Monetary Policy with Profit-Driven Inflation*

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Abstract

Following evidence on the role of firm profits in the current inflation surge, we develop a New Keynesian model where inflation arises because of the presence of reservation profits on the supply side. We use this framework to investigate the positive and normative implications of cost push shocks, focusing on energy price shocks. We first show that these shocks lead to inefficiently large supply contractions and thereby inefficiently large (profit-driven) inflation, as firms which retrench do not internalise the social costs of doing so. Second, we show that optimal monetary policy follows a pecking order. It first aims at shielding the supply side from the fallout of the shock, thereby undoing the negative retrenchment externality. It then splits the burden of the shock between supply and demand, when insulating the supply side is too costly. Finally, when the energy price shock is very large, monetary policy loses traction. Budget-neutral fiscal interventions, e.g. redistribution from high- to low-income households and/or from high- to low-profit firms, can then restore monetary policy effectiveness.

Keywords: Energy price shocks, price stickiness, reservation profits, optimal monetary policy, corporate tax.

JEL Classification codes: D21, E23, E31, E32, E52, E62, H24, H25.

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1 Introduction

On the surface, the current inflation surge bears many resemblances with the experience of the seventies (Walsh (2022)). In both instances, inflation experienced a sudden increase (see Figure 1 below), quickly reaching (close to) double digit levels, on the back of large adverse commodity shocks. In both cases, the original commodity shock quickly transmitted to good and service prices, generating broad based inflationary pressures (Pallara et al. (2023)). In both cases also, fiscal policy stepped in, to cushion the impact of the shock, possibly worsening the inflation problem (Beck-Friis and Clarida (2023)). Last but not least, the experience of the seventies has been flagged as a reminder of the cost of un-rooting inflation, short of early and decisive policy action (Kose et al. (2022)).[1]

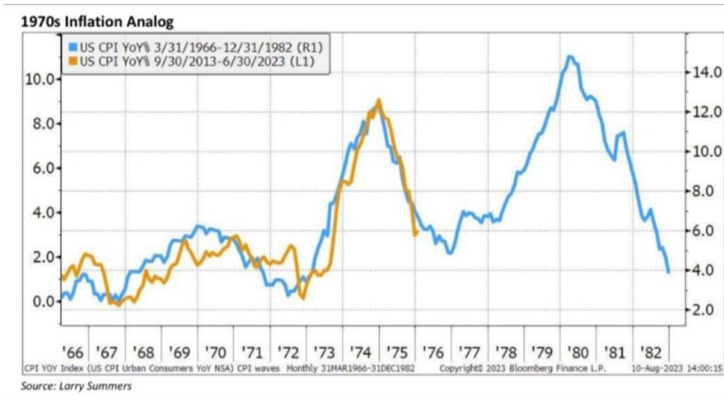


Figure 1: The current inflation burst looks very much like that of the seventies

In this paper, we argue that this comparison is ill-advised because the drivers of the recent inflation surge, have been fundamentally different from those in action back in the seventies. Chief among these differences —between the seventies and the recent period—, are firms' profits and their relationship with inflation. In a nutshell, while firm's profits behaved as a shock absorber back in the seventies, thereby limiting overall inflation, they have been more like an amplifying force, if not a source of inflation, altogether, in the recent inflation burst.

Back in the seventies, firms' profits were relatively low, especially during the second half of the decade, (when inflation has particularly high). After tax profits of non-financial

[1]Sluggish labour productivity is another common feature of the seventies and the current inflation burst.

firms for instance were roughly stable around 7.2% of value added for much of that period (Figure 2 below). And if cyclical variations did shift firm profits temporarily up and down, there was no clear trend at least until the end of the last century. However, things changed dramatically at the turn of the new millennium as firms' profits have since then followed an increasing trend, that the post-covid period only contributed to amplify.[2] To given an order of magnitude, firms' profitability —before tax— has increased by around 60% in about to decades from (10.0 to 16.0%) while over the same period, after tax firm profitability almost doubled (from 7.0 to almost 14.0%).

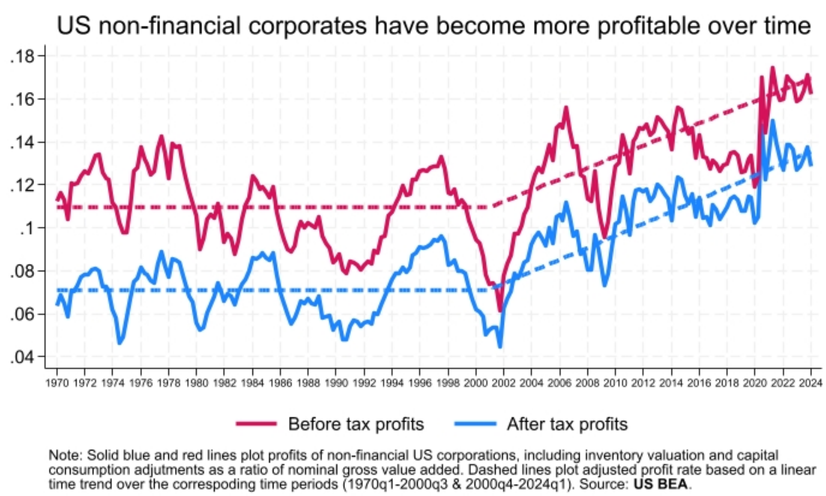


Figure 2: An inflation burst on the back of unprecedentedly high firm profitability

Mirroring the secular increase in firm profitability, the drivers of inflation have also changed accordingly. As we show with more details in the next section, if unit labour costs (ULC) were the main driver of inflation back in the seventies, more recently, unit profits (UP) have taken this role. The empirical evidence even points to UP having turned into a **leading** indicator of GDP inflation. We find evidence of this structural change for the United States, but the data also suggests a similar pattern in other countries like Canada or Germany. And as is clear, this evidence is fully consistent with corporate profits (i) now accounting for a larger share of gross domestic income and (ii) having increased significantly post-Covid.

[2] To give a sense of the increase in firms profits since the Covid crisis, after-tax profits of US non-financial corporate businesses reached about 2 trillion USD since mid-2021, compared with 1.1 trillion USD in 2019. see https://fred.stlouisfed.org/series/NFCPATAX.

Then building on these stylised facts, we first note that the standard New Keynesian workhorse model is unable to generate a positive correlation between profits and inflation as observed in the data, let alone turning profits into a driver of inflation.[3] This is because shocks that raise inflation typically reduce profits when firms face sticky prices, as prices do not increase in line with the marginal cost of production.[4] Moreover, in standard models, policy has no influence on firms' margins, as markups only depend on fundamental parameters. As a result, whether policy should be conducted differently when inflation is driven by costs or by profits, remains largely unexplored territory.

To address these shortcomings, we propose a model that builds on the New Keynesian framework, which we extend to allow for "profit-driven" inflation. We do so by introducing reservation profits on the supply side, so that firms operate only if they can break-even.[5] Shocks that reduce profits therefore contract the supply side, thereby giving rise to so-called "profit-driven" inflation. Using this framework, we investigate the impact of, and the optimal policy response to, cost push shocks, taking the example of energy price shocks.

In this model, energy price shocks are inflationary for three reasons. First, because energy is a consumption good. Second, because energy is a production input. Firms therefore charge higher prices, when energy is more expensive. Last, as higher energy prices eat into profits, some firms prefer to retrench. Incidentally, this reduces the number of consumption good varieties supplied on the market, raising the price of the composite consumption good, and leading to additional inflationary pressures.[6]

Based on this framework, we derive three main results. First, (positive) energy price

[3] This is true with the exception of markup shocks, which tend to increase both profits and inflation, although in a somewhat mechanical way. In addition, Bibliie and Kanzig (2023) shows that profits, even when pro-cyclical, cannot amplify aggregate demand shocks within the standard New Keynesian model.

[4] Note however that profits can be procyclical despite countercyclical markups when the number of producers/products is itself procyclical (see Bilbiie et al. (2014)). See also Nekarda and Ramey (2020) for an extensive discussion of the cylical properties of markups.

[5] We arbitrarily set reservation profits at zero to simplify the model's exposition. But this can be generalised with little implications for the main results described hereafter.

[6] There are several real-life interpretations to the firms' retrenchment effect. The literal one is firm exit: firms stop production and go out of business, at least temporarily, when they make losses. Another —more realistic— interpretation is that firms run their business at the product level, and only operate products, for which they can make enough profits. A last —more remote— interpretation of the model, is that firms that remain in operation are actually able to raise their prices

shocks lead to inefficiently large supply contractions, and thereby inefficiently high (profit-driven) inflation. The reason is as follows: Energy price shocks typically weigh on firms' profits, especially those of firms with sticky prices as these firms are unable to adjust the price of their output in line with the price of their inputs. As a consequence, some sticky price firms prefer to retrench, which ensures that those still operating are able to break-even. In the meantime, flexible price firms —which are able to set prices optimally— typically earn strictly positive profits. However, when sticky price firms retrench, they do so on the basis of private profits, while ignoring the implications for households' welfare. In particular, firms do not take into account the inflationary impact of their retrenchment decision on the price of the composite consumption good. As a result, positive energy price shocks lead to inefficiently large supply contractions, and thereby to inefficiently high "retrenchment- or profit-driven" inflation.

Second, in response to a positive energy price shock, we show that optimal monetary policy follows a pecking order. It first and foremost aims at insulating the supply side from the fallout of the energy price shock, which effectively neutralises the retrenchment inefficiency. This argument therefore goes against the standard view that monetary policy can, under some conditions, ignore —or look through— supply shocks. Rather, to the extent these shocks create inefficiencies —here on the supply side— the central bank should actually address them, with a view to limit, to the extent possible, their negative consequences.

To neutralise, or at least, limit the retrenchment inefficiency, the central bank needs to tighten policy, and the more so the larger the degree of price stickiness in the economy. When the central bank raises the interest rate, this dampens the increase in the price of energy. This also depresses aggregate demand, and thereby the wage rate. Both these forces reduce the marginal cost and hence the cost of the sticky price distortion. As a result, profits of sticky price firms fall by less and fewer firms need to retrench. As is clear, the larger the degree of price stickiness in the economy, the larger the benefits of a tight policy as a larger number of firms are subject to the risk of retrenchment.[7]

[7] The argument developed in this framework therefore goes against the standard view that monetary policy can ignore —or look through— supply shocks, insofar as inflation expectations remain anchored. Rather our framework highlights that to the extent supply shocks create inefficiencies on the supply side,

That said, insulating the supply side —from the fallout of the energy price shock— with high interest rates comes at the cost of depressed aggregate demand. Hence, when the energy price shock is large, so is the cost of insulating the supply side —in terms of aggregate demand compression. The optimal policy then splits the burden of the energy price shock, effectively letting some sticky price firms retrench, and generating some profit-driven inflationary pressures.

