s lemma. Since there is no closed form solution for the firm, equity, or debt value, Du, Elkamhi, and Ericsson (2019) calibrate their stochastic volatility model without jumps.

In a model where corporate securities are options on a firm's assets, option contracts on these can be viewed as options on options, or compound options (Geske, 1979). Doshi, Ericsson, Fournier, and Seo (2022) extend the compound option pricing model of Geske (1979) by allowing for idiosyncratic volatility and asset value jumps in the stochastic volatility model of Du, Elkamhi, and Ericsson (2019). Doshi, Ericsson, Fournier, and Seo (2022) also adopt the simulation approach in Du, Elkamhi, and Ericsson (2019) and show that their model jointly explains the level and time variation of both equity index (SPX) and credit index (CDX) option prices well OOS. In the model of Doshi, Ericsson, Fournier, and Seo (2022), aggregate unlevered asset return and variance shocks are the only two sources of priced risk. Thus, financial instruments such as the equity index, credit protection index, and equity/credit index options derive their risk premia from these two sources. However, each instrument has exposure to its own specific states of the world, and hence differs in its loading on the common sources of risk. As a result, while sources of risk are shared across markets, each instrument is priced quite differently.

Based on the above theoretical models, our joint factor model for bonds, stocks, and options can be justified within a compound option pricing model with stochastic asset price and asset volatility dynamics. As shown by Du, Elkamhi, and Ericsson (2019) and Doshi, Ericsson, Fournier, and Seo (2022), it is very difficult to estimate directly the asset value and asset volatility dynamics proposed in their model. Hence, both studies follow a calibration or simulation approach. In addition to the complications about the estimation of stochastic-volatility/jump type models for stock and bond returns, we argue for the presence of sizeable information flow between equity option and underlying stock markets. Thus, changes in equity option prices may have a significant impact on the future prices of individual stocks and bonds, which eventually influence the firm's asset returns. To incorporate these complicated dynamics in the option, stock, and bond markets and their impact on future firm values, we rely on joint IPCA with a large set of stock, bond, and option characteristics to back out a joint risk factor model for the firm's asset returns. As in Doshi, Ericsson, Fournier, and Seo (2022), we explicitly allow for heterogeneity in the sensitivity of bonds, options, and stocks to the common set of risk factors.

The benefits of a joint factor model are manifold: first and foremost, we retain a parsimonious factor structure across many asset classes, yielding a lower number of common factors, as soon as the included asset classes are (partially) integrated. Kozak, Nagel, and Santosh (2020) advocate for focusing on a small number of factors. Estimating the *common* factor structure for many assets serves the purpose so that the resulting factor structure is valid for pricing all these asset classes. Second, a joint factor model allows us to estimate a tangency portfolio, which incorporates the covariance structure between asset classes. This in turn informs us about the dynamics of the stochastic discount factor, which spans the joint pricing of multiple asset classes. From a trading perspective, the tangency portfolio across asset classes informs researchers and practitioners alike about relative investment opportunities in the three markets. Third, we learn about commonalities and differences in the risk-return tradeoff of different asset classes in a unified model. By investigating the importance of the latent factors and observable characteristics in instrumenting betas at the level of the asset class, we can assess the relative degree of integration of bond, option, and stock returns of the same firm. Our proposed extension of IPCA (Joint IPCA) combines these benefits in a simple and intuitive model setup.

3 Econometric Methodology

One contribution of this study is in extending the well-established IPCA by Kelly, Pruitt, and Su (2019). We accommodate asset class-level differences in how characteristics instrument the variation in factor sensitivities, while maintaining a joint factor structure across bonds, options, and stocks. Consider an asset i, which is part of one of three asset classes, $AC \in$ [Bonds, Options, Stocks]. We can express asset i's excess return R^{AC}_{it+1} as:

$$R^{AC}_{it+1} = \beta^{AC}_{it}{}' F_{t+1} + \varepsilon_{it+1}, \qquad \beta^{AC}_{it} = z'_{it}\Gamma^{AC}_{\beta}. \tag{9}$$

Individual returns are driven by K latent factors, F_{t+1}, through factor loadings β^{AC}_{it}, which we parameterize as a linear function of L observable characteristics z_{it}. The mapping function from characteristics to betas is given by an $L \times K$ matrix Γ^{AC}_{β} that is specific to asset class AC. If there are N_{t+1} assets with available data, then we can express the asset pricing equation as:

$$R^{AC}_{t+1} = \beta^{AC}_{t} F_{t+1} + \varepsilon^{AC}_{t+1}, \tag{10}$$

where $\beta_t^{AC} = Z_t \Gamma_\beta^{AC}$ is a $N_{t+1} \times K$ matrix of betas using the $N_{t+1} \times L$ matrix Z of characteristics.

It is easy to see that the factor sensitivity of the stock and bonds of the same firm need not be the same. In the classic Merton-type firm model (Merton, 1974), a firm's stock can be modeled as a call option on the firm's assets, while its debt is the combination of a risk-less bond and a written put. The sensitivities of these two option portfolios to the same set of factors will naturally differ. Likewise, delta-hedged equity option returns capture differences in the expectation and realization of variance and jump risks, among others. Their factor sensitivity is therefore also going to differ from the sensitivity of the underlying stock, which itself is not directly exposed to these higher-order terms.

The innovation in our paper, therefore, is to allow for differences in the mapping function Γ_β^{AC} for each asset class. Instead of forcing Γ_β to be the same for bonds, options and stocks, we allow for class-level variation in how asset characteristics inform us about the risk-return tradeoff.

Consider three $N_{t+1} \times 1$ vector of returns, R_{t+1}^B, R_{t+1}^O, and R_{t+1}^S, representing returns for bonds, options, and stocks, respectively. The factor sensitivities are given by $\beta_t^B = Z_t \Gamma_\beta^B$, $\beta_t^O = Z_t \Gamma_\beta^O$, and $\beta_t^S = Z_t \Gamma_\beta^S$ for the three asset classes. Note that since the set of L characteristics is the same for each asset class, we allow for stock/option characteristics to influence bond returns in addition to bond characteristics, and so on.

It will be convenient to stack the three return vectors together into one $3N_{t+1} \times 1$ vector R_{t+1} as:

$$R_{t+1} \equiv \begin{bmatrix} R_{t+1}^B \\ R_{t+1}^O \\ R_{t+1}^S \end{bmatrix} = \begin{bmatrix} \beta_t^B \\ \beta_t^O \\ \beta_t^S \end{bmatrix} F_{t+1} + \begin{bmatrix} \varepsilon_{t+1}^B \\ \varepsilon_{t+1}^O \\ \varepsilon_{t+1}^S \end{bmatrix} = \beta_t F_{t+1} + \varepsilon_{t+1},$$

with

$$\beta_t \equiv \begin{bmatrix} \beta_t^B \\ \beta_t^O \\ \beta_t^S \end{bmatrix} = \begin{bmatrix} Z_t \Gamma_\beta^B \\ Z_t \Gamma_\beta^O \\ Z_t \Gamma_\beta^S \end{bmatrix} = \mathcal{Z}_t \Gamma_\beta, \qquad \mathcal{Z}_t \equiv \begin{bmatrix} Z_t & 0 & 0 \\ 0 & Z_t & 0 \\ 0 & 0 & Z_t \end{bmatrix}, \; \Gamma_\beta \equiv \begin{bmatrix} \Gamma_\beta^B \\ \Gamma_\beta^O \\ \Gamma_\beta^S \end{bmatrix}, \tag{11}$$

where β_t is a $3N_{t+1} \times K$ matrix of loadings, $\mathcal{Z}_t$ is a $3N_{t+1} \times 3L$ matrix of stacked characteristics, and Γ_β is the $3L \times K$ mapping matrix from characteristics to loadings (0 is a $N_{t+1} \times L$ matrix of zeros). Eq. (11) is our central Joint IPCA asset pricing equation.

It is useful to define a $3L \times 3L$ matrix W_t as (0 below is a $L \times L$ matrix of zeros):

$$W_t = \mathcal{Z}_t' \mathcal{Z}_t / N_{t+1} = \begin{bmatrix} Z_t' Z_t & 0 & 0 \\ 0 & Z_t' Z_t & 0 \\ 0 & 0 & Z_t' Z_t \end{bmatrix} / N_{t+1}, \tag{12}$$

and a $3L \times 1$ matrix X_{t+1} as:

$$X_{t+1} \equiv \begin{bmatrix} X_{t+1}^B \\ X_{t+1}^O \\ X_{t+1}^S \end{bmatrix} = \mathcal{Z}_t' R_{t+1} / N_{t+1} = \begin{bmatrix} Z_t' R_{t+1}^B \\ Z_t' R_{t+1}^O \\ Z_t' R_{t+1}^S \end{bmatrix} / N_{t+1}. \tag{13}$$

It is readily seen that X_{t+1} are the returns of CMPs. Since we have L characteristics and three asset classes, we have $3L$ such portfolios.

With X_{t+1} and W_t at hand, the first order conditions for Eq. (11) are:

$$\widehat{F}_{t+1} = \left(\widehat{\Gamma}_\beta' W_t \widehat{\Gamma}_\beta \right)^{-1} \widehat{\Gamma}_\beta' X_{t+1} \tag{14}$$

$$\text{vec}\left(\widehat{\Gamma}_\beta \right) = \left(\sum_{t=1}^{T-1} W_t \otimes \widehat{F}_{t+1} \widehat{F}_{t+1}' \right)^{-1} \left(\sum_{t=1}^{T-1} X_{t+1} \otimes \widehat{F}_{t+1} \right).$$

While this system of first-order conditions still does not admit a closed-form solution, it is quickly solvable using an alternating least-squares procedure. Latent factor realizations are obtained from month-by-month cross-sectional regressions of the stacked vector of the excess returns of all assets R_{t+1} on β_t (Fama and MacBeth, 1973). Γ_β are the coefficients of regressing CMP returns on factors F_{t+1} interacted with asset characteristics $\mathcal{Z}_t$. Given the structure of the system, the estimation of Γ_βs is essentially three separate regressions, one for each asset class.

Identifying a unique set of parameters is important in latent factor models, as they are identified only up to a rotation. Models $\Gamma_\beta F_{t+1}$ and $\Gamma_\beta R R^{-1} F_{t+1}$ are identical for any rotation matrix R. Following Kelly, Pruitt, and Su (2019), we impose the normalization that $\Gamma_\beta' \Gamma_\beta$ is the identity matrix, that the unconditional second moment matrix of F_{t+1} is diagonal with descending diagonal entries, and that the time-series average of F_{t+1} is positive. The identification assumptions do not restrict the model's ability to explain returns of bonds, options, and stocks, but merely serve as a way to pin down unique parameters of the model.

4 Data

4.1 Returns & Characteristics

Returns. Our analyses use returns of three different asset classes. Excess stock returns (corrected for delisting) are from CRSP.

For corporate bond returns, we rely directly on the WRDS Corporate Bond Database for the sample period from August 2002 to December 2021. Corporate bond returns are directly from WRDS based on "RET_EOM" (e.g., returns computed using the last price at which bond was traded in a given month), which defines corporate bond return in month t as

$$R_{it} = \frac{P_{it} + A_{it} + C_{it}}{P_{it-1} + A_{it-1}} - 1, \tag{15}$$

where P_{it} is the last price at which bond was traded in month t, A_{it} is accrued interest on the same day of bond prices, and C_{it} the coupon payment in month t, if any.

Finally, we consider daily delta-hedged option returns following Bali, Beckmeyer, Moerke, and Weigert (2023). Let the option contract's value be denoted by O and the value of the underlying stock as S. Then, the delta-hedged dollar gain over the period $(t, t+1)$ is given by:

$$\Pi_{t+1} = O_{t+1} - O_t - \sum_{n=0}^{N-1} \Delta_{t_n} \left(S_{t_{n+1}} - S_{t_n}\right) - \sum_{n=0}^{N-1} \frac{R_{ft_n}}{365} \left(O_{t_n} - \Delta_{t_n} S_{t_n}\right). \tag{16}$$

We scale the dollar gain by the initial value of the investment portfolio (Cao and Han, 2013):

$$R_{t+1} = \frac{\Pi_{t+1}}{|\Delta_t S_t - O_t|}. \tag{17}$$

We retain option contracts that are at-the-money,[3] for which the bid is positive, the offer exceeds the bid, the mid price is at least \$0.125, the relative quoted spread is at most 50% of the mid, the implied volatility is available and the open interest is positive. Furthermore, option prices need to adhere to American option bounds. Lastly, we require that the put (call) prices are monotonically increasing (decreasing) in the contract's strike price, iteratively retaining those contracts with the larger trading volume and a strike price closer to the current price of the underlying.

The returns are winsorized at the 1% level per asset class. We want to understand the

[3] Defined by $\left|\frac{\ln K/S}{iv \times \sqrt{ttm}}\right| \leq 1$, where iv is the contract's implied volatility, ttm its time-to-maturity, K its strike price, and S the price of its underlying stock.

common structure underlying average not extreme returns. Limited by the availability of bond return data through TRACE, we start our sample in August of 2002. Our sample period includes the great financial crisis in 2008/2009, as well as the Covid-selloff at the beginning of 2020.

Contract Selection. For each firm, we select a representative bond and option. Without this step, given that many firms have hundred of bonds and options, information about options and bonds would invariably dominate that of stocks in the joint model estimation. Choosing one bond and one option also facilitates the comparison of the model's ability to price assets of different classes. For each firm, we choose the bond with the largest amount outstanding that matures within two to eight years. We find that the majority of bonds fall within this maturity range, which puts the bonds for different firms on an equal footing. For options, we select the contracts that expire in roughly 50 days and retain the expiration date on which the majority of options expire. This is typically the third Friday of the month after the next.

As we require information for each of the three asset classes for a firm to be included in our sample, we restrict our analysis to the largest and most liquid stocks, which are both optionable and have actively traded bonds. In total, we have valid bond, option, and stock return observations for 1,337 unique firms.

Characteristics. Bond-level characteristics are taken from Bali, Goyal, Huang, Jiang, and Wen (2022), option-level characteristics from Bali, Beckmeyer, Moerke, and Weigert (2023), and stock-level characteristics from Jensen, Kelly, and Pedersen (2021) In total, this gives us 264 firm characteristics, of which 40 are based on the bond, 71 on the option, and 153 on the stock. For each characteristic, we impose a limit on how often it can be missing.[4] Specifically, we require that each characteristic is available for at least 2/3 of the firms in the average month. This filter drops seven characteristics. Following standard practice in the literature, we rank each characteristic cross-sectionally and standardize its values to lie between -0.5 and 0.5 for each month.

To weed out characteristics that essentially convey the same information, we identify those pairs that have correlation, $|\rho| \geq 95\%$. From each identified pair, we retain that characteristic which is available for more asset$\times$month observations. In total, this drops eight characteristics from our dataset, for a total of 249 firm characteristics. A complete

[4]See Beckmeyer and Wiedemann (2023), Bryzgalova, Lerner, Lettau, and Pelger (2022), and Freyberger, Höppner, Neuhierl, and Weber (2022) for a discussion of missing data in cross-sectional asset pricing.

list of the 264 characteristics is provided in Appendix A, including their original academic source, as well as a classification of the characteristic's type, and whether it is included in the final dataset.

4.2 Characteristic-Managed Portfolios Across Asset Classes

Our joint dataset covers a large number of bond, option, and stock characteristics. We now provide first evidence of the benefits associated with a joint consideration of this information set when making investment decisions in either of the three asset classes. For each of the $l = 1, \ldots, L$ characteristics, we compute the investment performance of the associated CMPs, calculated as in Eq. (13) using bonds, options, or stocks as the investable assets. Panel A of Table 1 reports the twelve CMPs of bonds with the largest absolute SRs. We also provide the resulting SRs for the CMPs of options and stocks using the same 12 characteristics. The remaining panels replicate this exercise for the top-12 CMPs of options and stocks.