Third and last, policies that promote redistribution either on the demand side (within the household sector) or on the supply side (within the corporate sector) can help restore monetary policy effectiveness. The intuition is very simple. When the economy faces a very large energy price shock, the opportunity cost of keeping even a tiny number of sticky price firms in operation, becomes too large relative to the benefits. Monetary policy then has no alternative than to allow all sticky price firms to retrench. In the meantime, it looses any traction on the economy. Now when fiscal policy redistributes income towards constrained households, i.e. those who consume their current income, this typically lifts aggregate demand up, giving room for the central bank to tighten policy and bring some sticky price firms back in operation. Similarly, imposing a tax on profitable (flexible price) firms and extending a subsidy to unprofitable (sticky price) firms contributes to keep more sticky price firms in operation. In practise, the economy then functions as if price stickiness were more pervasive, which again gives more leeway for monetary policy to steer the economy in the appropriate direction.

1.1 The literature

Our paper relates to several strands of the literature. First, a number of studies and newspaper articles have looked into the drivers of the current inflation surge and more specifically at whether and to what extent high profits are responsible for high inflation, i.e., the question around the so-called ”greedflation”. Chassany (2023) for instance looks into measures of expected corporate profitability and shows they have increased significantly after the Covid crisis, for both US and European listed firms. Focusing on inflation in the Euro Area, Hansen et al. (2023) show that import prices and profits account together for

the central bank should actually respond to such shocks.

about 85% of consumer price inflation in the Euro Area, with wages only accounting for the remaining 15%. Similarly in a recent speech, Lagarde (2023) noted that profit margins have expanded in many sectors while real wages have taken a big hit as nominal wages failed to keep up with inflation, despite tight labour markets. In the case of the US, Glover et al. (2023) provide evidence that markup growth accounted for more than half of 2021 inflation. Specifically, markups grew by 3.4% in 2021, against inflation at 5.8%.[8] That said, the surge in corporate profits is by no means universal. For instance, Zemaityte and Walker (2023) finds no evidence of a rise in overall profits of UK companies.

Second, a theoretical and empirical literature investigates the impact of monetary policy when markups or firm entry are endogenous and, therefore, respond to different policy settings. In their first paper, Bilbiie et al. (2007) document how monetary policy transmits to the real economy in the presence of endogenous entry.[9] Closer to our paper, Bilbiie et al. (2012) shows that monetary policy should target long-run deflation when the equilibrium features too little entry, but long-run inflation when there is too much entry relative to the social optimum. From an empirical standpoint, Hartwig and Lieberknecht (2020) shows that expansionary monetary policy can propel net entry in the short-run. However, in the medium to long run, the least productive firms that could still operate thanks to the policy stimulus become unprofitable and exit overshoots. Our paper also relates to papers investigating how different competition environments affect monetary policy effectiveness (e.g. Wang and Werning (2022) or Duval et al. (2023)) as well as those looking into the supply-side effects of monetary policy (Baqaee et al. (2024)).

Third, our paper relates to the literature on wage price spirals. While this literature dating back to mid-eighties underscores the staggered nature of wage and price formation processes (Blanchard (1986)), more recent contributions have highlighted that such spirals are consistent with falling real wages (see Lorenzoni and Werning (2023)) as is currently the case, or the benefits of a cooperative approach to allocating income losses stemming from adverse energy price shocks (see Arce et al. (2023)).

[8] Considering the period 2021-2022, Gerinovics and Metelli (2023) come up with lower estimates, as markup growth accounted for about 25% of overall inflation, according to their calculations.

[9] Specifically, they show how the free-entry condition links the value of new products with inflation dynamics, through the marginal cost and the markup and also how the return to new product creation depends on the return on bonds, which in turn relates to monetary policy via interest rate setting.

Last, our paper contributes to the literature investigating the impact of energy shocks. As is the case in the seminal paper of Blanchard and Gali (2007a), energy price shocks do not imply in our model any trade-off for monetary policy as they tend to move the output and the inflation gaps in the same direction. Our paper therefore follows the course of Blanchard and Gali (2007b) whereby monetary policy faces a ”divine coincidence”. However, we differ in the implications for optimal monetary policy insofar as supply being endogenous, closing both gaps is hardly ever part of the optimal policy. More recent contributions have looked into the impact of energy price shocks in the context of heterogeneous agents models (Chan et al. (2022), Auclert et al. (2023) or Pieroni (2023)). But none includes an analysis of the supply side effects of such shocks, nor what this implies for monetary policy.

1.2 The road map of the paper

The rest of the paper proceeds as follows. The next section provides motivating evidence on the changing drivers of inflation. Section 3 turns to the model, laying out the main building blocks and deriving the equilibrium with flexible prices and free entry. The sticky price equilibrium is then described in section 4, while section 5 derives the general equilibrium under sticky prices and optimal monetary policy. Section 6 looks into alternative fiscal policy instruments and how they affect monetary policy effectiveness. Finally, conclusions are drawn in section 7.

2 The changing role of profits in inflation

In this section, we document some stylised facts that point towards the growing importance of profits for inflation. To this end, we draw on data from the OECD Quarterly National Accounts. Specifically, let nominal GDP, PY, write as the sum of nominal labour compensation wL, nominal capital compensation rK, and nominal taxes T, the so-called income approach to decomposing GDP: $PY = wL+rK+T$. Then, denoting ULC as $\ell_u = wL/Y$, UP as $k_u = rK/Y$, and unit taxes as $t_u = T/Y$ we can write GDP inflation as:

$$\frac{dP}{P} = \frac{wL}{PY}.\frac{d\ell_u}{\ell_u} + \frac{rK}{PY}.\frac{dk_u}{k_u} + \frac{T}{PY}.\frac{dt_u}{t_u} \tag{1}$$

The first term on the right-hand side of equation (1) represents the contribution of ULC to GDP inflation, while the second represents the contribution of UP to GDP inflation.[10,11] To fix ideas, when labour accounts for about two thirds of GDP (and capital for one third) and the economy is on a balanced growth path, then ULC should contribute about twice more than UP to GDP inflation. In practise though, things can be quite different.

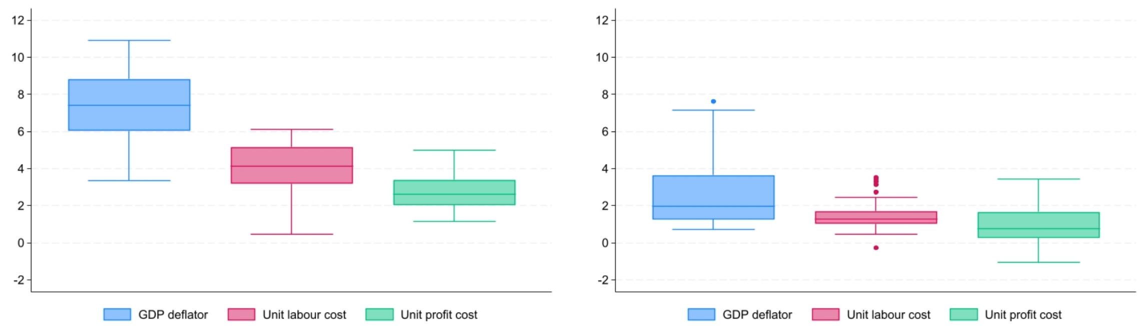
For instance, applying this decomposition to the experience of the US in the seventies shows that both ULC and UP contributed significantly to inflation, even if ULC were quantitatively speaking, more important (Panel (a) in Figure 3). Over this decade, median GDP inflation was about 7.5% while the median contributions of ULC and UP were respectively 4% and 3.5%.[12] By contrast, in the more recent period and especially since Covid, the contribution of UP inflation has become comparable to that of ULC inflation (Panel (b) in Figure 3). In addition, the contribution of UP inflation has become significantly more volatile, consistent with the view that UP inflation now accounts for a larger fraction of fluctuations in GDP inflation.

Second, in the seventies, GDP inflation and ULC inflation displayed a strong positive correlation. For example the correlation between current ULC inflation and subsequent GDP inflation used to be as high as 70% up to 2 quarters ahead (Panel (a) in Figure 4). In other words, ULC inflation used to be a leading indicator for GDP inflation. Similarly, the correlation between past inflation and current ULC inflation also used to be relatively strong, more than 50% up to 2 quarter lags, implying that GDP inflation was also, to some

[10] Hahn (2019) applies this decomposition to pre-covid Euro Area inflation, showing that the contribution of UP has generally been small relative to that of ULC. More recently, Benassy-Quéré (2023) estimates that by end-2022, more than half of Euro Area Inflation was due to UP inflation, while Mojon et al. (2023) uses the same decomposition to assess possible paths for disinflation.

[11] Note that we ignore in this exercise taxes, as well as the residual component of the GDP. Karabarbounis and Neiman (2018) provides an extensive discussion of the GDP residual component, in particular to what extent it can be attributed to extra-profits.

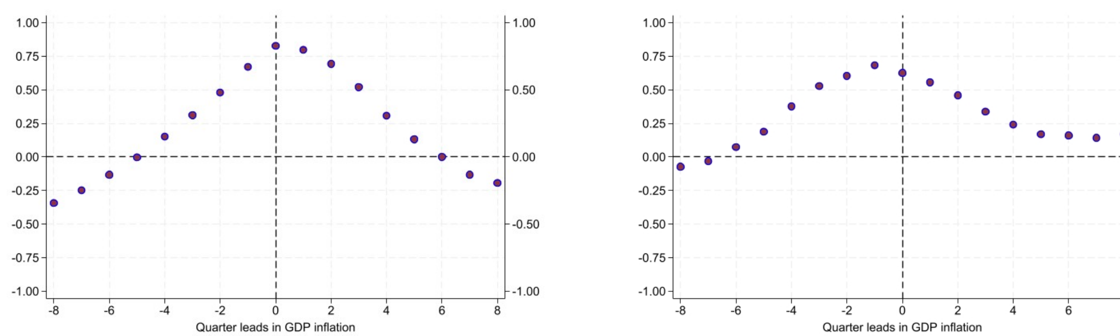
[12] Consistent with GDP inflation being mainly driven by ULC inflation, the variance of GDP inflation and the variance of the contribution of ULC inflation were comparable, while the variance of the contribution of UP inflation was more limited.



(a) : 1973q1-1982q4 (b) : 2014q1-2023q2

Figure 3: Unit labour costs have become less important for inflation. Figure 2 plots the distributions of y-o-y GDP inflation (in blue) and the contributions of ULC inflation (in pink) and UP inflation (in green) for the United States following decomposition (1), from 1973q1 to 1982q4 (Panel (a)) and from 2014q1 to 2023q2 (Panel (b)). GDP inflation is expressed in percent, contributions are expressed in percentage points. **Source**: OECD Quarterly National Accounts.

extent a leading indicator for ULC inflation. This two-way relationship between GDP and ULC inflation illustrates the so-called wage-price spiral, whereby higher ULC forces firms to raise prices, which in turn triggers demands for wage increases and cost of living adjustments.