Panel A of Table 1 shows that the SRs of CMPs of bonds are large. For example, a long-short bond portfolio sorted on the stock's short-term reversal (S_ret_1_0) achieves an in-sample (IS) SR of 3.89. In contrast, using the same information in the options or stock market fails to generate a significant SR.[5] The average absolute SR of the top-12 CMPs of bonds is high at 2.45. Sorting bonds on the stock's (idiosyncratic) skewness also generates highly profitable investment strategies, as does information about the option's put-call ratio (O_pcratio) and changes in the implied volatility curve (O_dciv and O_dpiv, see An, Ang, Bali, and Cakici, 2014). Interestingly, none of the top-12 CMPs of bonds use bond characteristics as the conditioning variable. Not reported in the table, we find that the most profitable bond characteristic for CMPs of bonds is bond-based short-term reversal (B_rev), which manages to generate only the 23rd largest SR. This is already first indicative evidence of the importance of considering characteristics *of the firm*, not only of the bond, when deciding to invest in corporate bonds. We also find that the characteristics of the top-12 CMPs of bonds are for the most part unable to generate meaningful investment performance for CMPs of options or stocks. The average absolute SR of CMPs of options (stocks) amounts to 0.36 (0.32), lower than that of an investment in the stock market. Only a sort on the option's delta generates a SR for the associated CMP of options that is significant at the 5% level.

The SRs of CMPs of options (Panel B of Table 1) are lower than those of the CMPs of bonds, but still sizable with an average absolute SR of 1.53. Sorting on implied volatility

[5] We test for statistical significance of the SR following Lo (2002).

Table 1: Characteristic-managed Portfolios

The table shows annualized SRs of the 12 characteristic-managed portfolios that generate the largest absolute SRs for bonds in Panel A, options in Panel B, and stocks in Panel C. ***, **, * denote significance at the 1%, 5%, 10% level using the significance test for SRs of Lo (2002).

	Bonds	Options	Stocks
Panel A: Top 12 CMPs of Bonds			
const	1.60***	−0.23	0.61*
O_pifht	−1.77***	0.26	−0.67*
S_seas_1_1na	1.92***	−0.74*	0.26
S_rskew_21d	2.05***	−0.03	−0.39
O_delta	2.15***	1.37***	0.13
O_dciv	−2.32***	−0.66*	−0.06
O_dpiv	−2.40***	−0.44	−0.37
S_iskew_capm_21d	2.51***	0.11	−0.30
O_pcratio	−2.84***	0.01	−0.40
S_iskew_ff3_21d	2.95***	−0.04	−0.32
S_iskew_hxz4_21d	3.06***	0.18	−0.31
S_ret_1_0	3.89***	−0.24	−0.06
Abs. Mean	2.45	0.36	0.32
Panel B: Top 12 CMPs of Options			
O_vol	−0.62	−1.37***	−0.55*
O_delta	2.15***	1.37***	0.13
O_pilliq	0.50	1.38***	0.55
S_div12m_me	−1.37***	1.43***	−0.14
O_embedlev	−0.42	1.49***	−0.02
O_ivrv	1.03***	−1.49***	−0.04
O_rnk30	−0.37	1.50***	−0.25
O_oi	−0.48	−1.52***	−0.61**
O_modos	0.73*	1.57***	0.58*
O_so	1.00***	1.69***	0.68**
O_dso	0.10	1.77***	0.42
O_ivd	−0.62*	1.82***	0.16
Abs. Mean	0.78	1.53	0.34
Panel C: Top 12 CMPs of Stocks			
const	1.60***	−0.23	0.61*
O_oi	−0.48	−1.52***	−0.61**
S_sti_gr1a	−1.55***	0.38	−0.62*
S_sale_bev	−0.19	−0.24	0.64**
O_pifht	−1.77***	0.26	−0.67*
O_so	1.00***	1.69***	0.68**
O_vs_level	1.42***	−0.81***	0.69**
O_atm_dcivpiv	0.78***	−0.61*	0.69**
S_seas_6_10na	−0.02	−0.34	−0.77**
O_atm_civpiv	0.16	−0.92***	0.87***
O_shrtfee	0.39	0.25	−1.01***
O_fric	−0.14	0.06	1.02***
Abs. Mean	0.79	0.61	0.74

duration (Schlag, Thimme, and Weber, 2021) produces the largest SR of 1.82. Four (two) out of the 12 most important characteristics for options also generate significant SRs for CMPs of bonds (stocks). For example, sorting on the firm's stock-to-option volume (O_so, see Roll, Schwartz, and Subrahmanyam, 2010) results in a SR of 1.0 for the CMP of bonds, 1.69 for the CMP of options, and 0.68 for the CMP of stocks, all of which are significant at the 5% level. 11 out of the top-12 characteristics for CMPs of options use option information in their construction.

The top-12 CMPs of stocks (Panel C of Table 1) have an average absolute SR of 0.74. In general, option-based information is most valuable for stock-based investments, with option frictions (O_fric, see Hiraki and Skiadopoulos, 2020) and implied short-selling fees (O_shrtfee, see Muravyev, Pearson, and Pollet, 2022) both generating an absolute SR of about 1.0. Muravyev, Pearson, and Pollet (2022) show that the implied shorting fee explains much of the outperformance of common stock-market anomalies. Sorting stocks on this characteristic alone is already a profitable investment strategy in the absence of fees and implementation costs. For CMPs of options or bonds, however, this information on its own is insignificant. Among the characteristics for the top-12 CMPs of stocks, we find the largest degree of predictability across asset classes. Six (four) of the 12 characteristics also generate significant SRs for CMPs of bonds (options). However, only nine CMPs of stocks have a significant SR.

Overall, out of the 249 characteristics, 144 generate an insignificant SR at the 5% level for all of the three asset classes, 82 have a significant SR for one asset class, 22 for two, and only the stock-to-option volume (O_so) and the weighted put-call spread of Cremers and Weinbaum (2010) (O_vs_level) is significant for bonds, options, and stocks. We use these 107 (82 + 22 + 2 + a constant) characteristics as inputs to modeling the risk-return tradeoff for each firm. Of these 107 characteristics, 14 (44, 47) are derived from the firm's bond (option, stock) plus the constant. We use the restricted set of characteristics to avoid overfitting on IS information. Our procedure represents an ex-ante feature selection step common in the field of machine learning. The estimation of joint IPCA outlined in Section 3 consequently seeks to explain the returns of the 107×3 CMPs that on their own offer the most valuable investment advice. We can report that we have fitted models on the entire set of 249 characteristics with very similar results.

5 A Joint Factor Model

5.1 Performance of Joint IPCA

Performance Metrics. We evaluate the model's IS and OOS performance using the metrics proposed by Kelly, Palhares, and Pruitt (2023):

$$\text{Total } R^2 = 1 - \frac{\sum_{i,t}\left(R_{it+1} - \widehat{\beta}_{it}\widehat{F}_{t+1}\right)^2}{\sum_{i,t} R_{it+1}^2}$$

$$\text{XS } R^2 = \frac{1}{T}\sum\nolimits_t R_t^2, \text{ where } R_t^2 = 1 - \frac{\sum_i\left(R_{it+1} - \widehat{\beta}_{it}\widehat{F}_{t+1}\right)^2}{\sum_i R_{it+1}^2}$$

$$\text{Relative Pricing Error} = \frac{\sum_i \alpha_i^2}{\sum_i \overline{R}_i^2}, \text{ where } \alpha_i = \frac{1}{T}\sum\nolimits_t \left(R_{it+1} - \widehat{\beta}_{it}\widehat{F}_{t+1}\right). \tag{18}$$

Total R^2 quantifies the model's success in explaining average returns for the three asset classes. It aggregates information both over months t and across assets i and compares the amount of variation in asset returns explained by joint IPCA's that is not already explained by a simple benchmark of predicting a zero return. Gu, Kelly, and Xiu (2020) argue that a historical mean tends to underperform a zero-forecast for single stocks OOS, which inflates a competing model's Total R^2. Next, we quantify how well a model explains cross-sectional returns. XS R^2 is similar to the average R^2 of Fama and MacBeth (1973) cross-sectional regressions performed each month. Finally, we record the average relative pricing error, which denotes how well a candidate model explains differences in average returns across assets. We prefer models that generate large R^2s and small relative pricing errors.

To assess the model's ability to explain average returns OOS, we estimate Joint IPCA using information until month t, which gives us the time t estimate of $\widehat{\Gamma}_{\beta,t}^{AC}\ \forall\, AC$ and therefore each asset's $\widehat{\beta}_{it}$. Then, we calculate the OOS factor realizations $\widehat{F}_{t+1}$ with a cross-sectional regression, as described in Eq. (14), using betas estimated through time t and asset return information realized in $t+1$. Consequently, the factors are obtained using asset weights which are known already in month t. We require at least 90 months of historical data to estimate the model. Therefore, our OOS test begins in February 2010.

In- and Out-of-Sample Performance. We vary the number of latent factors K of the joint ICPA model. We consider $K \in [1, 3, 5, 6]$. The general consensus in the literature is to focus on parsimonious models with a low number of factors (Kozak, Nagel, and Santosh, 2020). Fama and French (2018) and Kelly, Palhares, and Pruitt (2023) advocate for at most six and five factors for the U.S. equity and corporate bond markets, respectively. It is important to note that joint IPCA has the additional benefit of estimating factors designed to explain returns of all three asset classes simultaneously. If bond, option, and stock markets are (partially) integrated, this will require a lower total number of factors to explain average returns for all three asset classes, yielding a parsimonious factor model applicable to each of the classes.

Table 2: In- and Out-Of-Sample Metrics

The table shows IS and OOS performance metrics of joint IPCA models with K factors. The definition of each performance metric is given in Eq. (18). We consider $K \in [1, 3, 5, 6]$ factors. The IS period runs from August 2002 through December 2021, the OOS period starts in February 2010, as we require at least 60 months of training data.

$K \rightarrow$	1		3		5		6	
	IS	OOS	IS	OOS	IS	OOS	IS	OOS
Panel A: Total R^2								
Total	0.14	0.13	0.26	0.25	0.30	0.30	0.31	0.30
Bonds	0.09	0.04	0.30	0.29	0.36	0.38	0.37	0.38
Options	0.11	0.09	0.19	0.16	0.21	0.20	0.22	0.20
Stocks	0.22	0.24	0.29	0.32	0.31	0.33	0.32	0.34
Panel B: XS R^2								
Total	0.10	0.11	0.19	0.20	0.23	0.23	0.23	0.24
Bonds	0.05	−0.02	0.21	0.20	0.28	0.29	0.29	0.30
Options	0.06	0.06	0.11	0.10	0.13	0.12	0.14	0.12
Stocks	0.14	0.16	0.19	0.22	0.21	0.23	0.23	0.24
Panel C: Relative Pricing Error								
Total	0.85	0.93	0.74	0.82	0.71	0.81	0.71	0.79
Bonds	0.76	0.79	0.55	0.57	0.53	0.54	0.53	0.53
Options	0.97	1.09	0.91	1.01	0.89	1.01	0.89	0.99
Stocks	0.82	0.85	0.78	0.82	0.73	0.82	0.73	0.77

We show the IS and OOS performance metrics described in Eq. (18) in Table 2. We calculate these metrics for all three asset classes combined as well as separately for bonds, options, and stocks. We find that a single factor explains 14% of the total return variation, which varies between 9% for bonds, 11% for options, and 22% for stocks. The Total R^2 increases significantly for $K = 3$, explaining 19% of the variation of option returns, 29% of the stock return variation, and 30% of the bond return variation. Further increasing K leads to more modest but still noticeable improvements: a six-factor model explains 31% of the total variation. The Total R^2 is again lowest for options at 22% and largest for bonds at 37%. Stocks land in the middle with six factors explaining 32% of stock return variation.

OOS Total R^2s are remarkably close to their IS counterparts. The parsimonious structure of joint IPCA guards against overfitting and explains bond, option, and stock returns well IS and OOS. For the same six-factor model, we continue to explain 30% of the variation in returns including no forward-looking information.

The fraction of cross-sectional variation explained (XS R^2) is generally comparable to the Total R^2s. The comparable magnitudes tend to be slightly smaller, but the general trend is that option returns are the hardest to price and bond returns the easiest to price. The same applies to the relative pricing errors in Panel C of Table 2.

We find that we can explain more variation of stock returns than that reported by Kelly, Pruitt, and Su (2019). The reason for this is twofold: first, our sample is restricted to the largest and most liquid stocks, as we require that the stock is both optionable and that the firm's bonds are traded and recorded in the TRACE database. It is well-known that returns of large stocks tend to be better explained by common factor models. Second, we start our sample in 2004, limited by the availability of corporate bond data. The R^2s for options are larger than those reported in Goyal and Saretto (2022). We consider option returns that are delta-hedged daily, as opposed to an initial delta-hedge favored by Goyal and Saretto (2022). Also, the same reason that applies to stocks also applies here: we focus on a representative option for the largest firms, which are both optionable and have actively traded bonds available. The performance metrics for bonds are slightly lower than those reported by Kelly, Palhares, and Pruitt (2023), but within the same ballpark.

Given the best performance of a model with six joint factors, we will focus our subsequent analysis on the case of $K = 6$. Following Kelly, Pruitt, and Su (2019), we also test a version of our joint IPCA that allows for an characteristic-based intercept in Eq. (11). Appendix B shows that a five- to six-factor model is sufficient to drive out the explanatory power of firm characteristics that is not already picked up by the systematic factors.

Expected vs. Realized Returns. Figure 1 shows that joint IPCA explains well the realized returns of the CMPs described in Section 4.2. The IS fit is shown in the left panel, the OOS fit in the right panel. We consider $K = 6$ latent common factors. We compare the model-implied expected return for each CMP with the average realized excess return over the sample period. For comparability, we normalize all portfolios to have 10% annualized volatility. For CMPs of bond, option, *and* stocks, the figure shows that the Joint IPCA produces a scatter plot that is closely aligned with the 45°-line, demonstrating small IS and OOS pricing errors. The IS fit is best for bonds and stocks, with a slight tilt in the slope for options: realized option returns tend to be slightly less variable than expected by the model. The OOS fit remains remarkably stable, with joint IPCA explaining average returns well with a symmetric dispersion around the 45°-line.

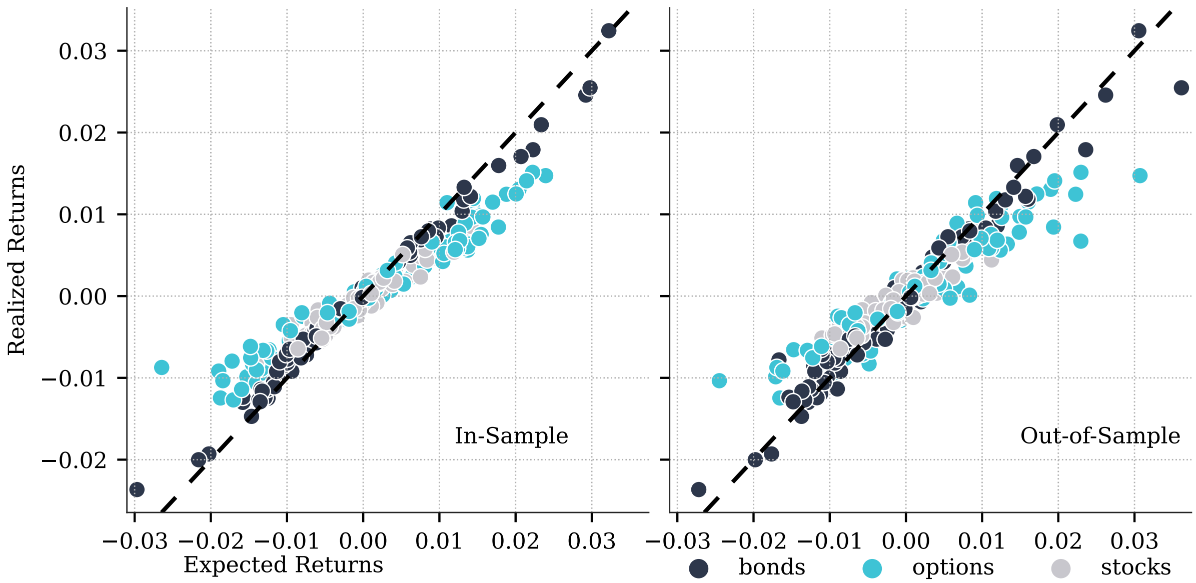

Figure 1: Expected vs. Realized Returns

The figure shows a scatter of returns expected by the six-factor joint IPCA model versus average realized returns of the 107 × 3 characteristic-managed portfolios described in Section 4.2 (107 for each asset class). In the left panel, we show the results for the IS period from August 2002 through December 2021. The right panel shows the results for iteratively fitting a model with no forward-looking information. The OOS period begins in February 2010. We distinguish CMPs of bonds, options, and stocks through different colors.

Explaining Aggregate Returns. Our factor model is constructed to explain returns across asset classes which puts us in the unique position to learn about the underlying joint factor structure. As an approximation, we now investigate the predictability of *aggregate returns*, for which we assume an equal investment in a firm's bond, option, and stock:

$$R_{it+1}^{firm} = \left(R_{it+1}^{B} + R_{it+1}^{O} + R_{it+1}^{S}\right)/3. \tag{19}$$

The left panel of Figure 2 shows the histogram of unconditional average aggregate returns in dark blue color and unconditional alphas after adjusting for risk using our $K = 6$ factor joint IPCA model in teal color. We only show the 7.6% of alphas that remain significant after the risk adjustment. Overall, the joint IPCA model explains aggregate returns well, which is also evident in the resulting R^2s shown in the right panel of Figure 2. The average firm-level R^2 is 28.6%.

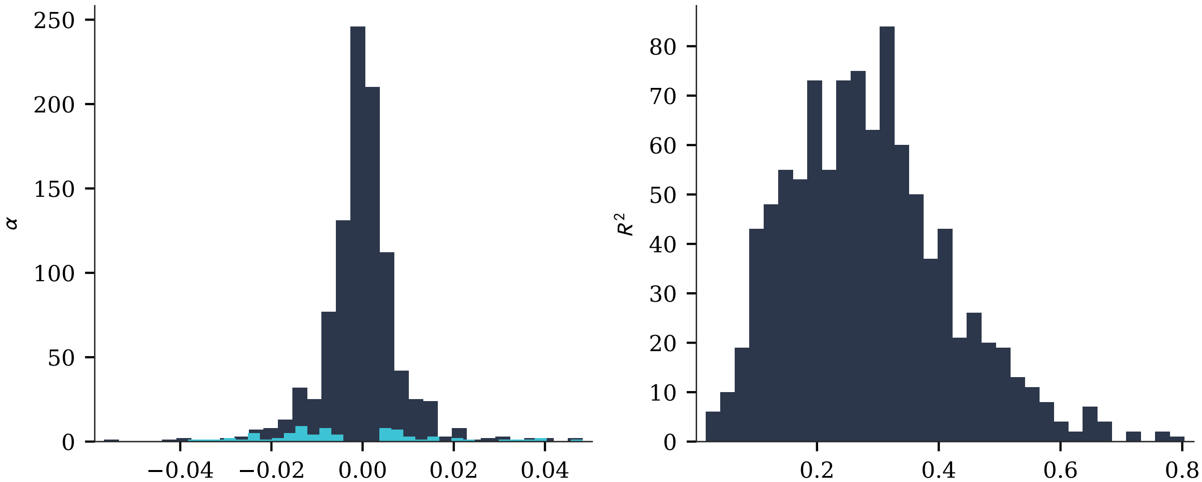

Figure 2: Unconditional αs and Time-Series R^2 of Aggregate Returns

The left panel of the figure shows average aggregate returns as defined in Eq. (19) in dark blue color and their unconditional αs that remain significant at the 5%-level after adjusting for risk using the $K = 6$ factor joint IPCA model in teal color. The right panel shows the resulting R^2s.

5.2 Stochastic Discount Factor

We next seek to understand the implications that joint IPCA has for the stochastic discount factor. For that, we calculate the mean-variance efficient (MVE) tangency portfolio implied by a $K = 6$ joint factor model. Joint IPCA can exploit the covariance between the asset classes to form efficient mean-variance portfolios *across asset classes*. The MVE portfolio is of particular interest, as shocks to its returns are directly proportional to shocks to the firm-level stochastic discount factor M (Cochrane, 2009):

$$M_{t+1} - \mathbb{E}_t[M_{t+1}] = b \times \left(R^{MVE}_{t+1} - \mathbb{E}_t[R^{MVE}_{t+1}] \right), \tag{20}$$

Returns of the MVE portfolio inform us about the risks most correlated with the marginal utility of the marginal investor that is simultaneously active in bonds, options, and stocks.

Table 3: Sharpe Ratio of the Tangency Portfolio

The table shows the IS and OOS SRs of the tangency portfolio implied by a K-factor joint IPCA model. The IS period runs from August 2002 through December 2021, the OOS period starts in February 2010, as we require at least 60 months of historic data.

$K \rightarrow$	1	3	5	6
IS	1.09	2.81	5.91	6.91
OOS	1.16	2.91	5.55	6.68

We report IS and OOS SRs of the tangency portfolios implied by a K-factor joint IPCA model in Table 3. We again consider $K \in [1, 3, 5, 6]$. A single factor generates a SR of 1.09 IS and 1.16 OOS. Increasing the number of factors monotonically increases the resulting IS and OOS performance: a six-factor model generates a SR of 6.91 IS and 6.68 OOS, with no performance degradation OOS, suggesting a remarkable stability in the usefulness of the extracted information from the three asset classes. All SRs are significant at the 1% level using the statistical test of Lo (2002).[6]

Eq. (20) shows that shocks to the tangency portfolio are directly proportional to shocks to the stochastic discount factor. Figure 3 therefore overlays the tangency portfolio's returns in $t+1$ over time with the VIX at time t. Martin (2017) shows that an option portfolio similar to that of the VIX can be used to derive a lower bound on the expected market return. The correlation between the VIX_t and R^{MVE}_{t+1} is 0.48: whenever the VIX is low, so are the returns of the tangency portfolio in the next month. In times of crises, both the VIX and the returns to the tangency portfolio tend to spike. For example, in the first half of 2009, the tangency portfolio has return of 8.9% with a VIX at around 40. During the Covid-selloff in March 2020 we find the largest return of the MVE portfolio of 12%, with the VIX above 50.

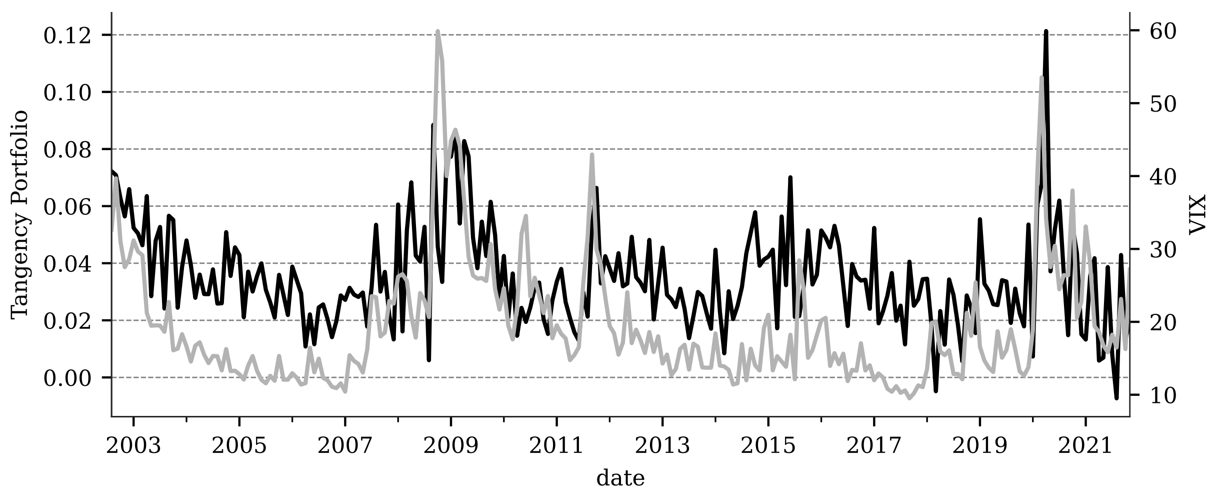

Figure 3: Tangency Portfolio Over Time

The figure shows the returns at time $t+1$ of the tangency portfolio for a $K = 6$ factor joint IPCA model in black. We overlay the VIX at time t in gray. Results are displayed for the IS model fit to show a longer history. Results for the OOS fit are comparable.

Decomposition to three asset classes. Since the MVE portfolio is a portfolio of latent factors, F_{t+1}, and the factors themselves are linear combinations of individual security returns

[6] In Section 6 we compare these SRs to those obtained within the individual asset classes.

(first row of Eq. (11)), we can express the tangency portfolio as:

$$R_{t+1}^{MVE} = \sum_{i \in \{B,O,S\}} w_{it} R_{it+1} = \sum_{i=1}^{N_{t+1}} w_{it}^{B} R_{it+1}^{B} + \sum_{i=1}^{N_{t+1}} w_{it}^{O} R_{it+1}^{O} + \sum_{i=1}^{N_{t+1}} w_{it}^{S} R_{it+1}^{S}, \tag{21}$$

where w_{it} is asset i's weight in the MVE portfolio.

Using Eq. (21), we can identify the contribution of each asset class to the returns of the tangency portfolio. We decompose the returns of the OOS tangency portfolio obtained with $K = 6$ latent factors in Panel A of Table 4. The portfolio has an average monthly return of 2.59% with a volatility of 1.34%, resulting in the OOS SR of 6.68 as discussed above. The portfolio's return is positively skewed (1.74) and the portfolio has small drawdowns of at most −0.72%. The remaining columns of Panel A in Table 4 show how the performance is attributable to investments in corporate bonds, options, and stocks. The sub-portfolio of bonds has an average return of 1.71% (SR of 6.01). The sub-portfolio of options contributes an average return of 0.67% (SR of 3.84), and the sub-portfolio of stocks has an average return of 0.22% (SR of 0.99).

Table 4: Return and Variance Decomposition of Joint Tangency Portfolios

The table shows a decomposition of the joint tangency portfolio's return profile. We provide average monthly returns, standard deviations (Std), annualized Sharpe ratios (SR), skewness (Skew), kurtosis (Kurt), the maximum drawdown (MDD), the relative turnover (TO) as defined in Eq. (22), and transaction costs in return units assuming relative implementation costs of 35bps (TC) in Panel A, both for the joint tangency portfolio and sub-portfolios invested in the corresponding bond, option, and stock components. Panel B provides correlation coefficients for the tangency portfolio and the sub-portfolios. All results are based on OOS estimates. The OOS period ranges from February 2010 through December 2021.

	Joint	Bonds	Options	Stocks
		Panel A: Returns		
Return	2.59	1.71	0.67	0.22
Std	1.34	0.98	0.60	0.77
SR	6.68	6.01	3.84	0.99
Skew	1.74	2.12	3.80	−0.31
Kurt	8.88	8.65	28.03	1.59
MDD	−0.72	−0.17	−0.72	−3.23
TO	1.11	2.42	1.41	0.72
TC	1.60	0.85	0.50	0.25
		Panel B: Correlation		
Bonds	0.73			
Options	0.37	−0.01		
Stocks	0.52	0.01	−0.13	

Trading in options (Ofek, Richardson, and Whitelaw, 2004) as well as in corporate bond markets is known to be expensive (Bessembinder, Spatt, and Venkataraman, 2020). To understand if the proposed tangency portfolio would be implementable in real-time, we measure the portfolio's monthly turnover and transaction costs. For this, we define the portfolio's relative turnover as:

$$\text{Turnover} = \sum_t \left(\sum_i |w_{it} - w_{it-1}|\right) \Big/ T. \tag{22}$$

Transaction costs are assumed to be proportional in the amount of trading. Kelly, Palhares, and Pruitt (2023) choose the upper bound of the transaction cost estimates for corporate bonds by Choi, Huh, and Shin (2021) who recommend a one-way cost of 17–19bps. Frazzini, Israel, and Moskowitz (2018) show that AQR's average implementation costs for trading in large stocks amounts to roughly 15bps with significant variation over time. For options trading, Muravyev and Pearson (2020) suggest that institutional investors are able to achieve much better execution than implied by bid-ask spreads. We consider a relatively high level of transaction costs of 35bps—roughly twice the estimates proposed by Choi, Huh, and Shin (2021) and Frazzini, Israel, and Moskowitz (2018). For simplicity, we use the same estimates for bonds, options, and stocks. The tangency portfolio's monthly turnover is relatively high at 111%, resulting in total transaction costs of 1.60% per month. As a result, the net-of-fees SR reduces to 2.56.

We find that the net return and SR of the sub-portfolio of stocks is negative with these high levels of transaction costs. Nevertheless, Panel B of Table 4 shows that there are important diversification benefits of investing jointly in bonds, stocks, and options. We show the correlations between the tangency portfolio and the three sub-portfolios. The returns of the tangency portfolio are modestly correlated with each of the sub-portfolios, as is to expected since the tangency portfolio is the sum of the three sub-portfolios.

The correlations between the different class-level sub-portfolios are close to zero or even negative, highlighting that investors can earn significant diversification benefits when incorporating information about the joint dependence structure of the three asset classes. The sub-portfolios of stocks and bonds are modestly correlated with a correlation of 0.01; sub-portfolios of stock and options have a correlation of -0.13, and sub-portfolios of option and bond have a correlation of -0.01.