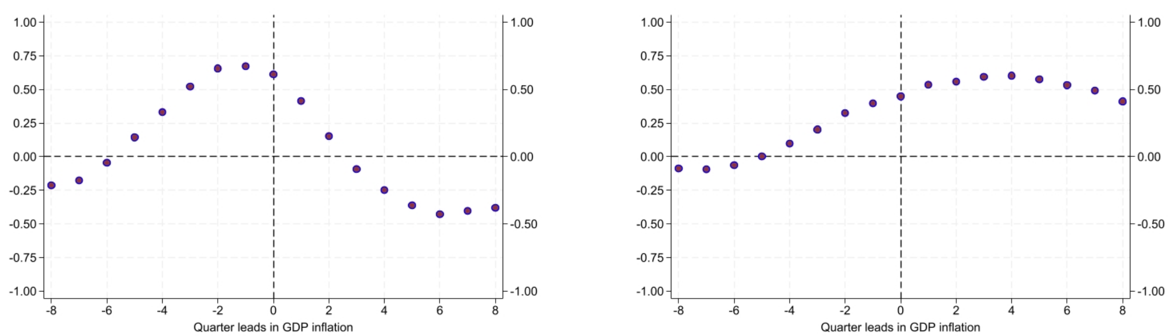


(a) : 1973q1-1982q4 (b) : 2014q1-2023q2

Figure 4: Unit labour costs used to be a leading indicator of GDP inflation. Figure 3 plots the lead/lag correlations between y-o-y ULC inflation and y-o-y GDP inflation for the United States from 1973q1 to 1982q4 (Panel (a)) and from 2014q1 to 2023q2 (Panel (b)). Dots to the left (to the right) of the vertical dashed line display correlations with lags (leads) of GDP inflation. **Source**: OECD Quarterly National Accounts.

Looking at the same correlations for the more recent period however shows some significant changes (Panel (b) in Figure 4). First, correlations between ULC inflation and past or future GDP inflation have all fallen relative to the seventies, suggesting that ULC inflation and GDP inflation matter less for each other. Second, the drop has been particularly pronounced for correlations between current ULC inflation and subsequent GDP inflation. For instance, at a 2-quarter ahead horizon, this correlation has fallen below 50% and has turned statistically insignificant from 3 quarters onward. To put it in a nutshell, this second piece of evidence suggests that ULC and hence wages are far less important for inflation than they used to be.

Third, running a similar type of exercise for UP also shows striking, but opposite, changes between the seventies and the more recent period. In the seventies, UP were essentially a lagging indicator of inflation, as the strong positive correlation between current UP inflation and lagged GDP inflation indicates (Panel (a) in Figure 5). Back in the seventies, higher inflation therefore signalled higher UP down the road, likely because inflation —which was primarily driven by increases in labour costs— would typically force at some point, firms to adjust prices upwards, in order to restore profits. Conversely, the correlation between current UP inflation and subsequent GDP inflation was, back in the seventies, basically zero, except over the very short-run (1 quarter ahead).



(a) : 1973q1-1982q4

(b) : 2014q1-2023q2

Figure 5: Unit profits have turned into a leading indicator of GDP inflation. Figure 4 plots the lead/lag correlations between y-o-y UP inflation and y-o-y GDP inflation for the United States for the period running from 1973q1 to 1982q4 (Panel (a)) and for the period running from 2014q1 to 2023q2 (Panel (b)). Dots to the left (to the right) of the vertical dashed line display correlations with lags (leads) of GDP inflation. **Source**: OECD Quarterly National Accounts.

These patterns have changed dramatically in the more recent period. Over the last decade, UP have essentially turned into a leading indicator of inflation, the correlation between current UP inflation and subsequent GDP inflation being positive and statistically significant up to 6 quarters ahead (Panel (b) in Figure 5). Interestingly computing these lead/lad correlations over a shorter time window (in order to focus on the recent post-Covid inflation surge) provides similar if not stronger results, in terms of the leading properties of UP inflation relative to GDP inflation. Consistently, Figure 9 in Appendix A.1, provides evidence, using rolling windows, that the correlation between current UP inflation and 1-year ahead inflation shoots up —reaching the highest levels ever over the last 50 years of data— right after the outbreak of the Covid pandemic. Comparing the seventies to the recent period, the conclusion in a nutshell, is therefore that UP have replaced ULC as a leading indicator of inflation.[13]

3 The model

The model builds on a simplified version of the New Keynesian framework, which we extend to incorporate reservation profits on the supply side. In this framework, heterogeneous households supply labour to firms and consume goods that firms produce under monopolistic competition.[14] In addition, firms, which produce distinct varieties of the consumption good, operate only if they can break-even, which affects the set of products that households can consume. The model focuses on the impact of inflationary shocks, taking the example of energy price shocks. To make these shocks relevant, we introduce energy on the demand side (as a consumption good for households) and on the supply side (as a production input for firms). Let us now describe the model's main assumptions more systematically, starting with households.

[13] Appendix A.2 presents similar evidence for Canada and Germany suggesting that UP have also been a leading indicator for GDP inflation, in each of these two countries over the recent period. The evidence for Germany is consistent with reports on the existence of a profit-price spirals (see Brzeski and Biehl (2023)). Other countries where UP have been leading inflation over the recent period, include Belgium, the Netherlands, Spain, Sweden or Switzerland.

[14] We restrict "fundamental" (ex ante) heterogeneity to the household sector. Conversely, on the firm side, we assume ex ante homogeneity. Yet ex post, firms will be heterogeneous as some firms will be able to reset their prices and some will not.

3.1 The demand side

3.1.1 Household preferences

Households live indefinitely. Each period, they consume energy —denoted E— and a composite consumption good —denoted C. In addition they are endowed each period with some quantity of energy and some quantity of labour —the latter being normalised to one for all households. Households' preferences U write as:

$$U = \sum_t \beta^t u\left(E_t; C_t\right) \text{ with } u\left(E_t; C_t\right) = \delta_h \ln E_t + (1 - \delta_h) \ln C_t \tag{2}$$

β is the rate at which households discount the future, and δ_h is the weight of energy in households' total consumption. The composite consumption good C, is in turn, a CES aggregation of the different varieties v (of the composite consumption good) produced by the different firms in the economy:

$$C_t = \left[\int_0^{N_t} \left[C_{vt}\right]^{1-\frac{1}{\eta}} dv \right]^{\frac{1}{1-\frac{1}{\eta}}} \tag{3}$$

N_t denotes the measure of consumption goods available at time t and $\eta > 1$ is the elasticity of substitution between the different consumption goods.[15] Let p_{et} denote the price of energy at time t and p_{vt} the price of consumption good variety v at time t. The price of the composite consumption good p_{ct} and the general price level P_t then satisfy:

$$p_{ct}^{1-\eta} = \int_0^{N_t} p_{vt}^{1-\eta} dv \quad \text{and} \quad \ln P_t = \delta_h \ln p_{et} + (1 - \delta_h) \ln p_{ct} \tag{4}$$

[15] The assumption $\eta > 1$ ensures that the elasticity of substitution between the different varieties of the consumption good is larger than the elasticity of substitution between the composite consumption good and energy.

3.1.2 Household demands for energy and consumption goods

Let D_t denote households' nominal expenditures at time t. Then households' demands for energy E_t^* and for the composite consumption good C_t^*, respectively write as:

$$p_{et}E_t^* = \delta_h D_t \quad \text{and} \quad p_{ct}C_t^* = (1-\delta_h)\, D_t \tag{5}$$

In addition, given the total amount C_t^* spent on consumption goods, the demand C_{st}^* for consumption good s satisfies:

$$C_{vt}^* = \left[\frac{p_{vt}}{p_{ct}}\right]^{-\eta} C_t^* = (1-\delta_h)\left[\frac{p_{vt}}{p_{ct}}\right]^{-\eta} \frac{D_t}{p_{ct}} \tag{6}$$

3.1.3 Household heterogeneity

There are two types of households, *constrained* and *unconstrained*. *Constrained* households are endowed every period with a quantity of energy e_c. They have no ownership rights over firms producing consumption goods, nor can they save nor borrow. Conversely, *unconstrained* households are endowed every period with a quantity of energy $e_u > e_c$, own the firms producing consumption goods and can freely lend and borrow. A fraction ϕ of households are constrained. Constrained and unconstrained households' expenditures, respectively denoted D_t^c and D_t^u, therefore write as:

$$D_{ct} = w_t + p_{et}e_c \quad \text{and} \quad D_{ut} = w_t + p_{et}e_u + \frac{1}{1-\phi}\int_0^{N_t} \pi_{vt} dv - [s_t - (1+i_{t-1})\, s_{t-1}] \tag{7}$$

The expenditures D_{ct} of an constrained households are simply the sum of labour income w_t and the value of the energy endowment $p_{et}e_c$. Similarly, the expenditures D_{ut} of an unconstrained households are the sum of labour income w_t, the value of the energy endowment $p_{et}e_u$ and the firms' profits minus net savings, π_{vt} denoting the profits of the firm producing variety v of the composite consumption good, and i_t the nominal interest rate on date-t savings s_t. Last, unconstrained households set their net savings

$s_t - (1 + i_{t-1}) s_{t-1}$, consistent with the standard Euler equation:

$$D_t^u = \frac{1}{\beta (1 + i_t)} D_{t+1}^u \tag{8}$$

3.2 The supply side

We now turn to the description of the supply side. Here, we make two key assumptions. First, we introduce energy as a production input and allow firms' energy demand to be non-homothetic. Second, we introduce reservation profits so that the measure N_t of operating firms will vary with demand and supply conditions. Let us in turn give more details on how both operate.

3.2.1 Energy fixed costs and firms' break-even condition

Firms produce consumption goods out of labour and energy, subject to an energy fixed cost. Specifically, output of variety v at time t, denoted y_{vt}, writes as:

$$\ln y_{vt} = \delta_f \ln \left(E_{vt} - e_f \right) + \left(1 - \delta_f \right) \ln L_{vt} \tag{9}$$

Here, E_{vt} denotes energy consumption, L_{vt} the labour hired from households and e_f the energy fixed cost. Finally δ_f is the share of energy in a firm's total variable inputs. Based on these notations, the profit π_{vt} of the firm producing variety v of the composite consumption good at time t is simply the difference between the value of the output $p_{vt} y_{vt}$ and the cost of inputs $(w_t L_{vt} + p_{et} E_{vt})$, in addition to the energy fixed cost $p_{et} e_f$:

$$\pi_{vt} = p_{vt} y_{vt} - w_t L_{vt} - p_{et} E_{vt} - p_{et} e_f \tag{10}$$

We then make the key assumption that firms producing consumption goods operate only if profits are non-negative. In other words, operating profits $p_{vt} y_{vt} - (w_t L_{vt} + p_{et} E_{vt})$ need to cover for the energy fixed cost $p_{et} e_f$.