We can also analyze the distribution of portfolio weights placed in each of the three asset classes. We separately consider long and short portfolio weights within each class AC, i.e., $w_{AC,t}^{\text{long}} = \sum_{i \in AC} w_{it} \mathbb{1}_{w_{it}>0}$ and $w_{AC,t}^{\text{short}} = \sum_{i \in AC} w_{it} \mathbb{1}_{w_{it}<0}$. Figure 4 shows that, on average, the tangency portfolio has a symmetric long and short investment in corporate bonds. We find a clear tilt towards shorting options, potentially to harness the variance risk premium

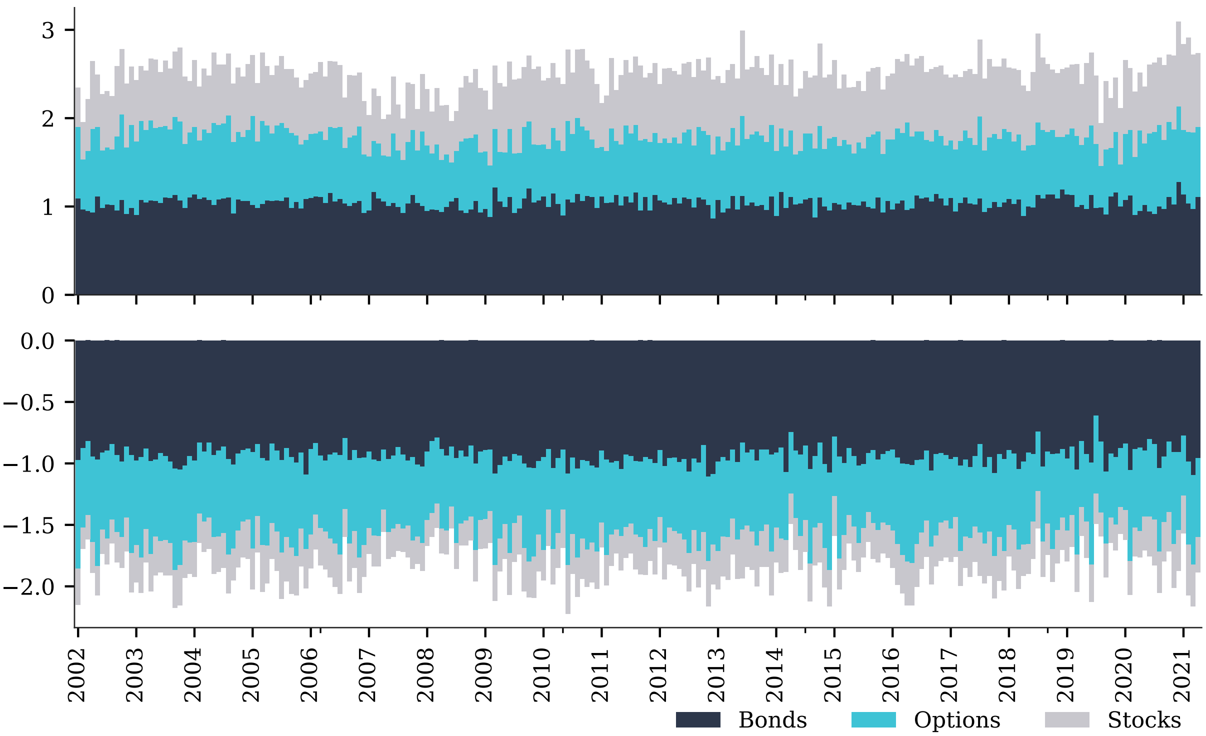

Figure 4: Tangency Portfolio Weights by Asset Class

The figure shows the distribution of long and short portfolio weights placed in each asset class over time, defined as $w^{\text{long}}_{AC,t} = \sum_{i \in AC} w_{it} \mathbb{1}_{w_{it}>0}$ and $w^{\text{short}}_{AC,t} = \sum_{i \in AC} w_{it} \mathbb{1}_{w_{it}<0}$, for $AC \in$ [bonds, options, stocks]. Results are displayed for the IS model fit to show a longer history. Results for the OOS fit are comparable.

embedded in equity options (Goyal and Saretto, 2009). The asset class weights in stocks are primarily positive. Overall, class-specific weights are fairly stable over time.

6 The Integration of Bonds, Options, and Stocks

A central question is the degree to which the different markets for bonds, options, and stocks are integrated. Table 2 already shows that the joint IPCA factor model simultaneously explains the returns of bonds, options, and stocks well. In this section, we dig deeper into common sources of return predictability.

6.1 Commonality in Predictability

Integrated Predictions. As a first step, we investigate if explanatory power of the factor model is *shared* across asset classes. If markets are (partially) integrated, we expect to observe shared patterns of predictability, with more predictable bond and option returns whenever the firm's stock return is easy to predict, for example. For each firm in our sample, we compute the Total R^2 using a joint $K = 6$ factor model separately for the firm's bond, option, and stock. To be able to compute a meaningful measure of variation, we require the data for a firm to be available for at least 24 months.

Table 5: Sorts on Asset Class Predictability

The table shows commonality in asset class-level predictability, measured by the Total R^2. As an example, in Panel A, we sort firms into deciles by their bond-level predictability and record the average R^2 of a $K = 6$ factor joint IPCA model. We then also record the average R^2 for the remaining two asset classes. We consider firms with at least 24 months of return data available. We repeat the procedure by sorting on option R^2 in Panel B and stock R^2 in Panel C.

	1	2	3	4	5	6	7	8	9	10	10-1
Panel A: Portfolios Sorted by Bonds' Total R^2											
Bonds	0.11	0.25	0.30	0.35	0.39	0.43	0.47	0.51	0.56	0.64	0.53
Options	0.17	0.18	0.20	0.22	0.19	0.25	0.25	0.26	0.30	0.30	0.13
Stocks	0.24	0.27	0.31	0.28	0.28	0.33	0.34	0.34	0.34	0.35	0.11
Panel B: Portfolios Sorted by Options' Total R^2											
Bonds	0.32	0.37	0.35	0.39	0.39	0.40	0.41	0.45	0.46	0.48	0.16
Options	0.01	0.08	0.13	0.17	0.20	0.24	0.28	0.33	0.39	0.49	0.47
Stocks	0.20	0.23	0.25	0.28	0.32	0.32	0.35	0.36	0.39	0.39	0.18
Panel B: Portfolios Sorted by Stocks' Total R^2											
Bonds	0.32	0.39	0.37	0.38	0.41	0.43	0.43	0.42	0.45	0.45	0.13
Options	0.15	0.17	0.17	0.20	0.21	0.22	0.24	0.31	0.31	0.35	0.19
Stocks	0.06	0.16	0.21	0.25	0.29	0.33	0.37	0.41	0.46	0.54	0.48

Next, we sort firms into decile portfolios by their bonds' Total R^2 and record the average bond, option, and stock R^2 for each decile. We also show the R^2 spread, as the difference between the average R^2 for the least predictable decile (1) and the most predictable decile (10). Panel A of Table 5 shows the results for this sort. We find that the explanatory power of the joint IPCA model is only 11% for decile 1 bonds but increases significantly to 64% for decile 10 bonds. By construction, the 10−1 predictability is high at 53%. More interesting for our purposes, we also find a positive 10−1 spread in Total R^2 for the two other asset classes. For example, options of firms in decile 1 have an average R^2 of 17%, compared to an average R^2 of 30% for options of firms in decile 10. For stocks, the 10−1 spread is somewhat smaller at 11%. This result shows that sorting on how well joint IPCA explains bond returns also produces a meaningful explanatory spread for options and stocks.

We repeat this exercise by sorting on options explanatory power and show the results in

Panel B of Table 5. The 10−1 explanatory spread for options is 47% and ranges between 1% and 49% from the bottom to the top decile. We find a strong commonality in the explanatory pattern between options, stocks and bonds: sorting on option Total R^2 produces a bond spread of 16% and a stock spread of 18%. Finally, Panel C of Table 5 sorts on stocks Total R^2. We find that the 10−1 spread for stock is 48%, for options is 19%, and for bonds is 13%, suggesting that stocks and options tend to be more integrated than stocks and bonds.

As another manifestation of commonality across the three asset classes, we calculate the Total R^2 of the 107 CMPs for each asset class and show bivariate scatter plots of these R^2s of CMPs of one asset class versus that of CMPs of another asset class in Figure 5. A strong correlation is evident in these plots showing that the explanatory power is shared across CMPs of bonds, options, and stocks. For example, regressing the R^2s of CMPs of stocks on the R^2s of CMPs of bonds gives a slope coefficient (β) of 1.16. This regression explains 63% of the variation in explanatory R^2s. We also find a large agreement in the explanatory power of CMP returns for CMPs of bonds vs. options in the middle and stocks vs. options in the right panel of Figure 5.

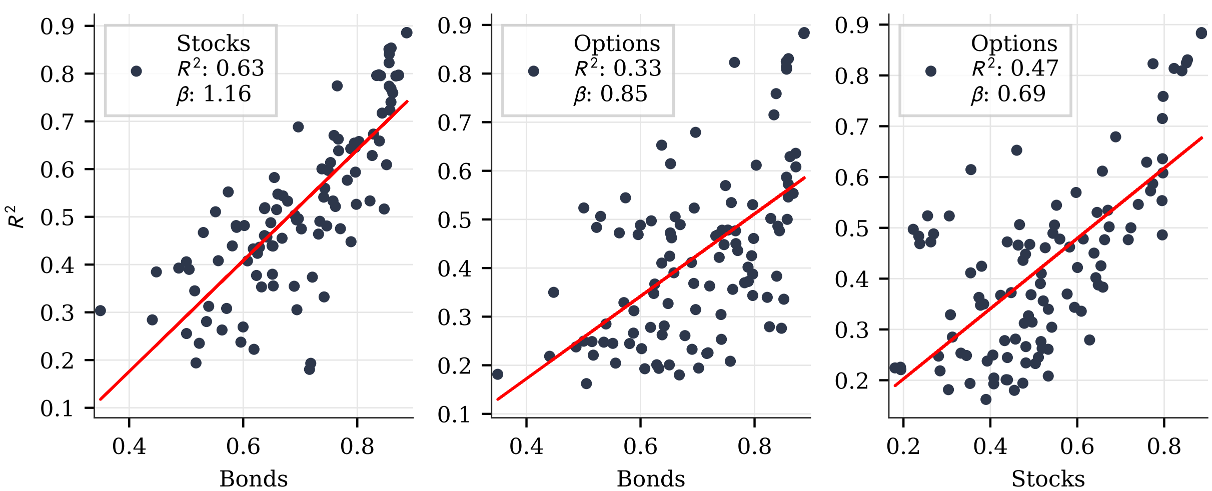

Figure 5: Commonality in the Predictability of Characteristic-Managed Portfolios

The figure shows the Total R^2 for a $K = 6$ factor joint IPCA model for the 107×3 CMPs of bonds, options, and stocks. The left plot is a scatter of the resulting R^2s for CMPs of bonds compared to CMPs of stocks. The middle (right) plot repeats this exercise for CMPs of bonds (stocks) on the x-axis and options on the y-axis.

Factor Importance. Our $K = 6$ joint IPCA factors jointly explain the returns of bonds, options, and stocks. But, are all six factors required for each asset class? For instance, if factors F1-F2 are important for bonds, F3-F4 for options, and F5-F6 for stocks, then this would imply weak integration across the three asset classes (even if the three asset classes share the same set of characteristics to model time-varying betas). The evidence so far in this section, showing commonality in the explanatory power across the three asset classes, suggests that this is an unlikely possibility. Nevertheless, we formally investigate the commonality picked up by the six factors.

To do so, we iteratively "turn off" the influence of a factor by setting its realizations to zero. We then document the resulting drop in Total R^2 across asset classes, as well as individually for the subsample of bonds, options, and stocks. We start with the factor with the highest mean return (factor F1) and work our way down to the factor with the smallest mean return (factor F6). We provide the results on the *relative* reduction in Total R^2 in Table 6. The panel on the left shows the results when restricting the influence of one factor at a time, the panel on the right shows the cumulative effect, i.e., the second row shows the effect of simultaneously turning off factors F1 and F2.

Table 6: Factor Influence on Explaining Bond, Option, and Stock Returns

The table shows the relative reduction in Total R^2 when "turning off" the influence of the kth factor. We do so by setting each of that factor's realizations to zero. The left panel shows the impact of turning off factors one-by-one, the right panel shows the cumulative effect. Factors are ordered with the first factor having the largest average return.

K	Factor Influence				Cumulative Factor Influence			
↓	Bonds	Options	Stocks	Total	Bonds	Options	Stocks	Total
1	−0.27	−0.23	−0.12	−0.21	−0.27	−0.23	−0.12	−0.21
2	−0.05	−0.61	−0.27	−0.26	−0.33	−0.85	−0.39	−0.47
3	−0.04	−0.08	−0.08	−0.06	−0.37	−0.93	−0.48	−0.54
4	−0.45	−0.03	−0.02	−0.20	−0.81	−0.95	−0.50	−0.74
5	−0.02	−0.04	−0.52	−0.20	−0.84	−0.99	−1.00	−0.93
6	−0.19	−0.01	−0.02	−0.09	−1.00	−1.00	−1.00	−1.00

Table 6 shows that turning off the influence of the first factor decreases the bond R^2 by 27 percent (from 35% to 26%). For options (stocks) the relative decrease when turning off factor F1 amounts to 23 percent (12 percent), highlighting a high degree of commonality in the importance of the first factor. In general, we find that factors F1, F4, and F6 are most important for explaining bond returns. Together, they are responsible for roughly 90 percent of the model's bond return explanatory power. The same factors are also important for understanding option and stock returns, making up around 27 percent of the model's option and 16 percent of the model's stock return Total R^2.

The most important factors for explaining option returns are F1 and F2, which together make up 84 percent of the model's option return predictability. Consistent with the commonality between option and stock explanatory power shown in Table 5, the two factors are also responsible for 39 percent of the model's stock return predictability as well as 32 percent of the model's bond return predictability, and thus for 47 percent of the model's total explanatory power.

The three most important factors for stock return predictability are factors F1, F2, and F5. They capture 91 percent of the model's explanatory power for stocks, 88 percent for options, and 34 percent for bonds. These results again indicate a large degree of integration between the options and stock market, and a lower integration of corporate bonds. The last column "Total" shows that all six factors are important for explaining firm returns.

Characteristics Importance. The specification in Eq. (11) has two main ingredients: latent factor realizations F_{t+1} and the Γ_β matrix, which maps observable characteristics to heterogeneity in factor sensitivities (betas). Combining the information from Γ_β and the factors, we can understand the relative importance of each input characteristic in describing expected returns. For this, we introduce an importance and a sensitivity measure, which extend the characteristic importance proposed by Kelly, Pruitt, and Su (2019). The lth row of Γ_β, γ_l, describes how characteristic l influences an asset's sensitivity to each of the K factors. Combined with the average return of the K factors this informs us about a characteristic's *Importance* defined as:

$$\mathit{Importance}(z_l) = |\gamma_l|' \, \overline{F}. \tag{23}$$

Importance measures the average absolute influence of the lth characteristic on an asset's β, weighted by each factor's average influence on expected returns. Similarly, we can assess the *Sensitivity* of the model's expected returns to a unit change in characteristic l:

$$\mathit{Sensitivity}(z_l) = \gamma_l' \overline{F}. \tag{24}$$

As characteristics are rank-standardized between -0.5 and 0.5, *Sensitivity* gives us the model-expected return spread of two stocks with a unit difference in l and with otherwise equal characteristics.

In Panel A of Table 7, we provide the ranks of *Importance* and the values for *Sensitivity* for the twelve most important characteristics for explaining bond returns.[7] Among the twelve most important characteristics for explaining bond returns are four stock characteristics, four option characteristics, and four bond characteristics (the last is the constant). The

[7] Table 1 shows the most important characteristics that have the highest SRs for CMPs. However, a characteristic that generates high SR need not necessarily be the one that is important for describing the risk-return tradeoff in a joint IPCA model.

most important characteristics for bond returns are short-term information about the stock's past return distribution, which includes short-term reversal (S_ret_1_0 and S_ret_3_1) and relative returns (S_prc_highprc_252d and S_rmax5_rvol_21d). Larger short-term stock returns are associated with higher expected bond returns, larger intermediate-term and short-term maximum stock returns in contrast with lower expected bond returns.

It is interesting to note that while there is no bond characteristic in list of top-12 CMPs of bonds in Section 4.2, information about the bond's short-term return reversal (B_rev), the overall bond market return (const), and the bond's expected shortfall (B_ES5) are important for modeling expected bond returns. The three characteristics are expected to generate an average return spread of −1.40%, 0.65%, and 0.67%, respectively. We also report the *Importance* rank and *Sensitivity* for the other two asset classes in Panel A of Table 7. Out of the twelve most important characteristics for explaining bond returns, five also show up in the top-12 characteristics for explaining option returns and six for explaining stock returns, highlighting that not only factor-level information is shared among bonds, options, and stocks (see Table 6), but also information about an asset's sensitivity to these factors. As an example, the option's spread between implied and realized volatility (O_ivrv) shows up as a highly influential characteristic for each asset class.

In Panel B of Table 7, we show the most influential characteristics for explaining option returns. The most important is the option's implied volatility (O_iv). It is not only vital for explaining variation in options' sensitivity to the common risk factors, its implied return spread is also large at −1.54% per month. Option's embedded leverage and open interest are also important for describing the option's risk sensitivity. The ratio of implied to realized volatility enters as the tenth most important characteristic for explaining option returns. Higher implied relative to realized volatilities are associated with lower option returns (Goyal and Saretto, 2009). Other important characteristics include option demand pressure, measured by the ratio of dollar open interest to market capitalization of the underlying stock, the option's volume (O_vol) and Delta (O_delta). We also find a large influence of information about the stock's short-term maximum return (S_rmax5_21d and S_rmax5_rvol_21d). Five (seven) out of the top-12 characteristics for explaining option returns also show up in the top-12 for bonds (stocks).

The most important characteristics for explaining stock returns (Panel C of Table 7) are almost all derived from the firm's options. Option illiquidity (O_pifht) is important, as is the option's volume, open interest, and demand pressure. Furthermore, we find a large influence of the option's relative expensiveness measured by the spread between implied and realized volatility. For stock-level characteristics, we find a large influence of the constant (the average

Table 7: Importance of Characteristics

The table shows *Importance* ranks (Eq. (23)) and *Sensitivity* values (Eq. (24)) for the top-12 most important characteristics, measured by their *Importance*, for bonds (Panel A), options (B), and stocks (C).

	Bonds		Options		Stocks	
	Imp.	Sens.	Imp.	Sens.	Imp.	Sens.
Panel A: Top Characteristics for Bonds						
S_ret_1_0	1	1.78	23	−0.18	34	−0.07
B_rev	2	−1.40	103	−0.03	69	0.06
S_rmax5_21d	3	−1.17	3	1.04	15	−0.06
S_prc_highprc_252d	4	−1.04	17	−0.34	5	−0.16
O_ivrv_ratio	5	−0.85	10	−0.29	2	0.02
const	6	0.65	7	−0.07	1	0.28
B_ES5	7	0.67	42	0.07	22	−0.20
O_ivrv	8	0.54	4	0.05	3	−0.05
S_ret_3_1	9	0.62	18	−0.23	77	−0.02
B_vol	10	−0.39	67	−0.04	24	0.18
O_vol	11	0.38	8	−0.69	6	0.42
O_pifht	12	0.26	58	−0.02	4	0.04
Panel B: Top Characteristics for Options						
O_iv	19	0.39	1	−1.54	10	−0.10
O_embedlev	87	−0.01	2	0.05	47	−0.07
S_rmax5_21d	3	−1.17	3	1.04	15	−0.06
O_ivrv	8	0.54	4	0.05	3	−0.05
O_doi	40	0.12	5	−0.21	9	−0.19
O_oi	50	−0.10	6	0.05	16	0.13
const	6	0.65	7	−0.07	1	0.28
O_vol	11	0.38	8	−0.69	6	0.42
O_demand_pressure	86	0.05	9	0.43	12	0.19
O_ivrv_ratio	5	−0.85	10	−0.29	2	0.02
S_rmax5_rvol_21d	39	−0.06	11	−0.65	18	−0.02
O_delta	98	−0.03	12	0.65	84	0.02
Panel C: Top Characteristics for Stocks						
const	6	0.65	7	−0.07	1	0.28
O_ivrv_ratio	5	−0.85	10	−0.29	2	0.02
O_ivrv	8	0.54	4	0.05	3	−0.05
O_pifht	12	0.26	58	−0.02	4	0.04
S_prc_highprc_252d	4	−1.04	17	−0.34	5	−0.16
O_vol	11	0.38	8	−0.69	6	0.42
O_toi	15	−0.46	14	−0.18	7	−0.41
S_corr_1260d	55	0.04	30	0.13	8	−0.00
O_doi	40	0.12	5	−0.21	9	−0.19
O_iv	19	0.39	1	−1.54	10	−0.10
O_modos	75	0.01	35	0.08	11	0.16
O_demand_pressure	86	0.05	9	0.43	12	0.19

stock return), price-to-high momentum (S_prc_highprc_252d) of George and Hwang (2004), as well as the stock's correlation to the market return (S_corr_1260d).

Overall, the overlap in characteristics explaining returns of all three asset classes shows that the joint factor structure that we extract provides a parsimonious factor model with the ability to simultaneously explain the returns of the three asset classes. Return and characteristics information from the other two asset classes is beneficial to understand the risk-return trade-off of the third asset class.

6.2 Joint vs. Single IPCA

The joint factor structure that we can extract with joint IPCA is beneficial to explain returns of bonds, options, and stocks, while maintaining a parsimonious model with a low number of latent factors. We now highlight that joint IPCA is better able to explain average returns than individual IPCA models estimated for a single asset class. We compare $K = 6$ factor models.

Unconditional Alphas. We compare our joint IPCA versus three individual IPCA models estimated for each asset class in their explanatory power for 107×3 CMPs described in Section 4.2. We calculate and compare unconditional alphas as this provides a common testing ground for how well the latent factors are able to explain returns of each of the three asset classes. The results are presented in Table 8.

Table 8: Unconditional Alphas of Joint and Single IPCA Models

The table shows how many of the 107×3 CMPs defined in Section 4.2 have significant average returns per asset class, and how many unconditional alphas remain significant after adjusting for risk either using the joint IPCA model, or individual IPCA models estimated for a single asset class. We also consider a combined model with two factors estimated from each asset class, which matches the number of latent factors for the other models ($2+2+2$). We use Newey and West (1987) standard errors with 12 lags.

		Unconditional alphas				
	Average	Joint	Single IPCA			
	returns	IPCA	6 Bond	6 Option	6 Stock	$2+2+2$
Bonds	83	5	4	59	82	38
Options	59	2	20	8	58	22
Stocks	27	0	4	8	13	3
Σ	169	7	28	75	153	63

Out of the 107 CMPs of bonds, 83 produce average returns which are significant at the 5% level. We use Newey and West (1987) standard errors with 12 lags to account for serial correlation and heteroskedasticity in returns. 59 CMPs of options and 27 CMPS of stocks have significant average returns for a total of 169 CMPs with significant full-sample returns, or a little over half of the 107×3 CMPs that we analyze.

Joint IPCA does an impressive job of explaining average returns across asset classes: it leaves only 7 alphas unexplained: 5 for bonds, 2 for options, and none for stocks. Looking at single IPCA models estimated on individual asset classes, we find that the six bond-based factors fail to explain 4 CMPs of bonds, 20 CMPs of options, and 4 CMPs of stocks, for a total of 28 CMPs with significant alphas. The option-level IPCA performs worse: it leaves 59, 8, and 8 CMP alphas of bonds, options, and stocks, respectively, as statistically significant. The worst model is the one estimated exclusively on stock returns: the single IPCA with six stock-level factors fails to explain 82, 58, and 13 CMPs of bonds, options, and stocks, respectively. Finally, we also consider a model that matches the number of $K = 6$ factors, but extracts 2 factors per asset class. This model performs reasonably well but still leaves a total of 63 CMP returns unexplained.

Sharpe Ratios. Next, we analyze how well each of the models describes the mean-variance frontier. Table 9 shows that the tangency portfolio of the joint IPCA has an annualized SR of 6.91, compared to 6.52 for the bond-based, 4.98 for the option-based, and 1.26 for the stock-based IPCA model. The combined model with two factors estimated from each of the asset classes has a tangency portfolio with a SR of 4.49.

Table 9: Comparison of Sharpe Ratios for Joint vs. Single IPCA Models

The table compares the SRs of the tangency portfolios for the joint IPCA model as well as individual IPCA models estimated for a single asset class. We assess the statistical significance of joint IPCA's outperformance, by regressing the returns of its tangency portfolio on a constant and each of the single IPCA tangency portfolio returns, after fixing each portfolio's full-sample standard deviation to 10% per year. The resulting t-statistics in parenthesis are computed with Newey and West (1987) standard errors with 12 lags. We also consider a combined model with two factors estimated from each asset class, which matches the number of latent factors for the other models $(2 + 2 + 2)$.

	Joint	Single IPCA			
	IPCA	6 Bond	6 Option	6 Stock	$2+2+2$
Sharpe Ratio	6.91	6.52	4.98	1.26	4.49
Outperformance		1.76	3.43	5.43	3.15
		(6.90)	(13.31)	(11.73)	(10.84)

The second row of Table 9 shows that the joint IPCA's tangency portfolio significantly outperforms its competitors. For this, we fix the full-sample standard deviation of the returns of each tangency portfolio to 10% per year, and regress joint IPCA's tangency portfolio returns on a constant (alpha) and each of the single IPCA tangency portfolio returns. The outperformance is measured by the alpha estimates and corresponding t-statistics that are computed with Newey and West (1987) standard errors with twelve lags. In all cases we find a highly significant SR outperformance of the joint tangency portfolio, ranging between 1.76 and 5.43.

Latent Factor Correlation. As a final comparison between joint and single IPCA models, we compute the correlations between the six joint factors and each of the six single factors. This allows us to understand common patterns in joint and single factors. Differences in the included information may inform us about the reason for joint IPCA's outperformance.

Consistent with the analysis of the influence of each factor in Table 6, we find in Table 10 that joint factors F1, F4, and F6 are highly correlated with one or more single bond factors. Joint factor F1 is also highly correlated to option factors O2 and O6, as well as stock factor S4. Joint factors F1 and F2 are highly correlated with option-based factors, and joint factors F1, F2, F3, and F5 are correlated with stock-based factors, which again is consistent with the evidence on the influence of each factor in Table 6.

Table 10: Correlation of Joint and Single Latent Factors

The table shows the Pearson correlation coefficient between the six joint IPCA factors (F1 to F6) and each of the six latent factors obtained from individual IPCA models estimated on a single asset class. The largest absolute correlation coefficient per row is highlighted in boldface.

	F1	F2	F3	F4	F5	F6
			Panel A: Latent Bond Factors			
$B1$	**0.68**	0.14	−0.40	−0.09	0.00	0.04
$B2$	−0.13	−0.06	0.36	0.33	0.07	**-0.51**
$B3$	−0.33	0.05	0.32	**0.33**	0.23	0.12
$B4$	**-0.22**	−0.02	0.13	−0.12	0.12	−0.17
$B5$	−0.05	−0.11	−0.02	0.20	−0.01	**0.77**
$B6$	0.19	−0.41	−0.24	**0.76**	−0.23	−0.09
			Panel B: Latent Option Factors			
$O1$	−0.04	**0.57**	−0.10	−0.14	0.01	0.07
$O2$	0.69	**-0.82**	0.25	−0.18	−0.13	−0.04
$O3$	−0.13	**0.26**	−0.19	−0.01	0.18	−0.20
$O4$	−0.06	**0.21**	0.12	−0.06	0.12	0.07
$O5$	−0.04	0.05	−0.23	**0.23**	−0.22	−0.04
$O6$	**0.46**	−0.08	−0.45	−0.08	0.17	0.19
			Panel C: Latent Stock Factors			
$S1$	0.03	−0.18	**0.51**	0.13	−0.12	0.18
$S2$	0.15	0.27	−0.16	−0.24	**-0.31**	0.16
$S3$	0.20	−0.26	**0.28**	−0.07	0.11	0.16
$S4$	0.52	−0.54	−0.22	−0.07	**0.65**	−0.01
$S5$	−0.13	−0.01	**0.22**	−0.00	0.14	0.01
$S6$	0.18	−0.08	**0.22**	−0.18	−0.04	0.08

7 Interpreting Latent Factors

7.1 Macroeconomic Sensitivity

It is instructive to understand how the joint latent factors, capable of pricing bonds, options and stocks simultaneously, relate to observable macroeconomic indicators. We, therefore, regress each of the $K = 6$ latent factors on several established macroeconomic variables in Table 11. We include innovations in the Chicago Fed National Activity Index (CFNAI), which is a leading indicator of U.S. economic activity extracted from a broad range of individual macroeconomic variables, innovations in the macroeconomic uncertainty measure (UNC) of Jurado, Ludvigson, and Ng (2015), and in the intermediary capital ratio (ICR) of He, Kelly, and Manela (2017).

We find that our first IPCA factor is significantly exposed to macroeconomic risks. A one standard deviation increase in CFNAI increases its return by 36bps, a one standard deviation decrease in UNC by 71bps, and a one standard deviation increase in ICR by 110bps. This is consistent with predictions of the intertemporal capital asset pricing model (ICAPM) (Merton, 1973), in that high (low, high) levels of CFNAI (UNC, ICR) signal better investment opportunities in the future. The latent factors are constructed to have a positive full-sample mean, suggesting that exposure to the first latent factor exposes the investor to considerable macroeconomic risks (Maio and Santa-Clara, 2012).

Table 11: Regressing Latent Factors on Macroeconomic Indicators

The table shows the results of regressing each of the $K = 6$ latent factors on innovations of the Chicago Fed National Activity Index (CFNAI), the macroeconomic uncertainty index (UNC) of Jurado, Ludvigson, and Ng (2015) and the intermediary capital ratio (ICR) of He, Kelly, and Manela (2017). The three macroeconomic indicators are normalized by their full-sample standard deviation. *** (**, *) denote statistical significance at the 1% (5%, 10%) level. We use Newey and West (1987) standard errors with twelve lags.

	F1	F2	F3	F4	F5	F6
const.	4.03***	1.54***	0.54*	0.49*	0.07	0.00
CFNAI	0.36***	0.04	−0.26***	0.29***	−0.23***	0.28***
UNC	−0.71***	1.18***	0.15	−0.03	0.52***	0.20*
ICR	1.10***	−2.14***	0.08	−0.03	1.86***	0.21
Adj.R^2	0.25	0.43	0.01	0.00	0.32	0.03

The second factor is positively exposed to UNC and negatively to ICR, suggesting that it is a hedge against macroeconomic uncertainty and intermediary capital risks. Factors F3, F4, and F6 are also related to innovations in CFNAI but the respective regressions' adjusted R^2 is miniscule. Finally, factor F5 is negatively exposed to innovations in CFNAI, and positively exposed to innovations in UNC and ICR, suggesting that it captures the spread between overall macroeconomic risks and risks of the intermediary sector.

7.2 Joint IPCA and Benchmark Factor Models

We have thus far shown that the joint IPCA model outperforms IPCA models estimated for single asset classes. We now compare the joint model with the performance of various benchmark factor models, which have been proposed in the literature for either of the three asset classes. To the best of our knowledge, joint IPCA is the first attempt at finding a *joint* factor model, capable of pricing bonds, options, and stocks simultaneously. We include the Fama and French (2015) five-factor model augmented with momentum (Carhart, 1997) as the leading factor model for the stock market (FF6) and the Kelly, Palhares, and Pruitt (2023) five-factor model for bonds (KPP). There is still no consensus about the "best" factor model for options, such that we resort to the two straddle-based factors which build on Coval and Shumway (2001) (CS). We also consider a combination of the three factor models, which in total includes 13 factors $(6+5+2)$.

Table 12: Unconditional Alphas of Joint IPCA vs. Benchmark Factor Models

The table shows how many of the 3×107 characteristic-managed portfolio defined in Section 4.2 have significant average returns per asset class, and how many unconditional alphas remain significant after adjusting for risk either using the joint IPCA model, or benchmark factor models. We consider the Fama and French (2015) five-factor model augmented with momentum (Carhart, 1997) (FF6), the Kelly, Palhares, and Pruitt (2023) five-factor bond model (KPP), and the two straddle-based factors inspired by Coval and Shumway (2001) (CS). We also consider a combination of the three factor models (Comb.), which in total includes 13 factors (6 + 5 + 2). We use Newey and West (1987) standard errors with twelve lags.