3.2.2 Firms' choices

Working with backward induction, the demand for energy and the demand for labour from the firm producing variety v of the composite consumption good respectively write as:

$$p_{et}E_{vt} = p_{et}e_f + \delta_f mc_t y_{vt} \quad \text{and} \quad w_t L_{vt} = \left(1-\delta_f\right) mc_t y_{vt} \tag{11}$$

In expressions (11), mc_t denotes the marginal cost of production, and satisfies $\ln mc_t = \delta_f \ln p_{et} + \left(1-\delta_f\right)\ln w_t$. Then the firm producing variety v of the composite consumption good sets its price p_{vt} to maximise operating profits net of the energy fixed cost, output being demand-determined:

$$\begin{aligned} &\max_{p_{vt}} \quad \pi_{vt} = \left[p_{vt} - mc_t\right] y_{vt} - p_{et}e_f \\ &\text{s.t.} \begin{cases} y_{vt} = \left[\frac{p_{vt}}{p_{ct}}\right]^{-\frac{\mu}{\mu-1}} C_t^* \\ p_{ct}C_t^* = (1-\delta_h)\, D_t \quad \text{and} \quad D_t = \phi D_{ct} + \left(1-\phi\right) D_{ut} \end{cases} \end{aligned} \tag{12}$$

Firms then set their prices as a markup over the marginal cost: $p_{vt} = \mu mc_t$, where $\mu = \eta/(\eta-1)$. Then considering a symmetric equilibrium, the expression for firms' individual profits π_t simplifies as

$$\pi_t = (1-\delta_h)\,\frac{\mu-1}{\mu}\frac{D_t}{N_t} - p_{et}e_f \tag{13}$$

Expression (13) shows that firms' profits are decreasing, everything else equal, in the number of operating firms N_t. Intuitively, a larger number of operating firms means a smaller slice of the market for each individual and hence lower operating profits.

3.3 The free entry steady-state

In the free entry equilibrium, the break-even condition $\pi_t = 0$ determines the measure of operating firms N_t, as a function of household nominal demand D_t and the price of energy p_{et}. In addition, in the free entry equilibrium, households' aggregate net savings should be zero. As a result, household aggregate demand D_t is simply the sum of aggregate labour income w_t and the aggregate energy endowment, i.e. $D_t = w_t + e$, with $e = \phi e_c + (1-\phi)e_u$. Last, the wage rate w_t should clear the labour market. Based on firms' demand for labour

(11), the wage rate should satisfy $w_t = (1-\delta)D_t/\mu$, where $1-\delta = (1-\delta_h)(1-\delta_f)$. With these expressions at hand and denoting $\theta = \mu/(\phi(1-\delta))$, we can derive the following result.

Proposition 1 *Denoting p_e the steady-state price of energy, the measure of operating firms N, household aggregate demand D and the wage rate w satisfy at the steady state:*

$$N = \frac{\mu - 1}{(1-\delta_f)(\phi\theta - 1)} \frac{e}{e_f} \quad \text{and} \quad D = \frac{\phi\theta}{\phi\theta - 1} p_e e \quad \text{and} \quad w = \frac{1}{\phi\theta - 1} p_e e \tag{14}$$

Proof 1 *Substituting the expression for the wage rate into the expression for aggregate demand and applying the result into the expression for wages and the number of firms in operation yields expressions (14).*

In the free entry equilibrium, nominal demand D and the wage rate w both depend positively on households' total energy endowment e, while the number of operating firms N depends positively on households' energy endowment e, and negatively on firms' energy fixed cost e_f. Based on expressions (14), the price of the composite consumption good p_c and the level of consumption C of the composite good at the steady state satisfy respectively:

$$p_c = \frac{1}{N^{\mu-1}} \mu \left[w^{1-\delta_f} p_e^{\delta_f} \right] \quad \text{and} \quad C = (1-\delta_h) N^{\mu-1} \left[w/p_e \right]^{\delta_f} \tag{15}$$

A larger number of operating firms N therefore raises the level of consumption C and reduces the relative price of consumption goods. Moreover, the first expression in (15) shows that the relative price of consumption goods increases with households' energy endowment e if and only if $1-\delta_f \geq \mu - 1$. When this condition holds, the positive demand effect of a higher energy endowment e dominates the positive supply effect stemming from more products being offered on the market in response to increased demand. In what follows, we will assume that this condition holds so that higher demand always coincides with a higher relative price of consumption goods.[16]

[16] Another way to think about this condition is that it imposes that the elasticity of aggregate supply to households' income should be lower than the elasticity of aggregate demand to households' income.

4 The sticky price equilibrium

Let us now introduce some nominal rigidity and assume that some firms are unable to reset their prices every period. To look into what this implies for the functioning of the economy and policy making, we proceed as follows. We consider an economy at the steady state (as described above) that gets hit with an unexpected one-off shock that temporarily raises the global price of energy at time t from p_e to p^*_{et}. In the meantime, only a fraction n_f of the measure N of firms in operation can revise their prices and set them in line with the new price of energy. The other $N_s = (1 - n_f)N$ firms have to keep their prices at the previous period's level, i.e. at the steady state level. The case $n_f = 0$ therefore corresponds to the situation of full price rigidity and no firm can reset its price in light of the new cost conditions, while $n_f = 1$ is the case of full price flexibility. All firms can then reset their prices optimally.

4.1 Firms' profits under sticky prices

Firms facing an increase in the price of their inputs undergo a fall in their profits if they cannot raise the price of their output accordingly. As a result, sticky price firms i.e. those which cannot reset their prices, may run losses, in which case they would prefer to retrench, as they would not be able to break-even anymore.

Following the model's description above, the price charged by sticky price firms — which we denote p_{st}— is equal to the price charged at the steady state, i.e. $p_{st} = \mu mc$, mc denoting the marginal cost of production at the steady state. Conversely, the price charged by flexible price firms —which we denote p_{ft}— reflects the higher price of energy and hence the higher marginal cost of production, i.e. $p_{ft} = \mu mc_t$. Let us now denote m_t the ratio of the current to the steady state marginal cost of production, i.e. $m_t = mc_t/mc$ and d_t the ratio of current to steady state nominal demand, i.e. $d_t = D_t/D$. Then profits π_{ft} of flexible price firms and profits π_{st} of sticky price firms respectively write as:

$$\frac{\pi_{ft}}{p_e e_f} = \frac{d_t}{n_f + n_{st} m_t^{\frac{1}{\mu-1}}} - \frac{p_{et}}{p_e} \quad \text{and} \quad \frac{\pi_{st}}{p_e e_f} = \frac{\mu - m_t}{\mu - 1} \frac{m_t^{\frac{1}{\mu-1}} d_t}{n_f + n_{st} m_t^{\frac{1}{\mu-1}}} - \frac{p_{et}}{p_e} \tag{16}$$

Here n_{st} denotes the number of varieties produced by sticky price firms expressed as a ratio of the steady state number of varieties N. By construction, sticky price firms enjoy lower profits than flexible price firms, i.e. $\pi_{st} \leq \pi_{ft}$. As a consequence, when the energy price shock hits, sticky price firms are the first to run losses. And given that firms operate only if they break-even, a positive energy price shock may lead to a reduction in the number of varieties produced by sticky prices firms, hence the notation n_{st} in expressions (16), as the equilibrium number of varieties produced by sticky price firms may fall below the initial number. What determines n_{st}, and what this implies for policy making is the focus of the next sections.

4.2 Aggregate supply and the equilibrium number of varieties

When the global energy price shock hits, the relative number of varieties produced by sticky price firms falls from $1 - n_f$ to n_{st} so that the break-even constraint still holds, i.e. $\pi_{st} \geq 0$. Inverting this condition using expression (16) for the profits of sticky price firms, this yields:

$$n_{st} = \min\left\{1 - n_f; \left[\frac{\mu - m_t}{\mu - 1}\frac{d_t}{p_{et}/p_e} - n_f m_t^{-\frac{1}{\mu-1}}\right]^+\right\} \tag{17}$$

Expression (17) shows that the equilibrium measure of (varieties produced by) sticky price firms n_{st} increases with household aggregate demand d_t, as higher demand implies larger operating profits (provided the markup μ exceeds the marginal cost m_t). Expression (17) also shows that the impact of an increase in the marginal cost m_t can go both ways: A larger marginal cost typically reduces profits of sticky price firms as such firms are then further away from their profit maximising price. However, a higher marginal costs m_t also shifts demand towards sticky price firms, and the more so the larger the fraction n_f of flexible price firms. This demand reallocation effect then contributes to raise profits of sticky price firms. Finally, when some, but not all sticky price firms retrench, i.e. $0 < n_{st} < 1 - n_f$, profits of sticky price firms π_{st} are by definition zero, while profits of flexible price firms write as:

$$\pi_{ft} = \left[\frac{\mu - 1}{\mu - m_t} m_t^{-\frac{1}{\mu-1}} - 1\right] p_{et} e_f \tag{18}$$

As expression (18) shows, flexible price firms enjoy strictly positive profits whenever the current marginal cost deviates from its steady-state, i.e. when $m_t \neq 1$. A positive energy price shock that raises the marginal cost of production above its steady state level, therefore has opposite effects on sticky and flexible price firms. It shrinks the measure of the former, but it raises the profits of the latter, one being the flip side of the other.

4.3 Aggregate demand and the labour market

Having determined the relations governing the supply side of the economy, we can turn to the demand side, which will close the model. For this, we simply need to determine household aggregate demand D_t. Household demand depends on the equilibrium wage rate w_t, which in turn clears the labour market and equates the supply for labour from households and the demand for labour from firms. Labour supply is inelastic and equal to one, while labour demand L_t is the sum of labour demands from flexible and sticky price firms. Denoting ω_t the ratio of the current to the steady state nominal wage rate, i.e. $\omega_t = w_t/w$, the aggregate demand for labour L_t writes as:

$$L_t = \frac{m_t n_{st} p_{ft}^{\frac{1}{\mu-1}} + n_f p_{st}^{\frac{1}{\mu-1}}}{n_{st} p_{ft}^{\frac{1}{\mu-1}} + n_f p_{st}^{\frac{1}{\mu-1}}} \frac{d_t}{\omega_t} \tag{19}$$

According to expression (19), an energy price shock affects the demand for labour through three channels. First when energy becomes more expensive, firms substitute away from energy, thereby increasing the demand for labour. Second, higher energy prices lead flexible price firms to raise their prices relative to sticky price firms. This shifts demand towards sticky price firms, which contributes to raise the demand for labour, as sticky price firms typically sell more goods given their relatively low price. Thirdly however, higher energy prices lead some sticky price firms to retrench which contributes to reduce firms' aggregate demand for labour.

Using the expression for prices p_{st} and p_{ft} charged respectively by sticky and flexible price firms and expression (17) for the equilibrium number of sticky price firms n_{st}, and assuming $0 < n_{st} < 1 - n_f$, the expression for the equilibrium wage rate ω_t simplifies as:

$$\omega_t = m_t d_t - n_f (\mu - 1) \frac{p_{et}}{p_e} \frac{m_t - 1}{\mu - m_t} m_t^{-\frac{1}{\mu-1}} \tag{20}$$

Expression (20) shows that higher aggregate demand d_t is associated with a higher equilibrium wage rate ω_t. However a larger fraction of flexible price firms n_f tends to weigh on the wage rate (provided $1 < m_t < \mu$), as these firms tend to charge higher prices, sell fewer goods and hence have lower demand for inputs, including labour.