		Unconditional alphas				
	Average	Joint	Benchmark Factors			
	returns	IPCA	KPP	CS	FF6	Comb.
Bonds	83	5	18	62	82	17
Options	59	2	12	52	59	10
Stocks	27	0	3	12	15	2
Σ	169	7	33	126	156	29

We repeat the unconditional alpha analysis of Table 8 for the comparison between the joint IPCA model and the three benchmark factor models in Table 12. We have already shown that the joint ICPA leaves a statistically significant alpha in only 7 out of 169 CMPs. The performance of the benchmark factor models are much worse. The KPP bond model fares best but still leaves 33 CMP returns unexplained, 18 for bonds, 12 for options, and 3 for stocks. The CS option model performs much worse: it fails to explain the returns of 126 CMPs. The FF6 stock model fails to explain the returns of 156 CMPs. Even a combination of the 13 factors into a single model fails to explain the returns of 29 CMPs, of which 17 are of bonds, 10 of options, and 2 of stocks.

In Table 13, we show results from regressing each of the $K = 6$ latent factors on the twelve benchmark factors from the three models described above. One of the advantages of our joint IPCA specification is that it is able to extract information from the three asset classes simultaneously. In contrast, factor models have typically been confined to extracting information from a single asset class. Another advantage of the latent specification is that it does not require prior knowledge about which characteristic-sorted factors drive return differences in the cross-section. Instead, we extract a statistically optimal set of factors. The results in Table 13 impressively highlight this advantage: we cannot find a clear mapping between the six latent factors and the benchmark factors. Interestingly, while the macroeconomic indicators have low adjusted R^2 in explaining the returns of factors F3, F4, and F6 (see Table 11), we find that the benchmark factors are able to explain a much greater amount of the variation of factor returns (Adj. R^2), which ranges between 27% for latent factor F6 to 66% for latent factor F4.

Table 13: Regressing Latent Factors on Benchmark Factors

The table shows the results of regressing each of the $K = 6$ latent factors on a number of benchmark factors. We consider the Fama and French (2015) five-factor model augmented with momentum (Carhart, 1997), the Bai, Bali, and Wen (2019) four-factor bond model, and the two straddle-based factors of Coval and Shumway (2001). The benchmark factors are normalized by their full-sample standard deviation. *** (**, *) denote statistical significance at the 1% (5%, 10%) level. We use Newey and West (1987) standard errors with twelve lags.

	F1	F2	F3	F4	F5	F6
const.	1.20***	2.68***	1.33***	−0.36	0.50*	0.66
STRADDLE_INDEX	0.24	−0.72***	−0.20	−0.54*	−0.31	0.01
STRADDLE_STOCK	−0.96***	1.76***	−0.19	1.03**	1.03***	−0.04
MKT-RF	0.69***	−1.43***	1.68***	0.24	1.51***	0.24
SMB	0.25*	−0.11	−0.63***	0.14	0.57***	−0.13
HML	−0.63	−0.13	0.26	0.46*	0.46***	−0.10
RMW	−0.06	0.26	0.20	0.21	0.08	−0.00
CMA	0.21	0.16	−0.53***	−0.08	0.15	−0.20
MOM	−0.41**	0.05	1.53***	0.28	−0.66***	0.02
KPP1	1.04***	−0.04	−0.67***	−0.81***	−0.27**	−0.33
KPP2	0.47***	−0.40*	0.07	2.50***	−0.15	−0.32
KPP3	0.03	−1.17***	−0.53***	0.74***	−0.50***	−0.29
KPP4	1.21***	−0.29**	−1.03***	0.37***	−0.30***	0.26*
KPP5	0.10	−0.26	−0.02	0.86***	−0.10	0.82***
Adj.R^2	0.56	0.66	0.56	0.66	0.80	0.27

7.3 Replacing Factors

As a final analysis towards interpreting the $K = 6$ latent factors of our joint IPCA specification, we perform a factor-replacement exercise. For this, we first calculate the drop in the Total R^2 when setting all realizations of each factor separately to zero. We have discussed the implications of this exercise in Table 6. Denote the resulting Total R^2 as R^2_{zero}. Then, we regress each of the kth latent factor on a constant and either the three macroeconomic indicators CFNAI, UNC and ICR, or the 13 benchmark factors discussed above, subsumed in matrix $\boldsymbol{X}$:

$$F_{t,k} = \alpha_k + \boldsymbol{\beta}_k \boldsymbol{X}_t + \varepsilon_{t,k} \tag{25}$$

We replace the realizations of the kth factor with the fitted values from this regression:

$$\hat{F}_{t,k} = \alpha_k + \boldsymbol{\beta}_k \boldsymbol{X}_t, \tag{26}$$

and record the resulting Total R^2 when replacing factor kth's realizations with either macroeconomic information or information from the twelve benchmark factors. Denote the resulting Total R^2 as R^2_X. Finally, in Figure 6, we show the reduction in the model's Total R^2 relative to setting the realizations of the kth factor to zero:

$$\text{Relative Reduction} = \frac{R^2_X - R^2}{R^2_{\text{zero}} - R^2}. \tag{27}$$

A value of 1 indicates that replacing the factor's realizations with its projections on macroeconomic information/benchmark factors produces a Total R^2 as low as that achieved by simply setting its realizations to zero. A value of 0 instead indicates that replacing the factor's realizations works well and produces no loss in explanatory power. This exercise allows us to quantify the relative importance of the information embedded in macroeconomic indicators and benchmark factors for the six latent and shared factors.

Figure 6 shows that replacing the latent factors with their fitted values always reduces the model's ability to explain returns of bonds, options, and stocks (all Relative Reduction values are greater than zero). This again highlights that the joint IPCA factors optimally describe the risk-return trade-off across the three asset classes and pick up on important variation unexplained by macroeconomic information and information extracted from the asset classes individually. The figure also shows that the first factor, which explains most variation of bond, option, and stock returns and has the largest average return, is better replicated by macroeconomic information than by benchmark factors. This is despite the fact that benchmark factors explain a larger fraction of the factor's return variation (56% in

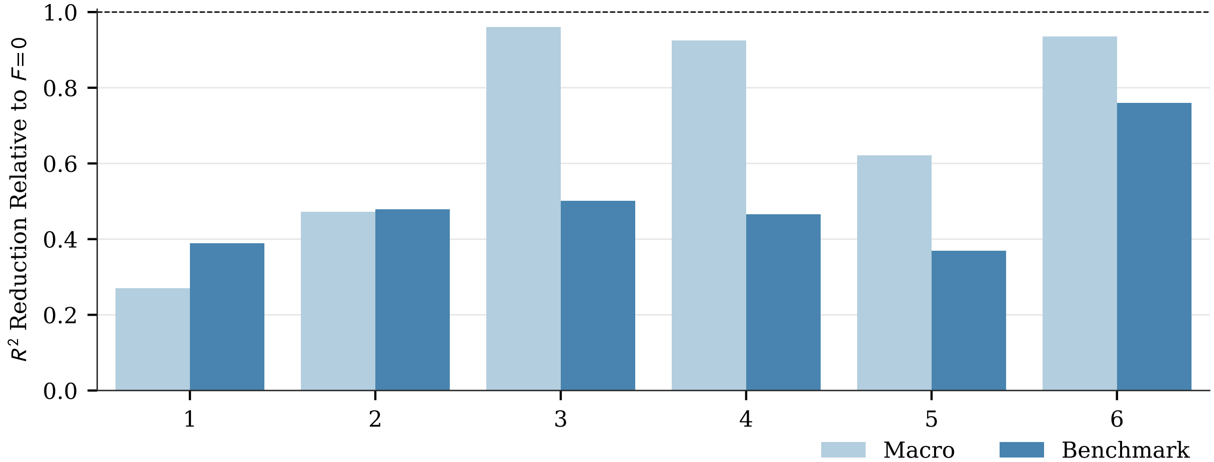

Figure 6: Replacing Latent Factors with Macroeconomic Indicators or Benchmark Factors

The figure shows the relative reduction in the Total R^2 (Eq. (27)) when replacing latent factor realizations with fitted values from regressing it on macroeconomic indicators or benchmark factors extracted from bonds, options, and stocks.

Table 13 versus 25% in Table 11), and highlights the importance of the factor replacement exercise to interpret the latent factors. The second factor is equally-well replicated by both information sources, and factors 3 to 6 are better replicated using benchmark factors.

8 Conclusion

We propose a factor model to jointly describe the risk-return tradeoff for bonds, options, and stocks. Just six shared factors are able to explain between 22% and 37% of the return variation of bonds, options, and stocks, and describe the conditional and unconditional pricing of each asset class well. The resulting tangency portfolio exploits important diversification benefits enjoyed when simultaneously modeling the risk-return tradeoff for the three asset classes, and achieves an IS and OOS SR above 6.0.

The parsimonious factor structure of joint IPCA better explains average returns across asset classes. Of 169 CMPs that have a significant average return over our sample period between August of 2002 and December of 2021, our six-factor joint IPCA model leaves only seven unexplained. In contrast, a six-factor bond-only IPCA model leaves 28 unexplained. Option- and stock-only IPCA models leave 75 and 153 unexplained, respectively. We also compare joint IPCA with prominent benchmark factor models put forth in the literature for the three asset classes. Even a combination of five bond, two option, and six stock factors fails to explain the return patterns of 29 CMPs, lagging far behind the explanatory power of joint IPCA.

We investigate patterns of commonality and find a high degree of integration between bonds, options, and stocks. Interestingly, we also find a high degree of integration between bonds and options, lending empirical credence to the idea of Merton (1974) structural credit risk model that bonds (and stocks) are options on a firm's assets and thus share many of the properties of equity options. While most research has thus far focused on the integration of bond and stock markets (see Du, Elkamhi, and Ericsson, 2019, as an example), our results call for the additional consideration of options and how the trading activity in equity options relates not only to the underlying stock but also to corporate bonds of the same firm. Doshi, Ericsson, Fournier, and Seo (2022) is a first step in this direction at the index level.

References

An, Byeong-Je, Andrew Ang, Turan G. Bali, and Nusret Cakici, 2014, The joint cross section of stocks and options, *Journal of Finance* 69, 2279–2337.

Bai, Jennie, Turan G. Bali, and Quan Wen, 2019, Common risk factors in the cross-section of corporate bond returns, *Journal of Financial Economics* 131, 619–642.

Bali, Turan G., Heiner Beckmeyer, Mathis Moerke, and Florian Weigert, 2023, Option return predictability with machine learning and big data, *Review of Financial Studies* Forthcoming.

Bali, Turan G., Jie Cao, Fousseni Chabi-Yo, Linjia Song, and Xintong Zhan, 2022, A Factor Model for Stock Options, *Available at SSRN 4308916*.

Bali, Turan G., Amit Goyal, Dashan Huang, Fuwei Jiang, and Quan Wen, 2022, Predicting corporate bond returns: Merton meets machine learning, *Georgetown McDonough School of Business Research Paper (3686164)* pp. 20–110.

Bali, Turan G., and Armen Hovakimian, 2009, Volatility spreads and expected stock returns, *Management Science* 55, 1797–1812.

Bates, David S., 1996, Jumps and stochastic volatility: Exchange rate processes implicit in deutsche mark options, *Review of Financial Studies* 9, 69–107.

Beckmeyer, Heiner, and Timo Wiedemann, 2023, Recovering missing firm characteristics with attention-based machine learning, *Available at SSRN 4003455*.

Bessembinder, Hendrik, Chester Spatt, and Kumar Venkataraman, 2020, A survey of the microstructure of fixed-income markets, *Journal of Financial and Quantitative Analysis* 55, 1–45.

Bryzgalova, Svetlana, Sven Lerner, Martin Lettau, and Markus Pelger, 2022, Missing financial data, *Available at SSRN 4106794*.

Cao, Jie, Amit Goyal, Xiao Xiao, and Xintong Zhan, 2023, Implied volatility changes and corporate bond returns, *Management Science* 69, 1375–1397.

Cao, Jie, and Bing Han, 2013, Cross section of option returns and idiosyncratic stock volatility, *Journal of Financial Economics* 108, 231–249.

Carhart, Mark M., 1997, On persistence in mutual fund performance, *Journal of Finance* 52, 57–82.

Choi, Jaewon, Yesol Huh, and Sean Seunghun Shin, 2021, Customer liquidity provision: Implications for corporate bond transaction costs, *Available at SSRN 2848344*.

Choi, Jaewon, and Yongjun Kim, 2018, Anomalies and market (dis) integration, *Journal of Monetary Economics* 100, 16–34.

Chordia, Tarun, Amit Goyal, Yoshio Nozawa, Avanidhar Subrahmanyam, and Qing Tong, 2017, Are capital market anomalies common to equity and corporate bond markets? An empirical investigation, *Journal of Financial and Quantitative Analysis* 52, 1301–1342.

Cochrane, John, 2009, *Asset pricing: Revised edition*. (Princeton university press).

Collin-Dufresne, Pierre, Benjamin Junge, and Anders B. Trolle, 2022, How integrated are credit and equity markets? Evidence from index options, *Swiss Finance Institute Research Paper*.

Coval, Joshua D., and Tyler Shumway, 2001, Expected option returns, *Journal of Finance* 56, 983–1009.

Cremers, Martijn, and David Weinbaum, 2010, Deviations from put-call parity and stock return predictability, *Journal of Financial and Quantitative Analysis* 45, 335–367.

Culp, Christopher L., Yoshio Nozawa, and Pietro Veronesi, 2018, Option-based credit spreads, *American Economic Review* 108, 454–488.

Doshi, Hitesh, Jan Ericsson, Mathieu Fournier, and Sang Byung Seo, 2022, Asset Variance Risk and Compound Option Prices, *Available at SSRN 3885357*.

Du, Du, Redouane Elkamhi, and Jan Ericsson, 2019, Time-Varying Asset Volatility and the Credit Spread Puzzle, *Journal of Finance* 74, 1841–1885.

Easley, David, Maureen O'Hara, and P.S. Srinivas, 1998, Option volume and stock prices: Evidence on where informed traders trade, *Journal of Finance* 53, 431–465.

Fama, Eugene F., and Kenneth R. French, 2015, A five-factor asset pricing model, *Journal of Financial Economics* 116, 1–22.

Fama, Eugene F., and Kenneth R. French, 2018, Choosing factors, *Journal of Financial Economics* 128, 234–252.

Fama, Eugene F., and James D. MacBeth, 1973, Risk, return, and equilibrium: Empirical tests, *Journal of Political Economy* 81, 607–636.

Fortet, Robert, 1943, Les functions aléatoires du type Markoff associées à certaines equations lineaires aux derivées partielles du type parabolique, *J. Math. Pures. Appl.* 22, 177–243.

Frazzini, Andrea, Ronen Israel, and Tobias J. Moskowitz, 2018, Trading costs, *Available at SSRN 3229719*.

Freyberger, Joachim, Björn Höppner, Andreas Neuhierl, and Michael Weber, 2022, Missing data in asset pricing panels, Working paper, National Bureau of Economic Research.

Gârleanu, Nicolae, Lasse Heje Pedersen, and Allen M. Poteshman, 2008, Demand-based option pricing, *Review of Financial Studies* 22, 4259–4299.

George, Thomas J., and Chuan-Yang Hwang, 2004, The 52-week high and momentum investing, *Journal of Finance* 59, 2145–2176.

Geske, Robert, 1979, The valuation of compound options, *Journal of Financial Economics* 7, 63–81.

Goyal, Amit, and Alessio Saretto, 2009, Cross-section of option returns and volatility, *Journal of Financial Economics* 94, 310–326.

Goyal, Amit, and Alessio Saretto, 2022, Are Equity Option Returns Abnormal? IPCA Says No, *IPCA Says No (August 19, 2022)*.

Gu, Shihao, Bryan T. Kelly, and Dacheng Xiu, 2020, Empirical asset pricing via machine learning, *Review of Financial Studies* 33, 2223–2273.

He, Zhiguo, Bryan T. Kelly, and Asaf Manela, 2017, Intermediary asset pricing: New evidence from many asset classes, *Journal of Financial Economics* 126, 1–35.

Heston, Steven L., 1993, A closed-form solution for options with stochastic volatility with applications to bond and currency options, *Review of Financial Studies* 6, 327–343.

Hiraki, Kazuhiro, and George Skiadopoulos, 2020, The Contribution of Frictions to Expected Returns, Working paper.

Horenstein, Alex R., Aurelio Vasquez, and Xiao Xiao, 2022, Common factors in equity option returns, *Available at SSRN 3290363*.

Hou, Kewei, Chen Xue, and Lu Zhang, 2015, Digesting anomalies: An investment approach, *Review of Financial Studies* 28, 650–705.

Jensen, Theis Ingerslev, Bryan T. Kelly, and Lasse Heje Pedersen, 2021, Is there a replication crisis in finance? Working paper, National Bureau of Economic Research.

Johnson, Travis L., and Eric C. So, 2012, The option to stock volume ratio and future returns, *Journal of Financial Economics* 106, 262–286.

Jurado, Kyle, Sydney C Ludvigson, and Serena Ng, 2015, Measuring uncertainty, *American Economic Review* 105, 1177–1216.

Kelly, Bryan, Diogo Palhares, and Seth Pruitt, 2023, Modeling corporate bond returns, *The Journal of Finance* 78, 1967–2008.

Kelly, Bryan T., Seth Pruitt, and Yinan Su, 2019, Characteristics are covariances: A unified model of risk and return, *Journal of Financial Economics* 134, 501–524.

Koijen, Ralph S.J., Hanno Lustig, and Stijn Van Nieuwerburgh, 2017, The cross-section and time series of stock and bond returns, *Journal of Monetary Economics* 88, 50–69.

Kozak, Serhiy, Stefan Nagel, and Shrihari Santosh, 2020, Shrinking the cross-section, *Journal of Financial Economics* 135, 271–292.

Leland, Hayne E., 1994, Corporate debt value, bond covenants, and optimal capital structure, *Journal of Finance* 49, 1213–1252.

Liu, Yukun, Aleh Tsyvinski, and Xi Wu, 2022, Common risk factors in cryptocurrency, *Journal of Finance* 77, 1133–1177.

Lo, Andrew W., 2002, The statistics of Sharpe ratios, *Financial Analysts Journal* 58, 36–52.

Lustig, Hanno, Nikolai Roussanov, and Adrien Verdelhan, 2011, Common risk factors in currency markets, *Review of Financial Studies* 24, 3731–3777.

Maio, Paulo, and Pedro Santa-Clara, 2012, Multifactor models and their consistency with the ICAPM, *Journal of Financial Economics* 106, 586–613.

Martin, Ian, 2017, What is the Expected Return on the Market? *Quarterly Journal of Economics* 132, 367–433.

Merton, Robert C, 1973, An intertemporal capital asset pricing model, *Econometrica: Journal of the Econometric Society* pp. 867–887.

Merton, Robert C., 1974, On the pricing of corporate debt: The risk structure of interest rates, *Journal of Finance* 29, 449–470.

Muravyev, Dmitriy, and Neil D. Pearson, 2020, Options trading costs are lower than you think, *Review of Financial Studies* 33, 4973–5014.

Muravyev, Dmitriy, Neil D. Pearson, and Joshua Matthew Pollet, 2022, Anomalies and Their Short-Sale Costs, *Available at SSRN 4266059*.

Neuhierl, Andreas, Xiaoxiao Tang, Rasmus Tangsgaard Varneskov, and Guofu Zhou, 2023, Option characteristics as cross-sectional predictors, *Available at SSRN 3795486*.

Newey, Whitney K., and Kenneth D. West, 1987, A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix, *Econometrica* 55, 703–708.

Ofek, Eli, Matthew Richardson, and Robert F. Whitelaw, 2004, Limited arbitrage and short sales restrictions: Evidence from the options markets, *Journal of Financial Economics* 74, 305–342.

Roll, Richard, Eduardo Schwartz, and Avanidhar Subrahmanyam, 2010, O/S: The relative trading activity in options and stock, *Journal of Financial Economics* 96, 1–17.

Sandulescu, Mirela, 2023, How Integrated Are Corporate Bond and Stock Markets? Working paper, Swiss Finance Institute.

Schlag, Christian, Julian Thimme, and Rüdiger Weber, 2021, Implied volatility duration: A measure for the timing of uncertainty resolution, *Journal of Financial Economics* 140, 127–144.

Szymanowska, Marta, Frans de Roon, Theo Nijman, and Rob van den Goorbergh, 2014, An anatomy of commodity futures risk premia, *Journal of Finance* 69, 453–482.

Xing, Yuhang, Xiaoyan Zhang, and Rui Zhao, 2010, What does the individual option volatility smirk tell us about future equity returns? *Journal of Financial and Quantitative Analysis* 45, 641–662.

A Firm Characteristics

The following table shows the whole set of 264 firm-level characteristics extracted from the firm's bonds, options, and stock. Alongside the characteristic's name, we provide a short description, its original source in the literature, whether it was extracted from information of the firm's bond, option, or stock. We also provide the reason for dropping the characteristic in the estimation of joint IPCA inEq. (11).

Feature	Description	Information Source	Source	Dropped?
B_Age	Age	Bonds		Insignificant
B_Amihud	Amihud measure of illiquidity	Bonds		Insignificant
B_AmountOut	Nominal amount outstanding	Bonds		Correlation
B_AvgBidAsk	Difference of average bid and ask prices	Bonds		Insignificant
B_BetaBond	Bond market beta	Bonds		
B_BetaDEF	Default beta	Bonds		Insignificant
B_BetaTERM	Term beta	Bonds		Insignificant
B_BetaUNC	Macroeconomic uncertainty beta	Bonds		Insignificant
B_BetaVIX	Volatility beta	Bonds		
B_ES10	Downside risk proxied by the 10% Expected Shortfall	Bonds		Correlation
B_ES5	Downside risk proxied by the 5% Expected Shortfall	Bonds		
B_GammaPS	Pastor and Stambaugh's liquidity measure	Bonds		Missingness
B_MOM12	Twelve-month momentum	Bonds		Insignificant
B_MOM6	Six-month momentum	Bonds		Insignificant
B_PHighLow	High-low spread estimator	Bonds		Insignificant
B_PIHighLow	An extended High-low spread estimator	Bonds		Insignificant
B_PILambda	Lambda	Bonds		Insignificant
B_PIRoll	An extended Roll's measure	Bonds		Insignificant
B_PZeros	Illiquidity measure based on zero returns	Bonds		
B_Rating	Credit rating	Bonds		
B_Roll	Roll's daily measure of illiquidity	Bonds		
B_Roundtrip	Round-trip transaction costs	Bonds		Insignificant
B_Size	Issuance size	Bonds		
B_StdAmihud	Std.dev of the Amihud measure	Bonds		Insignificant
B_TCRoll	Roll's intraday measure of illiquidity	Bonds		Insignificant
B_VaR10	Downside risk proxied by the 10% VaR	Bonds		
B_VaR5	Downside risk proxied by the 5% VaR	Bonds		
B_coskew	Co-skewness	Bonds		Insignificant
B_dur	Duration	Bonds		
B_illiq	Illiquidity	Bonds		
B_iskew	Idiosyncratic Skewness	Bonds		Insignificant
B_kurt	Kurtosis	Bonds		
B_ltr	Long-term reversal	Bonds		Missingness
B_mat	Time-to-maturity	Bonds		Insignificant
B_pfht	Modified illiquidity measure based on zero returns	Bonds		Insignificant
B_pifht	An extended FHT measure based on zero returns	Bonds		Insignificant
B_rev	Short-term reversal	Bonds		
B_skew	Skewness	Bonds		Insignificant
B_tciqr	Interquartile range	Bonds		Insignificant
B_vol	Volatility	Bonds		
O_ailliq	Absolute illiquidity	Options	Cao and Wei (2010)	
O_amihud	Amihud illiquidity per bucket	Options	Amihud (2002)	Correlation
O_atm_civpiv	At-the-money put vs. call implied volatility	Options		
O_atm_dcivpiv	Change in atm put vs. call implied volatility	Options	An Ang Bali and Cakici (2014)	
O_atm_iv	At-the-money implied volatility (maturity-specific)	Options		Insignificant

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Feature	Description	Information Source	Source	Dropped?
O_bucket_dvol	Option bucket dollar volume	Options		Correlation
O_bucket_vol	Option bucket volume	Options		Correlation
O_civpiv	Near-the-money put vs. call implied volatility	Options	Bali and Hovakimian (2009)	
O_dciv	Change in atm call implied volatility	Options	An Ang Bali and Cakici (2014)	
O_delta	Delta	Options	Buchner and Kelly (2020)	
O_demand_pressure	Option Demand Pressure	Options		
O_doi	Dollar open interest	Options		
O_dpiv	Change in atm put implied volatility	Options	An Ang Bali and Cakici (2014)	
O_dso	Stock vs. option volume in USD	Options	Roll Schwartz and Subrahmanyam (2010)	
O_dvol	Dollar trading volume	Options	Cao and Wei (2010)	Correlation
O_embedlev	Embedded Leverage	Options	Karakaya (2014)	
O_fric	Contribution of market frictions to expected returns	Options	Hiraki and Skiadopoulos (2020)	
O_gamma	Gamma	Options	Buchner and Kelly (2020)	
O_gammaps	Pastor and Stambaugh liquidity measure	Options	Pastor and Stambaugh (2003)	Insignificant
O_hkurt	Historic kurtosis	Options		
O_hskew	Historic skewness	Options		Insignificant
O_hvol	Historic Volatility	Options		Insignificant
O_illiq	Illiquidity	Options	Bao Pan and Wang (2011)	
O_iv	Implied volatility	Options	Buchner and Kelly (2020)	
O_iv_rank	Implied volatility rank vs. last year	Options		
O_ivarud30	Option implied variance asymmetry	Options	Huang and Li (2019)	Insignificant
O_ivd	Implied volatility duration	Options	Schlag Thimme and Weber (2020)	
O_ivrv	Implied volatility minus realized volatility	Options	Bali and Hovakimian (2009)	
O_ivrv_ratio	Implied volatility minus realized volatility ratio	Options		
O_ivslope	Implied volatility slope	Options	Vasquez (2017)	
O_ivvol	Volatility of atm volatility	Options	Baltussen van Bekkum and van der Grient (2018)	
O_m_degree	Standardized strike	Options		
O_mid	Option mid price	Options		Insignificant
O_modos	Modified stock vs. option volume	Options	Johnson and So (2012)	
O_nopt	Number of options trading	Options		
O_ocgo	Disposition Effect	Options	Bergsma Fodor and Tedford (2020)	
O_oi	Open interest	Options		
O_oistock	Open interest vs. stock volume	Options		
O_optspread	Option bid-ask spread	Options		Insignificant
O_pba	Proportional bid-ask spread	Options	Cao and Wei (2010)	Insignificant
O_pcpv	Put-call parity deviations	Options	Ofek Richardson and Whitelaw (2004)	
O_pcratio	Put-call ratio	Options	Blau Nguyen and Whitby (2014)	
O_pfht	Modified illiquidity measure based on zero returns	Options	Fong Holden and Trzcinka (2017)	Correlation
O_pifht	An extended FHT measured based on zero returns	Options		
O_pilliq	Percentage illiquidity	Options	Cao and Wei (2010)	
O_piroll	Extended Roll's measure	Options	Goyenko Holden and Trzcinka (2009)	
O_pzeros	Illiquidity measure based on zero returns	Options	Lesmond 1999	
O_rnk182	182-day risk-neutral kurtosis	Options		Insignificant
O_rnk273	273-day risk-neutral kurtosis	Options		Insignificant
O_rnk30	30-day risk-neutral kurtosis	Options		

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Feature	Description	Information Source	Source	Dropped?
O_rnk365	365-day risk-neutral kurtosis	Options		Insignificant
O_rnk91	91-day risk-neutral kurtosis	Options		
O_rns182	182-day risk-neutral skewness	Options	Borochin Chang and Wu (2020)	Insignificant
O_rns273	273-day risk-neutral skewness	Options	Borochin Chang and Wu (2020)	Insignificant
O_rns30	30-day risk-neutral skewness	Options	Borochin Chang and Wu (2020)	Insignificant
O_rns365	365-day risk-neutral skewness	Options	Borochin Chang and Wu (2020)	Insignificant
O_rns91	91-day risk-neutral skewness	Options	Borochin Chang and Wu (2020)	Insignificant
O_roll	Roll's measure of illiquidity	Options	Roll (1984)	
O_shrtfee	Implied shorting fees	Options	Muravyev and Pearson (2020)	
O_skewiv	IV skew	Options	Xing Zhang and Zhao (2010)	Insignificant
O_so	Stock vs. option volume	Options	Roll Schwartz and Subrahmanyam (2010)	
O_stdamihud	Standard deviation of Amihud's illiquidity measure	Options		
O_theta	Theta	Options	Buchner and Kelly (2020)	Insignificant
O_tlm30	Tail loss measure	Options	Vilkov and Xiao (2012)	
O_toi	Total option open interest	Options		
O_turnover	Option turnover	Options		Insignificant
O_vega	Vega	Options	Buchner and Kelly (2020)	Insignificant
O_vol	Trading volume in options	Options		
O_volga	Volga	Options	Buchner and Kelly (2020)	Insignificant
O_vs_change	Change in weighted put-call spread	Options	Cremers and Weinbaum (2010)	Insignificant
O_vs_level	Weighted put-call spread	Options	Cremers and Weinbaum (2010)	
S_age	Firm age	Underlying	Jiang Lee and Zhang (2005)	Insignificant
S_aliq_at	Liquidity of book assets	Underlying	Ortiz-Molina and Phillips (2014)	Insignificant
S_aliq_mat	Liquidity of market assets	Underlying	Ortiz-Molina and Phillips (2014)	Insignificant
S_ami_126d	Amihud Measure	Underlying	Amihud (2002)	Insignificant
S_at_be	Book leverage	Underlying	Fama and French (1992)	Insignificant
S_at_gr1	Asset Growth	Underlying	Cooper Gulen and Schill (2008)	
S_at_me	Assets-to-market	Underlying	Fama and French (1992)	Insignificant
S_at_turnover	Capital turnover	Underlying	Haugen and Baker (1996)	Insignificant
S_be_gr1a	Change in common equity	Underlying	Richardson et al. (2005)	Insignificant
S_be_me	Book-to-market equity	Underlying	Rosenberg Reid and Lanstein (1985)	Insignificant
S_beta_60m	Market Beta	Underlying	Fama and MacBeth (1973)	Insignificant
S_beta_dimson_21d	Dimson beta	Underlying	Dimson (1979)	Insignificant
S_betabab_1260d	Frazzini-Pedersen market beta	Underlying	Frazzini and Pedersen (2014)	Insignificant
S_betadown_252d	Downside beta	Underlying	Ang Chen and Xing (2006)	Insignificant
S_bev_mev	Book-to-market enterprise value	Underlying	Penman Richardson and Tuna (2007)	Insignificant
S_bidaskhl_21d	The high-low bid-ask spread	Underlying	Corwin and Schultz (2012)	Insignificant
S_capex_abn	Abnormal corporate investment	Underlying	Titman Wei and Xie (2004)	
S_capx_gr1	CAPEX growth (1 year)	Underlying	Xie (2001)	
S_capx_gr2	CAPEX growth (2 years)	Underlying	Anderson and Garcia-Feijoo (2006)	Insignificant
S_capx_gr3	CAPEX growth (3 years)	Underlying	Anderson and Garcia-Feijoo (2006)	
S_cash_at	Cash-to-assets	Underlying	Palazzo (2012)	
S_chcsho_12m	Net stock issues	Underlying	Pontiff and Woodgate (2008)	
S_coa_gr1a	Change in current operating assets	Underlying	Richardson et al. (2005)	Insignificant
S_col_gr1a	Change in current operating liabilities	Underlying	Richardson et al. (2005)	Insignificant