With the expression for the equilibrium wage rate ω_t at hand, we can now turn to aggregate demand D_t. Aggregate demand D_t is the sum of constrained and unconstrained households' demand: $D_t = \phi D_{ct} + \left(1 - \phi\right) D_{ut}$. Following expression (7), the former is simply the sum of labour income and the value of the energy endowment, $D_{ct} = w_t + p_{et} e_c$. Conversely, the latter is governed by the inter-temporal Euler equation (8). Given that the global price of energy is hit with an unexpected temporary shock, and given that UIP holds, the global and the local prices of energy p^*_{et} and p_{et} satisfy $\beta (1 + i_t) p_{et} = p^*_{et}$. Using this relationship, the Euler equation (8) governing unconstrained households' demand simplifies as

$$\frac{D^u_t}{p_{et}} = \frac{1}{p^*_{et}/p_e} \frac{D^u}{p_e} = \frac{1}{1 + \varepsilon_t} \frac{D^u}{p_e} \quad \text{with} \quad \frac{D^u}{p_e} = \frac{e}{\phi\theta - 1} + e_u \tag{21}$$

Here ε_t is the shock to global energy prices, i.e. $p^*_{et}/p_e = 1 + \varepsilon_t$. Expression (21) shows that an increase in global energy prices cuts demand from unconstrained households (expressed in energy units). This is because unconstrained households differ consumption into the future, as the temporary one-off shock to global energy prices implies an expected disinflation (in energy prices), and hence a higher expected real interest rate.

Last, using expression (20) for the equilibrium wage rate and expression (21) for unconstrained households' demand, total nominal demand in the economy writes as:

$$\theta d_t = \omega_t + \frac{p_{et}}{p_e} \frac{Z(\varepsilon_t)}{1 + \varepsilon_t} \quad \text{with} \quad Z(\varepsilon_t) = \theta - 1 + (\phi\theta - 1) \frac{e_c}{e} \varepsilon_t \tag{22}$$

Expression (22) summarises the conflicting effects of energy price shocks on demand from constrained vs. unconstrained households. As noted above, energy price shocks typically lead unconstrained households to cut on consumption, thereby dragging down aggregate demand. However, energy price shocks typically lead constrained households to increase consumption. This is because households —constrained and unconstrained— tend to substitute away from energy into consumption goods while firms tend to substitute away from energy into labour. Both these forces contribute to increase firms' demand for labour, and hence the equilibrium wage rate. Overall, the net balance of these two effects will depend on the relative weight ϕ of constrained vs. unconstrained households as well as on constrained households' relative energy endowment e_c/e. The next section which solves for the general equilibrium provides the specific conditions under which energy prices and household aggregate demand move in similar or opposite directions.

5 General equilibrium under sticky prices

To understand how the economy functions in general equilibrium and how policy should be conducted, we proceed in two steps. We first consider the case where all firms face sticky prices. By removing firm ex post heterogeneity, we make it easier to understand the main channels through which energy price shocks propagate through the economy, the possible inefficiencies that may arise as a result of such shocks and the possible mitigating role for policy, if any. In a second step, we come back to the general case with sticky and flexible price firms, deriving the main positive and normative implications of energy price shocks, as a first-order approximation relative to the case of universal sticky prices.

5.1 The case of full price rigidity

When all firms face sticky prices, energy price shocks affect inflation through the direct channel of energy as a consumption good, and through the retrenchment channel, the so-called extensive margin, as a higher marginal cost of production leads some sticky price firms to retrench. However, the usual cost-driven channel, the so-called intensive margin channel, by which a higher marginal cost of production leads to higher prices, does not show up as the assumption of universal sticky prices implies zero pass-through from marginal cost to individual prices. Moreover, with sticky price firms only, firms' profits are always zero, as sticky price firms break-even in equilibrium. Hence by construction, the framework with sticky price firms only, cannot generate any observable correlation between inflation and profits, even if some inflation stems from firms retrenching to meet the break-even constraint.[17]

5.1.1 Aggregate demand and aggregate supply

When there are only sticky price firms, i.e. $n_f = 0$, household aggregate demand d_t and the marginal cost m_t respectively write as separate functions of the primitives of the model, in particular the energy price shock ε_t and the nominal interest rate i_t. Specifically, household aggregate demand d_t satisfies:

$$\theta d_t = \frac{Z\left(\varepsilon_t\right) + (1+\varepsilon_t)d_t^{\frac{1}{\delta_f}}}{\beta(1+i_t)} \tag{23}$$

Considering small deviations from the steady state, expression (23) shows that household nominal demand typically decreases with the nominal interest rate i_t, but increases with the shock to global energy prices ε_t when $\theta\delta_f > 1$, which we assume will hold in what follows. Turning to the marginal cost of production m_t, it satisfies:

$$m_t^{\delta_f}(\theta - m_t)^{1-\delta_f} = \frac{Z(\varepsilon_t)^{1-\delta_f}(1+\varepsilon_t)^{\delta_f}}{\beta(1+i_t)} \tag{24}$$

Again, based on a first-order approximation around the steady state, expression (24) shows that the nominal marginal cost of production m_t increases with the energy shock ε_t and decreases with the nominal interest rate i_t. Then using these two expressions, one can easily re-write the equilibrium number of operating firms n_{st} as:

$$n_{st} = \min\left\{1; \left[\frac{1}{\mu-1}\frac{\mu - m_t}{\theta - m_t}\frac{Z(\varepsilon_t)}{1+\varepsilon_t}\right]^+\right\} \tag{25}$$

[17] Paradoxically, this framework gives rise to profit-driven inflation because firms faced with a fall in their profits, prefer to retrench, which leads to additional inflationary pressures. Put differently, the higher the reservation profits, the stronger the retrenchment effect and the higher the profit-driven inflation rate, everything else equal.

It then straightforward to note that the number of firms in operation n_{st} decreases with (positive) energy price shocks, but increases with the nominal interest rate. Following expression (24), a higher nominal interest rate reduces the marginal cost of production m_t. Then, given that $\theta > \mu$, a lower marginal cost unambiguously raises the number of firms in operation.

Based on these expressions, and assuming $0 < n_{st} < 1 - n_f$, the inflation gap and the output gap — which we write as the ratio of household demand expressed in energy units $d_t/(p_{et}/p_e)$, to the number of firms in operation n_{st}— respectively writes as:

$$p_t = \left[\frac{1+\varepsilon_t}{\beta(1+i_t)}\right]^{\delta_h} \left[(\mu-1)\frac{1+\varepsilon_t}{Z(\varepsilon_t)}\frac{\theta - m_t}{\mu - m_t}\right]^{(\mu-1)(1-\delta_h)} \quad \text{and} \quad y_t = \frac{d_t}{p_{et}/p_e}\frac{1}{n_{st}} = \frac{\mu-1}{\mu-m_t} \tag{26}$$

As is clear from expressions in (26), an energy price shock that raises the marginal cost of production m_t always opens a positive inflation gap and a positive output gap. While the inflationary impact of energy price shocks is straightforward, the impact on the output gap deserves some discussion. On the one hand, higher energy prices lead to a contraction in aggregate supply, i.e. to fewer firms in operation n_{st}, which contributes to open a positive output gap. On the other hand, higher energy prices reduce demand expressed in energy units, which contributes on the contrary to open a negative output gap. However, under full price rigidity, energy price shocks necessarily have a relatively small impact on aggregate demand. While unconstrained households' demand typically drops one-to-one with energy price shocks (in line with the increase in the expected real interest rate), demand from constrained households actually increases following a positive energy price shock. This is because the "real" wage rate w_t/p_{et} goes up as the energy price shock leads firms to switch from energy to labour and consumers to switch from energy to consumption goods. Hence, unless households are all unconstrained, aggregate demand tend to shrink less than one-to-one, following an energy price shock.

More generally, aggregate demand is less sensitive to the energy price shock when the fraction ϕ of constrained households is higher. Conversely on the supply side, given that all firms face sticky prices, energy price shocks raise the fixed cost of production and reduce operating profits for all firms. As a result, energy price shocks tend to have an large impact on the supply side and thereby lead many firms to retrench. This is why energy price shocks tend to cut supply relative to demand and open a positive output gap. Hence, absent any policy response, an energy price shock typically induces a positive correlation between the output and the inflation gaps.[18] This however may change when we factor in the policy response. We look at this in more detail in the following paragraph.

5.1.2 Optimal policy

Let us now consider a central bank that sets the nominal interest rate i_t to maximise household aggregate real income $\mathcal{R}$ in response to an energy price shock ε_t. The problem for the central bank writes as:

$$\max_{i_t} \quad \mathcal{R}(i_t) = d_t/p_t$$

$$\text{s.t.} \begin{cases} \theta d_t = \frac{1+\varepsilon_t}{\beta(1+i_t)} \left[\frac{Z(\varepsilon_t)}{1+\varepsilon_t} + d_t^{\frac{1}{\delta_f}} \right] \\ p_t = \left[\frac{1+\varepsilon_t}{\beta(1+i_t)} \right]^{\delta_h} \left[\frac{1+\varepsilon_t}{Z(\varepsilon_t)} \frac{\theta-m_t}{\mu-m_t} \right]^{(\mu-1)(1-\delta_h)} \\ m_t^{\delta_f} (\theta - m_t)^{1-\delta_f} = \frac{(1+\varepsilon_t)^{\delta_f} [Z(\varepsilon_t)]^{1-\delta_f}}{\beta(1+i_t)} \end{cases}$$

The central bank maximises household real income subject to three constraints: the aggregate demand equation, the general price level equation and the aggregate supply equation. Solving this problem, we can then derive the following result.

Proposition 2 *Under full price rigidity, the optimal policy response to an energy price shock ε_t consists in setting the nominal interest rate i_t such that*

$$\beta(1+i_t) = \frac{(1+\varepsilon_t)^{\delta_f} [Z(\varepsilon_t)]^{1-\delta_f}}{m_0^{\delta_f} (\theta - m_0)^{1-\delta_f}} \tag{27}$$

with $m_0 = \max \left\{ m_0(\varepsilon_t); m_0^* \right\}$, $m_0(\varepsilon_t)$ being such that all firms remain in operation, i.e. n_{st}=1,

[18] Note that this positive correlation between the output and inflation gaps still comes with a negative correlation between output and inflation. Positive energy price shocks still act as standard negative supply shocks, cutting quantities and raising prices.

and m_0^* satisfying:

$$\frac{(\mu-1)(\theta-\mu)}{\mu-m_0^*}=\frac{\theta\delta_f}{m_0^*}+\frac{\delta_h}{1-\delta_h} \tag{28}$$

Proof 2 *cf. appendix A.4*

The trade-off for the central bank setting the nominal interest rate is as follows: On the negative side, a high nominal interest rate reduces household demand. On the positive side, a high nominal interest rate reduces domestic energy prices and lowers the marginal cost of production, which reduces inflation directly, energy being a consumption good, and indirectly, by keeping more firms in operation.

On its own, and absent the latter indirect effect, the benefits of high interest rates in reducing domestic energy prices are not enough to compensate for the cost in terms of lower household demand. This is why the central bank sets the interest rate such that all firms remain in operation when the energy price shock is relatively small (lower region in Figure 6 below). In this case, the optimal policy targets a marginal cost of production which ensures that all firms can break-even despite the energy price shock. In other words, optimal policy insulates the supply side of the economy from the fall out of the energy shock and puts all the burden of the adjustment on the demand side.

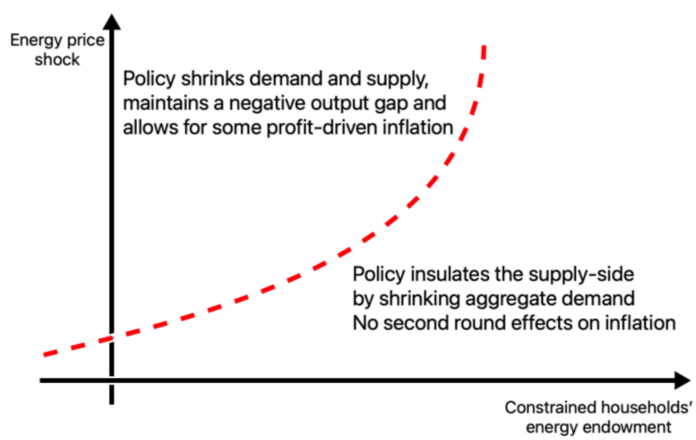


Figure 6: Optimal monetary policy under full price rigidity.

However, things turn different when the energy price shock is large. In this case, keeping all firms in operation requires a large cut in the marginal cost, which in turn needs a larger compression in household demand. Because this would be too costly for household real income, the optimal policy then shares the burden of the shock between supply and demand. Specifically, under the optimal policy, energy price shocks have the same proportional negative impact on aggregate supply and aggregate demand. According to Proposition 2, the number of operating firms and household demand expressed in energy units, then satisfy respectively:

$$n_{st} = \frac{1}{\mu - 1} \frac{\mu - m_0^*}{\theta - m_0^*} \frac{Z(\varepsilon_t)}{1 + \varepsilon_t} \quad \text{and} \quad \frac{d_t}{p_{et}/p_e} = \frac{1}{\theta - m_0^*} \frac{Z(\varepsilon_t)}{1 + \varepsilon_t} \tag{29}$$

In other words, supply and demand equally share the cost of energy price shocks (upper region in Figure 6 above). One can indeed easily check based on expressions (29) that both the number of firms in operation and household demand expressed in energy units are below their steady state values under the optimal policy, i.e. $n_{st} < 1$ and $d_t/(p_{et}/p_e) < 1$. In addition, the share the same negative elasticity w.r.t to the energy price shock ε_t.

Irrespective of the energy price shock hitting the economy being large or small, the optimal policy always maintains a negative output gap and a positive inflation gap.[19] When the shock is small and the optimal policy keeps all firms in operation, it does so by compressing aggregate demand, which implies a negative output gap. Similarly, when the shock is large and the optimal policy splits the cost of the energy price shock between supply and demand, expressions (29) show that aggregate demand always falls short of aggregate supply, hence again the negative output gap (remember that $m_0^* < 1$). Moreover, optimal monetary policy also keeps the inflation gap positive as it does not stabilise the price of energy in local currency, as is visible in expression (27). As a consequence, even in the case where all firms remain in operation, inflation is positive under the optimal policy. And this is even more true when the optimal policy allows for some firm retrenchment. Hence, in contrast to the case where policy is muted, under the optimal policy, a positive energy price shock induces a *negative* correlation between the output and the inflation gaps.

[19]Formally, the optimal policy always induces a positive inflation gap, provided $\mu\delta_f \leq 1$, which is likely to hold under any reasonable set of parameters. For instance, considering values for the markup around 1.2-1.3, the condition $\mu\delta_f \leq 1$ holds as long as the share of energy in firms' variable inputs does not exceed 75%. In practise this share barely reaches 10-15%.

The reason why the central bank maintains a negative output gap under the optimal policy is clear. Firms decide to remain in operation or to retrench on the basis of their private profits. As a consequence, they do not take into account the impact of their collective retrenchment decision on the price of the composite consumption good, and the resulting negative external effect on households' real income. Energy price shocks therefore lead too many firms to retrench relative to the social optimum. Monetary policy then corrects for this inefficiency by compressing aggregate demand and maintaining a negative output gap, as this is the only way to curtail firm incentives for retrenchment.[20] To put it differently, optimal monetary policy follows a pecking order: It first and foremost aims at fixing the retrenchment inefficiency on the supply side through tight policy. Second, when the cost of insulating the supply side becomes too large, it then splits the cost of energy price shock between supply and demand, allowing some firms to retrench.

Note also that the central bank maintains a positive inflation gap because there are no benefits to raising the interest rate beyond levels that shield the supply side from the fallout of the energy price shock (in the case of small shocks), or beyond the level that equally splits the burden of the shock between supply and demand (in the case of large shocks). Given that demand expressed in energy units already falls following an energy price shock, raising the interest rate all the way up to the point where it would stabilise domestic energy prices, would be counter-productive.

Finally, the energy price shock below which optimal policy keeps all firms in operation and insulates the supply side, is increasing in the energy endowment of the constrained households (red dotted schedule in Figure 6). When constrained households' energy endowment e_c is relatively large, an energy price shock has a relatively small negative impact on household demand as constrained households who consume current income benefit from a large windfall on their income. In the meantime, the energy price shock has a relatively large impact on the number of operating firms as high energy prices and resilient household demand raise the marginal cost of production. As a consequence, the set of shocks under which optimal monetary policy puts all the adjustment on the

[20] Obviously, the social planner could also address this inefficiency with additional tools, e.g. taxes and subsidies to firms. Section 6 looks into this possibility.

demand side and shields the supply side is relatively larger. Conversely, when the energy endowment of constrained households e_c is relatively low, an energy price shock produces a relatively large reduction in household demand as unconstrained households, who then account for the bulk of aggregate demand, respond to higher energy prices, by cutting consumption. Given the relatively large fall in household demand, the set of energy price shocks for which the optimal policy puts all the burden of the adjustment on the demand side and shields the supply side is more limited.

5.2 The general case of partial price rigidity

5.2.1 The positive analysis

Let us now come back to the general case where firms can have either flexible or sticky prices. The economy can then be described with two equations. First, the aggregate supply equation expresses the marginal cost of production m_t as a function of the energy shock ε_t and the nominal interest rate i_t:

$$\frac{\theta - m_t}{m_t}\left[\frac{\beta\,(1+i_t)}{1+\varepsilon_t}m_t\right]^{\frac{1}{1-\delta_f}} + \theta n_f\,(\mu-1)\,\frac{m_t-1}{\mu-m_t}m_t^{-\frac{\mu}{\mu-1}} = \frac{Z\,(\varepsilon_t)}{1+\varepsilon_t} \tag{30}$$

Equation (30) shows that a positive energy shock ε_t is always associated with a higher marginal cost of production m_t. Similarly, a higher nominal interest rate i_t is always associated with a lower marginal cost of production m_t. That said in both cases, a larger the fraction n_f of flexible price firms tends to weaken the response of the marginal cost m_t either to an energy price shock or to an interest rate shock.

Second, the aggregate demand equation expresses household nominal demand d_t as a function of the energy price shock ε_t, the nominal interest rate i_t, and the marginal cost of production m_t:

$$\theta d_t = \frac{1+\varepsilon_t}{\beta(1+i_t)}\left[\frac{Z\,(\varepsilon_t)}{1+\varepsilon_t} + \left[\frac{\beta\,(1+i_t)}{1+\varepsilon_t}m_t\right]^{\frac{1}{1-\delta_f}}\right] \tag{31}$$

As in the case for the marginal cost of production, household nominal demand d_t always decreases with the nominal interest rate i_t, provided $\theta\delta_f \geq 1$. In addition, a positive energy price shock raises household nominal demand when $\mu n_f \leq 1$, a condition we will assume to hold in what follows.[21]

To understand how the model with flexible price firms differs from the model with sticky price firms only, it is useful to consider the expression for the number of sticky price firms n_{st} at the equilibrium. Specifically, based on expression (30) for the marginal cost m_t and expression (31) for household nominal demand d_t, expression (17) for the equilibrium number of sticky price firms n_{st} simplifies as:

$$n_{st} = \min\left\{1 - n_f; \left[\frac{1}{\mu - 1}\frac{\mu - m_t}{\theta - m_t}\frac{Z(\varepsilon_t)}{1 + \varepsilon_t} - n_f\frac{\theta - 1}{\theta - m_t}m_t^{-\frac{1}{\mu-1}}\right]^+\right\} \tag{32}$$

The equilibrium number of sticky price firms n_{st} then satisfies two properties. First, as was the case under full price rigidity, it decreases with energy price shocks and increases with the nominal interest rate, as higher energy prices (or lower nominal interest rate) raise the marginal cost of production, and thereby reduce the number of sticky price firms that can be sustained on the market. Secondly however, a larger number of flexible price firms n_f tends to dampen the impact of energy price shocks or the nominal interest rate on the equilibrium number of sticky price firms. This dampening effect operates through two channels. One is that, as noted above, a larger number of flexible price firms n_f reduces the sensitivity of the marginal cost to energy prices or to the nominal interest rate. Another is that a larger number of flexible price firms implies a larger shift in market shares in favour of sticky price firms following an energy price (or an interest rate) shock. As a consequence, the profits of sticky price firms fall by less and the retrenchment effect is dampened.

With sticky and flexible price firms, inflation p_t and the output gap y_t —defined as the ratio of household demand in energy units to the weighted sum of sticky and flexible price

[21] As was the case for the condition under which optimal policy under full price rigidity always opens a positive inflation gap, this condition is also likely to be satisfied under any reasonable set of parameters. For instance, considering values for the markup around 1.2-1.3, the condition $\mu n_f \leq 1$ holds as long as the fraction of flexible price firms does not exceed 75%. If one assumes that prices are fixed for about 2 quarters, this implies a fraction of flexible price firms around 50%, therefore much below the 75% upper bound.

firms— respectively write as:

$$p_t = \left[\frac{p_{et}}{p_e}\right]^{\delta_h} \left[n_{st} + n_f m_t^{-\frac{1}{\mu-1}}\right]^{-(\mu-1)(1-\delta_h)} \quad \text{and} \quad y_t = \frac{d_t/(p_{et}/p_e)}{n_{st} + n_f m_t^{-\frac{1}{\mu-1}}} \tag{33}$$

As was the case with sticky price firms only, a positive energy price shock always opens a positive inflation gap. In addition, using expression (17) for the equilibrium number n_{st} of sticky price firms, and provided $0 < n_{st} < 1 - n_f$, the expression for the output gap simplifies as $y_t = \frac{\mu-1}{\mu-m_t}$, implying that a positive energy price shock also open a positive output gap. However, consistent with above remarks, both gaps decrease in the fraction of flexible price firms n_f, as the number of sticky price firms n_{st} and the marginal cost m_t are both less sensitive to energy price shocks, when the fraction n_f of flexible price firms is larger.[22] Having detailed how the supply and the demand sides of the economy respond to energy price shocks, we can now turn to the optimal policy response.