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Feature	Description	Information Source	Source	Dropped?
S_cop_at	Cash-based operating profits-to-book assets	Underlying		Insignificant
S_cop_atl1	Cash-based operating profits-to-lagged book assets	Underlying	Ball et al. (2016)	Insignificant
S_corr_1260d	Market correlation	Underlying	Assness, Frazzini, Gormsen, Pedersen (2020)	
S_coskew_21d	Coskewness	Underlying	Harvey and Siddique (2000)	Insignificant
S_cowc_gr1a	Change in current operating working capital	Underlying	Richardson et al. (2005)	Insignificant
S_dbnetis_at	Net debt issuance	Underlying	Bradshaw Richardson and Sloan (2006)	Insignificant
S_debt_gr3	Growth in book debt (3 years)	Underlying	Lyandres Sun and Zhang (2008)	Insignificant
S_debt_me	Debt-to-market	Underlying	Bhandari (1988)	Insignificant
S_dgp_dsale	Change gross margin minus change sales	Underlying	Abarbanell and Bushee (1998)	Insignificant
S_div12m_me	Dividend yield	Underlying	Litzenberger and Ramaswamy (1979)	
S_dolvol_126d	Dollar trading volume	Underlying	Brennan Chordia and Subrahmanyam (1998)	Insignificant
S_dolvol_var_126d	Coefficient of variation for dollar trading volume	Underlying	Chordia Subrahmanyam and Anshuman (2001)	Correlation
S_dsale_dinv	Change sales minus change Inventory	Underlying	Abarbanell and Bushee (1998)	Insignificant
S_dsale_drec	Change sales minus change receivables	Underlying	Abarbanell and Bushee (1998)	Insignificant
S_dsale_dsga	Change sales minus change SG&A	Underlying	Abarbanell and Bushee (1998)	Insignificant
S_earnings_variability	Earnings variability	Underlying	Francis et al. (2004)	
S_ebit_bev	Return on net operating assets	Underlying	Soliman (2008)	Insignificant
S_ebit_sale	Profit margin	Underlying	Soliman (2008)	
S_ebitda_mev	Ebitda-to-market enterprise value	Underlying	Loughran and Wellman (2011)	
S_emp_gr1	Hiring rate	Underlying	Belo Lin and Bazdresch (2014)	
S_eq_dur	Equity duration	Underlying	Dechow Sloan and Soliman (2004)	
S_eqnetis_at	Net equity issuance	Underlying	Bradshaw Richardson and Sloan (2006)	Insignificant
S_eqnpo_12m	Equity net payout	Underlying	Daniel and Titman (2006)	
S_eqnpo_me	Net payout yield	Underlying	Boudoukh et al. (2007)	
S_eqpo_me	Payout yield	Underlying	Boudoukh et al. (2007)	
S_f_score	Pitroski F-score	Underlying	Piotroski (2000)	
S_fcf_me	Free cash flow-to-price	Underlying	Lakonishok Shleifer and Vishny (1994)	Insignificant
S_fnl_gr1a	Change in financial liabilities	Underlying	Richardson et al. (2005)	Insignificant
S_gp_at	Gross profits-to-assets	Underlying	Novy-Marx (2013)	Insignificant
S_gp_atl1	Gross profits-to-lagged assets	Underlying		Insignificant
S_intrinsic_value	Intrinsic value-to-market	Underlying	Frankel and Lee (1998)	Insignificant
S_inv_gr1	Inventory growth	Underlying	Belo and Lin (2011)	
S_inv_gr1a	Inventory change	Underlying	Thomas and Zhang (2002)	
S_iskew_capm_21d	Idiosyncratic skewness from the CAPM	Underlying		
S_iskew_ff3_21d	Idiosyncratic skewness from the Fama-French 3-factor model	Underlying	Bali Engle and Murray (2016)	
S_iskew_hxz4_21d	Idiosyncratic skewness from the q-factor model	Underlying		
S_ivol_capm_21d	Idiosyncratic volatility from the CAPM (21 days)	Underlying		Insignificant
S_ivol_capm_252d	Idiosyncratic volatility from the CAPM (252 days)	Underlying	Ali Hwang and Trombley (2003)	Insignificant
S_ivol_ff3_21d	Idiosyncratic volatility from the Fama-French 3-factor model	Underlying	Ang et al. (2006)	Insignificant
S_ivol_hxz4_21d	Idiosyncratic volatility from the q-factor model	Underlying		Insignificant
S_kz_index	Kaplan-Zingales index	Underlying	Lamont Polk and Saa-Requejo (2001)	Insignificant
S_lnoa_gr1a	Change in long-term net operating assets	Underlying	Fairfield Whisenant and Yohn (2003)	
S_lti_gr1a	Change in long-term investments	Underlying	Richardson et al. (2005)	
S_market_equity	Market Equity	Underlying	Banz (1981)	Insignificant
S_mispricing_mgmt	Mispricing factor: Management	Underlying	Stambaugh and Yuan (2016)	Insignificant

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Feature	Description	Information Source	Source	Dropped?
S_mispricing_perf	Mispricing factor: Performance	Underlying	Stambaugh and Yuan (2016)	Insignificant
S_ncoa_gr1a	Change in noncurrent operating assets	Underlying	Richardson et al. (2005)	
S_ncol_gr1a	Change in noncurrent operating liabilities	Underlying	Richardson et al. (2005)	Insignificant
S_netdebt_me	Net debt-to-price	Underlying	Penman Richardson and Tuna (2007)	Insignificant
S_netis_at	Net total issuance	Underlying	Bradshaw Richardson and Sloan (2006)	Insignificant
S_nfna_gr1a	Change in net financial assets	Underlying	Richardson et al. (2005)	Insignificant
S_ni_ar1	Earnings persistence	Underlying	Francis et al. (2004)	Insignificant
S_ni_be	Return on equity	Underlying	Haugen and Baker (1996)	Insignificant
S_ni_inc8q	Number of consecutive quarters with earnings increases	Underlying	Barth Elliott and Finn (1999)	
S_ni_ivol	Earnings volatility	Underlying	Francis et al. (2004)	
S_ni_me	Earnings-to-price	Underlying	Basu (1983)	
S_niq_at	Quarterly return on assets	Underlying	Balakrishnan Bartov and Faurel (2010)	Insignificant
S_niq_at_chg1	Change in quarterly return on assets	Underlying		Insignificant
S_niq_be	Quarterly return on equity	Underlying	Hou Xue and Zhang (2015)	Insignificant
S_niq_be_chg1	Change in quarterly return on equity	Underlying		Insignificant
S_niq_su	Standardized earnings surprise	Underlying	Foster Olsen and Shevlin (1984)	Insignificant
S_nncoa_gr1a	Change in net noncurrent operating assets	Underlying	Richardson et al. (2005)	
S_noa_at	Net operating assets	Underlying	Hirshleifer et al. (2004)	Insignificant
S_noa_gr1a	Change in net operating assets	Underlying	Hirshleifer et al. (2004)	
S_o_score	Ohlson O-score	Underlying	Dichev (1998)	
S_oaccruals_at	Operating accruals	Underlying	Sloan (1996)	Insignificant
S_oaccruals_ni	Percent operating accruals	Underlying	Hafzalla Lundholm and Van Winkle (2011)	Insignificant
S_ocf_at	Operating cash flow to assets	Underlying	Bouchard, Krüger, Landier and Thesmar (2019)	Insignificant
S_ocf_at_chg1	Change in operating cash flow to assets	Underlying	Bouchard, Krüger, Landier and Thesmar (2019)	Insignificant
S_ocf_me	Operating cash flow-to-market	Underlying	Desai Rajgopal and Venkatachalam (2004)	Insignificant
S_ocfq_saleq_std	Cash flow volatility	Underlying	Huang (2009)	Insignificant
S_op_at	Operating profits-to-book assets	Underlying		Insignificant
S_op_atl1	Operating profits-to-lagged book assets	Underlying	Ball et al. (2016)	Insignificant
S_ope_be	Operating profits-to-book equity	Underlying	Fama and French (2015)	Insignificant
S_ope_bel1	Operating profits-to-lagged book equity	Underlying		Insignificant
S_opex_at	Operating leverage	Underlying	Novy-Marx (2011)	Insignificant
S_pi_nix	Taxable income-to-book income	Underlying	Lev and Nissim (2004)	Insignificant
S_ppeinv_gr1a	Change PPE and Inventory	Underlying	Lyandres Sun and Zhang (2008)	
S_prc	Price per share	Underlying	Miller and Scholes (1982)	Insignificant
S_prc_highprc_252d	Current price to high price over last year	Underlying	George and Hwang (2004)	
S_qmj	Quality minus Junk: Composite	Underlying	Assness, Frazzini and Pedersen (2018)	Insignificant
S_qmj_growth	Quality minus Junk: Growth	Underlying	Assness, Frazzini and Pedersen (2018)	Insignificant
S_qmj_prof	Quality minus Junk: Profitability	Underlying	Assness, Frazzini and Pedersen (2018)	Insignificant
S_qmj_safety	Quality minus Junk: Safety	Underlying	Assness, Frazzini and Pedersen (2018)	Insignificant
S_rd5_at	R&D capital-to-book assets	Underlying	Li (2011)	Missingness
S_rd_me	R&D-to-market	Underlying	Chan Lakonishok and Sougiannis (2001)	Missingness
S_rd_sale	R&D-to-sales	Underlying	Chan Lakonishok and Sougiannis (2001)	Missingness
S_resff3_12_1	Residual momentum t-12 to t-1	Underlying	Blitz Huij and Martens (2011)	Insignificant
S_resff3_6_1	Residual momentum t-6 to t-1	Underlying	Blitz Huij and Martens (2011)	
S_ret_12_1	Price momentum t-12 to t-1	Underlying	Fama and French (1996)	Insignificant

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Feature	Description	Information Source	Source	Dropped?
S_ret_12_7	Price momentum t-12 to t-7	Underlying	Novy-Marx (2012)	Insignificant
S_ret_1_0	Short-term reversal	Underlying	Jegadeesh (1990)	
S_ret_3_1	Price momentum t-3 to t-1	Underlying	Jegedeesh and Titman (1993)	
S_ret_60_12	Long-term reversal	Underlying	De Bondt and Thaler (1985)	
S_ret_6_1	Price momentum t-6 to t-1	Underlying	Jegadeesh and Titman (1993)	
S_ret_9_1	Price momentum t-9 to t-1	Underlying	Jegedeesh and Titman (1993)	Insignificant
S_rmax1_21d	Maximum daily return	Underlying	Bali Cakici and Whitelaw (2011)	Insignificant
S_rmax5_21d	Highest 5 days of return	Underlying	Bali, Brown, Murray and Tang (2017)	
S_rmax5_rvol_21d	Highest 5 days of return scaled by volatility	Underlying	Assness, Frazzini, Gormsen, Pedersen (2020)	
S_rskew_21d	Total skewness	Underlying	Bali Engle and Murray (2016)	
S_rvol_21d	Return volatility	Underlying	Ang et al. (2006)	Insignificant
S_sale_bev	Assets turnover	Underlying	Soliman (2008)	
S_sale_emp_gr1	Labor force efficiency	Underlying	Abarbanell and Bushee (1998)	Insignificant
S_sale_gr1	Sales Growth (1 year)	Underlying	Lakonishok Shleifer and Vishny (1994)	Insignificant
S_sale_gr3	Sales Growth (3 years)	Underlying	Lakonishok Shleifer and Vishny (1994)	Insignificant
S_sale_me	Sales-to-market	Underlying	Barbee Mukherji and Raines (1996)	Insignificant
S_saleq_gr1	Sales growth (1 quarter)	Underlying		Insignificant
S_saleq_su	Standardized Revenue surprise	Underlying	Jegadeesh and Livnat (2006)	Insignificant
S_seas_11_15an	Years 11-15 lagged returns, annual	Underlying	Heston and Sadka (2008)	Insignificant
S_seas_11_15na	Years 11-15 lagged returns, nonannual	Underlying	Heston and Sadka (2008)	Insignificant
S_seas_16_20an	Years 16-20 lagged returns, annual	Underlying	Heston and Sadka (2008)	Missingness
S_seas_16_20na	Years 16-20 lagged returns, nonannual	Underlying	Heston and Sadka (2008)	Missingness
S_seas_1_1an	Year 1-lagged return, annual	Underlying	Heston and Sadka (2008)	Insignificant
S_seas_1_1na	Year 1-lagged return, nonannual	Underlying	Heston and Sadka (2008)	
S_seas_2_5an	Years 2-5 lagged returns, annual	Underlying	Heston and Sadka (2008)	
S_seas_2_5na	Years 2-5 lagged returns, nonannual	Underlying	Heston and Sadka (2008)	Insignificant
S_seas_6_10an	Years 6-10 lagged returns, annual	Underlying	Heston and Sadka (2008)	Insignificant
S_seas_6_10na	Years 6-10 lagged returns, nonannual	Underlying	Heston and Sadka (2008)	
S_sti_gr1a	Change in short-term investments	Underlying	Richardson et al. (2005)	
S_taccruals_at	Total accruals	Underlying	Richardson et al. (2005)	Insignificant
S_taccruals_ni	Percent total accruals	Underlying	Hafzalla Lundholm and Van Winkle (2011)	Insignificant
S_tangibility	Asset tangibility	Underlying	Hahn and Lee (2009)	Insignificant
S_tax_gr1a	Tax expense surprise	Underlying	Thomas and Zhang (2011)	Insignificant
S_turnover_126d	Share turnover	Underlying	Datar Naik and Radcliffe (1998)	Insignificant
S_turnover_var_126d	Coefficient of variation for share turnover	Underlying	Chordia Subrahmanyam and Anshuman (2001)	
S_z_score	Altman Z-score	Underlying	Dichev (1998)	Insignificant
S_zero_trades_126d	Number of zero trades with turnover as tiebreaker (6 months)	Underlying	Liu (2006)	Insignificant
S_zero_trades_21d	Number of zero trades with turnover as tiebreaker (1 month)	Underlying	Liu (2006)	Insignificant
S_zero_trades_252d	Number of zero trades with turnover as tiebreaker (12 months)	Underlying	Liu (2006)	Insignificant

Done.

B Characteristics or Covariances

What is the sufficient number of latent factors K to drive out the explanatory power over returns that characteristics possess in excess of the systematic factor structure? To answer this question, we follow Kelly, Pruitt, and Su (2019) and estimate an unrestricted version of Eq. (11), which allows characteristics to directly influence expected returns:

$$r_{i,t+1}^{AC} = \alpha_{i,t} + \beta_{i,t} f_{t+1} + \varepsilon_{i,t+1}, \qquad \beta_{i,t} = z_{i,t}' \Gamma_\beta^{AC}; \quad \alpha_{i,t} = z_{i,t}' \Gamma_\alpha^{AC} \tag{B1}$$

We perform the bootstrap procedure outlined in Kelly, Pruitt, and Su (2019) to understand how many common factors are required for $\alpha_{i,t}$ to become jointly insignificant, i.e., for there to be no direct influence of characteristics on expected returns. In other words, we are looking for the smallest number of common factors, such that systematic variation drives out the explanatory power of asset-specific characteristics.

Table B2: Alpha Bootstrap p-values

The table shows the resulting p-values of the bootstrap detailed in Kelly, Pruitt, and Su (2019) to understand if the $\alpha_{i,t}$ in Eq. (B1) adds explanatory power to IPCA's systematic factor structure.

	K Factors			
	1	3	5	6
Bonds	0.00	0.00	0.22	0.55
Options	0.00	0.00	0.53	0.84
Stocks	0.58	0.26	0.89	0.67

Table B2 provides the p-values for Γ_α^{AC} for $AC \in$ [Bonds, Options, Stocks]. Five to six *joint* factors are sufficient to render all αs insignificant at the 5%-level. For stocks, three factors are sufficient for the p-value to exceed 5%.