5.2.2 Optimal policy

Let us now consider a central bank that sets the nominal interest rate i_t to maximise household aggregate real income $\mathcal{R}$. Then the problem for the central bank writes as:

$$\max_{i_t} \quad \mathcal{R}(i_t) = d_t/p_t$$

$$\text{s.t.} \begin{cases} \theta d_t = \frac{1+\varepsilon_t}{\beta(1+i_t)} \left[\frac{Z(\varepsilon_t)}{1+\varepsilon_t} + \left[\frac{\beta(1+i_t)}{1+\varepsilon_t} m_t\right]^{\frac{1}{1-\delta_f}}\right] \\ p_t = \left[\frac{1+\varepsilon_t}{\beta(1+i_t)}\right]^{\delta_h} \left[\frac{\mu-1}{\mu-m_t} \frac{1+\varepsilon_t}{\beta(1+i_t)} \frac{1}{d_t}\right]^{(\mu-1)(1-\delta_h)} \\ \frac{\theta-m_t}{m_t} \left[\frac{\beta(1+i_t)}{1+\varepsilon_t} m_t\right]^{\frac{1}{1-\delta_f}} + \theta n_f (\mu-1) \frac{m_t-1}{\mu-m_t} m_t^{-\frac{\mu}{\mu-1}} = \frac{Z(\varepsilon_t)}{1+\varepsilon_t} \end{cases}$$

As was the case under full price rigidity, the problem for the central bank consists in maximising household real income subject to the aggregate demand equation, the general price level equation and the aggregate supply equation. Each however differs from the case of full price rigidity. First, as noted above, the response of the marginal cost m_t to

[22] Pushing the argument to the limit, when there are only flexible price firms, an energy price shock does not move the output gap as prices then all adjust one-to-one to the new price of energy, leaving quantities unchanged and the output gap closed.

changes in the nominal interest rate i_t now depends on the degree of price flexibility n_f in the economy. Second, under partial price rigidity, aggregate demand writes as a function of the marginal cost of production m_t. How the supply side of the economy reacts to monetary policy therefore determines, at least partially, the response of the demand side. Last, under partial price rigidity, the general price level depends positively on the marginal cost — through the intensive and extensive margin channels—, but also negatively on aggregate demand, as higher demand helps, everything else equal, maintain a larger number of sticky price firms in operation, thereby pushing down the general price level. Based on these remarks, we can then derive the following result.

Proposition 3 *Provided the number of sticky price firms n_{st} satisfies $0 < n_{st} < 1 - n_f$, the optimal interest rate policy satisfies:*

$$\frac{\beta\left(1+i_t\right)}{1+\varepsilon_t} = \frac{1}{(m_t^*)^{\delta_f}(\theta - m_t^*)^{1-\delta_f}}\left[\frac{Z\left(\varepsilon_t\right)}{1+\varepsilon_t} - \theta n_f(\mu-1)\frac{m_t^*-1}{\mu - m_t^*}(m_t^*)^{-\frac{\mu}{\mu-1}}\right]^{1-\delta_f} \tag{34}$$

where the marginal cost of production m_t^* writes, up a first-order approximation, as:

$$m_t^* = m_0^* + n_f F(m_0^*)\frac{1+\varepsilon_t}{Z\left(\varepsilon_t\right)} \quad \text{with} \quad F\left(m_0^*\right) > 0 \tag{35}$$

Proof 3 *c.f. appendix*

Before discussing the main properties of the optimal policy as described in proposition 3, let us first note that this policy holds insofar as the equilibrium number of sticky price firms n_{st} is positive, but does not exceed the total number of sticky price firms in the economy, i.e. $0 < n_{st} < 1 - n_f$. Starting with the latter constraint, the equilibrium number of sticky price firms does not exceed the total number of sticky price firms in the economy when:

$$\frac{\mu-1}{\mu - m_t^*}\left[n_f(m_t^*)^{-\frac{1}{\mu-1}} + (1-n_f)\frac{\theta - m_t^*}{\theta-1}\right] \geq \frac{1}{\theta-1}\frac{Z(\varepsilon_t)}{1+\varepsilon_t} \tag{36}$$

Provided the fraction n_f of flexible price firms in the economy is not too large, the term on the left-hand side of condition (36) is increasing in the marginal cost m_t^*. Hence there exists ε_{min} such that there are fewer sticky price firms in equilibrium than in the economy, if and only if the energy price shock satisfies $\varepsilon_t \geq \varepsilon_{min}$. Conversely, whenever the energy price shock ε_t is sufficiently low, i.e. $\varepsilon_t \leq \varepsilon_{min}$, the optimal policy as described in proposition 3 would imply more sticky price firms than there are in the economy, which is not possible. The optimal policy therefore keeps all sticky price firms in operation when the energy price shock is relatively small, i.e. when $\varepsilon_t \leq \varepsilon_{min}$. Turning to the former constraint, the equilibrium number of sticky price firms is non-negative when:

$$n_f \frac{\mu - 1}{\mu - m_t^*}(m_t^*)^{-\frac{1}{\mu-1}} \leq \frac{1}{\theta - 1}\frac{Z(\varepsilon_t)}{1 + \varepsilon_t} \tag{37}$$

Again, provided the fraction n_f of flexible price firms is not too large, the term on the left-hand side of condition (37) is increasing in the marginal cost of production m_t^*. As a result, there exists ε_{max} such that the equilibrium number of sticky price firms in non-negative if and only if the energy price shock satisfies $\varepsilon_t \leq \varepsilon_{max}$. Conversely, whenever the energy price shock ε_t is sufficiently large, i.e. $\varepsilon_t \geq \varepsilon_{max}$, then the optimal policy as described in proposition 3 would imply a negative number of sticky price firms, which is not possible. The optimal policy therefore allows all sticky price firms to retrench when $\varepsilon_t \geq \varepsilon_{max}$.

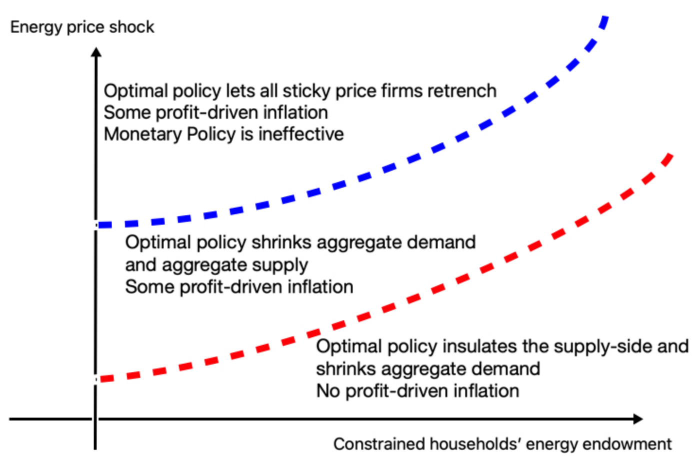


Figure 7: Optimal monetary policy under partial price rigidity.

To wrap up, the optimal policy under partial price rigidity can be described with the following taxonomy:

- First, when the economy faces small energy price shocks, i.e. $\varepsilon_t \leq \varepsilon_{min}$, the optimal policy insulates the supply side by keeping all (sticky price) firms in operation (region below the red dashed schedule in Figure 7). It does so by compressing the marginal cost, and the more so the lower the fraction n_f of flexible price firms in the economy. In so doing, it maintains a negative output gap, but a positive inflation gap.

- Second, when the economy faces intermediate energy price shocks, neither too low, nor too large, i.e. $\varepsilon_{min} \leq \varepsilon_t \leq \varepsilon_{max}$, then the optimal policy splits the burden of the shock, as it shrinks both supply and demand, in response to an energy price shock (region between the red and the blue dashed schedules in Figure 7). In addition, the output gap switches under the optimal policy, from negative to positive as the economy faces larger shocks. In contrast, the inflation gap is always positive.

- Last, when the economy faces large energy price shocks, i.e. $\varepsilon_t \geq \varepsilon_{max}$, then monetary policy becomes toothless, as all sticky price firms retrench under the optimal policy (region above the blue dashed schedule in Figure 7). In this case, both the output gap and the inflation gap are positive while the nominal interest rate becomes irrelevant, as the economy settles on a situation akin to a flexible price equilibrium.

5.3 Dissecting optimal policy under partial price rigidity

The optimal policy under partial price rigidity features three main properties. First, as in the case of full price rigidity, optimal monetary policy follows a pecking order: it first and foremost aims at insulating the supply side of the economy, by keeping all firms in operation. Here the logic is the same as under full price rigidity: Optimal monetary focuses on keeping firms in operation because too many firms decide to retrench following an energy price shock, as firms which retrench do so without factoring in the social costs, in terms of lower household welfare. Then, when fully insulating the supply side becomes too costly —in terms of demand compression—, optimal monetary policy splits the burden of the energy price shock between supply and demand, by shrinking both and thereby allowing for some ”profit-driven” inflation.

Second, while the optimal policy under full price rigidity always maintains a negative output gap by shrinking household demand relative to firm supply, under partial price rigidity, the optimal policy may imply a positive output gap, especially in the case of large energy price shocks. Under full price rigidity, the (ratio of the current to the steady state) marginal cost m_0 is always below one, implying that monetary policy actually cuts the marginal cost of production below its steady state value and thereby maintains a negative output gap. Conversely, under partial price rigidity, the optimal policy is such that the (ratio of the current to the steady state) marginal cost m_t^* *increases* with the energy price shock. As a consequence, optimal policy targets a marginal cost of production above its steady state value when the economy faces a large energy price shock, thereby implying a positive output gap.

Last, under partial price rigidity, optimal monetary policy may imply that all sticky price firms retrench, leaving the economy with only flexible price firms in operation, and monetary policy unable to steer the economy. This situation is obviously impossible in the case of full price rigidity, as it would imply supply going down to zero. But with partial price rigidity, it is typically optimal to let all sticky price firms retrench when the energy price shock is very large (as in the upper-left region in Figure 7).

When the central bank sets the interest rate, it balances three forces. A higher interest rate first reduces nominal demand. This reduces households' real income, both directly and indirectly as lower nominal demand leads more sticky price firms to retrench, which raises the general price level. However, a higher interest rate also reduces the price of energy in local currency which reduces the general price level and therefore raises, everything else equal, household real income. In addition, lower energy prices and lower nominal demand tend to reduce the marginal cost of production. This contributes to cut prices charged by flexible price firms, and to keep more sticky price firms on the market. This reduces the price of the composite consumption good and thereby contributes to cut the general price level and raise household real income.

Now, when the energy price shock hitting the economy is relatively small, the logic developed in the case of full price rigidity still applies: the opportunity cost of keeping all firms in operation —and neutralising the negative retrenchment externality— is limited relative to the social benefits. Optimal policy then keeps all firms in operation, which requires targeting a lower marginal cost, the larger the energy price shock.

Then, when the economy faces a larger energy price shock, it is not optimal anymore, as in the case of full price rigidity, to keep all firms in operation as the opportunity cost of doing so, in terms of foregone demand, becomes too large. However, the presence of flexible price firms changes monetary policy calculations in three ways. First, when there are flexible price firms, monetary policy is less effective in steering the marginal cost. Specifically, when the central bank raises the nominal interest rate, the marginal cost falls but less so, the higher the fraction of flexible price in the economy. Monetary policy is therefore less effective in preventing sticky price firms' retrenchment. Second, when there are flexible price firms, the equilibrium number of sticky price firms is less sensitive to the marginal cost. This is because a high marginal cost redistributes sales to the benefit of sticky price firms, thereby dampening the negative impact of a high marginal cost on the profits of sticky price firms. In other words the benefits of bringing down the marginal cost are typically lower, the higher the fraction of flexible price firms in the economy. Third and last, when there are flexible price firms, the scope for monetary policy to prevent sticky price firms' retrenchment is, by construction, more limited, as flexible price firms are not subject to retrenchment risks. Hence, for each of these three reasons, monetary policy targets a higher marginal cost, the larger the fraction of flexible price firm in the economy and/or the stronger the energy price shock. By implication, it can be optimal to maintain a positive output gap —typically when the economy faces a large energy price shock and/or when there are many flexible price firms—, contrary to the case of full price rigidity where monetary policy always maintains a negative output gap.

Finally, when monetary policy maintains a positive output gap, this induces a positive correlation between inflation and firms' profit. More specifically, under the optimal policy, energy price shocks, provided they are sufficiently large, tend to raise inflation, as the marginal cost of production goes up relative to the steady state. In addition, flexible price firms enjoy strictly positive profits as soon as the marginal cost of production moves from its steady state value, these positive profits being the flip side of sticky price firms' retrenchment. Inflation and profits therefore both increase relative to the steady state when the energy price shock is sufficiently large, leading to the positive correlation between profits and inflation observed in the data.

6 Corporate taxes and subsidies

When the energy price shock such that all sticky price firms retrench, monetary policy loses traction and cannot steer the economy anymore. Still, restoring some effectiveness for monetary policy is possible. One option is to redistribute income from *unconstrained* to *constrained* households. Given that the latter consume their current income, such redistribution would act to lift up demand, which would typically benefit sticky price firms. This would shift the blue schedule up in Figure 7 and narrow the set of shocks for which all sticky price firms stop operations.

An alternative would be to provide a subsidy to sticky price firms, i.e. to unprofitable firms, that would be funded by a tax on flexible price firms, i.e. on profitable firms. Implementing such a policy following an energy price shock would shore up sticky price firm profits and therefore help them stay in operation, thereby restoring some effectiveness for monetary policy. In addition it could reduce inflation following an energy price shock, if the supply-side effects associated with more firms being in operation dominate adverse effects associated with a higher marginal cost of production.

6.1 Corporate profits with taxes and subsidies

To look into the effects of such a tax/subsidy scheme, let us consider a set of lump-sum subsidies S and lump-sum taxes T, which for simplicity we write as fractions of the fixed cost of production: $S = \sigma p_{et} e_f$ and $T = \tau p_{et} e_f$, with $\sigma; \tau \geq 0$. Then, normalising the steady state fixed cost of production to one, i.e. $p_e e_f = 1$, profits of flexible and sticky price firms respectively write as:

$$\pi_{ft}(\tau) = \frac{d_t}{n_f + n_{st} m_t^{\frac{1}{\mu-1}}} - (1+\tau)\frac{p_{et}}{p_e} \quad \text{and} \quad \pi_{st}(\sigma) = \frac{\mu - m_t}{\mu - 1} \frac{m_t^{\frac{1}{\mu-1}}}{n_f + n_{st} m_t^{\frac{1}{\mu-1}}} d_t - (1-\sigma)\frac{p_{et}}{p_e} \tag{38}$$

Based on these expressions, flexible price firms earn larger profits than sticky price firms despite the tax/subsidy scheme, i.e. $\pi_{ft}(\tau) \geq \pi_{st}(\sigma)$, if and only if the sum of the tax and subsidy rate $\sigma + \tau$ does not exceed some upper bound:

$$\sigma + \tau \leq \frac{d_t/(p_{et}/p_e)}{n_f + n_{st} m_t^{\frac{1}{\mu-1}}} \left[1 - \frac{(\mu - m_t) m_t^{\frac{1}{\mu-1}}}{\mu - 1} \right] \tag{39}$$

When condition (39) holds, flexible price firms earn larger profits than sticky price firms, implying that positive energy price shocks still reduce the number of sticky price firms, but leave the number of flexible price firms unchanged. Specifically, the equilibrium number of sticky price firms satisfies:

$$n_{st}(\sigma) = \min \left\{ 1 - n_f ; \left[\frac{\mu - m_t}{\mu - 1} \frac{1}{1 - \sigma} \frac{d_t}{p_{et}/p_e} - n_f m_t^{-\frac{1}{\mu-1}} \right]^+ \right\} \tag{40}$$

With this expression at hand and assuming the equilibrium number of sticky price firms satisfies $0 < n_{st}(\sigma) < 1 - n_f$, the aggregate demand and aggregate supply equations respectively write as:

$$\theta d_t = \frac{1 + \varepsilon_t}{\beta(1 + i_t)} \left[\frac{Z(\varepsilon_t)}{1 + \varepsilon_t} + \left[\frac{\beta(1 + i_t)}{1 + \varepsilon_t} m_t \right]^{\frac{1}{1-\delta_f}} \right] \tag{41}$$

and

$$\frac{\theta - m_t}{m_t} \left[\frac{\beta(1 + i_t)}{1 + \varepsilon_t} m_t \right]^{\frac{1}{1-\delta_f}} + (1 - \sigma) n_f \theta (\mu - 1) \frac{m_t - 1}{\mu - m_t} m_t^{-\frac{\mu}{\mu-1}} = \frac{Z(\varepsilon_t)}{1 + \varepsilon_t} \tag{42}$$

As is clear from equation (41), introducing a tax/subsidy scheme, to the extent that it does not affect the margin of adjustment to energy price shocks, does not change the relationship governing the demand side, which is identical to the case with no tax/subsidy scheme. Conversely, and as would be expected, equation (42) which governs the supply side does change in response to the introduction of a tax/subsidy scheme on firms. Interestingly, the impact of the tax/subsidy scheme can be summarised in a very simple way. Under the tax/subsidy scheme, the supply side functions as if the economy had a lower fraction of flexible price firms, specifically a fraction $(1-\sigma)n_f$ instead of a fraction n_f. Finally, under the tax/subsidy scheme, the inflation equation, which reflects the balance between aggregate supply and aggregate demand writes as:

$$p_t(\sigma) = \left[\frac{p_{et}}{p_e}\right]^{\delta_h} \left[(1-\sigma)\frac{\mu - 1}{\mu - m_t}\frac{p_{et}/p_e}{d_t}\right]^{(\mu-1)(1-\delta_h)} \tag{43}$$

Equation (43) shows that the tax/subsidy scheme is dis-inflationary as it allows more (sticky price) firms to operate, which cuts the price of the composite consumption good. In the meantime however, because the tax/subsidy scheme works as if the economy had a lower fraction of flexible price firms (down from n_f to $(1-\sigma)n_f$), it also means that marginal cost of production m_t and household demand expressed in energy units $d_t/(p_{et}/p_e)$ are more sensitive to energy price shocks, which may counteract the dis-inflationary impact of subsidies to sticky price firms.

6.2 Optimal monetary policy with taxes and subsidies

Having determined aggregate demand, aggregate supply and inflation under the tax/subsidy scheme, we can now turn to optimal monetary policy. For this, let us denote $n_f(\sigma) = (1-\sigma)n_f$.

Proposition 4 *Assuming flexible price firms pay a lump-sum tax $T = \tau p_{et}/p_e$ and sticky price firms get a subsidy $S = \sigma p_{et}/p_e$, and provided the number of sticky price firms satisfies $0 < n_{st} < 1 - n_f$, the optimal interest rate policy satisfies:*

$$\beta(1+i_t) = \frac{1+\varepsilon_t}{m_t^*(\sigma)^{\delta_f}(\theta - m_t^*(\sigma))^{1-\delta_f}} \left[\frac{Z(\varepsilon_t)}{1+\varepsilon_t} - \theta(\mu-1)n_f(\sigma)\frac{m_t^*(\sigma)-1}{\mu - m_t^*(\sigma)}\left[m_t^*(\sigma)\right]^{-\frac{\mu}{\mu-1}}\right]^{1-\delta_f} \tag{44}$$

where the marginal cost of production $m_t^*(\sigma)$ writes, up to a first-order approximation, as

$$m_t^*(\sigma) = m_0^* + n_f(\sigma)F(m_0^*)\frac{1+\varepsilon_t}{Z(\varepsilon_t)} \quad \textit{with} \quad F\left(m_0^*\right) > 0 \tag{45}$$

Proof 4 *Applying the results of proposition 3 to the case where equation (41) governs aggregate demand, equation (42) governs aggregate supply and inflation is given by equation (43), yields proposition 4.*

Proposition 4 shows that, provided the marginal cost m_t is larger than one, i.e. provided the energy price shock ε_t is sufficiently large, then the presence of a tax/subsidy scheme as described above, typically leads the central bank to set a higher nominal interest rate, relative to the case where there is no such tax/subsidy scheme. As result, the central bank targets a lower marginal cost as is visible from equation (45). The reason for these properties is relatively simple. Given that the economy under the tax/subsidy scheme essentially functions as if there were fewer flexible price firms, this means that energy price shocks have a larger impact on the marginal cost of production and thereby on sticky price firms. To put it differently, the subsidy raises profits of sticky price firms, but also makes them more sensitive to energy price shocks. As consequence, the scope for inefficient retrenchment following an energy price shock is greater. To counteract this greater inefficiency, the central bank targets a lower marginal cost in response to a given energy price shock and does so by setting a higher nominal interest rate.

Then using expression (40) for the equilibrium number of sticky price firms and equations (41) and (42) for aggregate demand and aggregate supply, it follows that, under the optimal policy, the number of sticky firms at the equilibrium is non-negative if and only if

$$n_f \frac{\mu - 1}{\mu - m_t^*(\sigma)} \left[m_t^*(\sigma)\right]^{-\frac{1}{\mu-1}} \leq \frac{1}{1-\sigma} \frac{1}{\theta - 1} \frac{Z(\varepsilon_t)}{1 + \varepsilon_t} \tag{46}$$

Assuming as previously that the fraction n_f of flexible price firms is not too large, one can easily verify that the upper bound ε_{max} on energy price shocks, below which condition (46) holds, is increasing in the subsidy σ, i.e. $\partial\varepsilon_{max}/\partial\sigma > 0$. In other words, the presence of the tax/subsidy scheme actually widens the set of energy price shocks for which some sticky price firms are able to remain in operation under the optimal policy despite the energy price shock. Put differently, monetary policy is less likely to become ineffective the higher the subsidy σ to sticky price firms.

6.3 "Optimal" taxes and subsidies

A social planner setting the tax/subsidy scheme described above, faces two constraints. First, flexible price firms should still earn larger profits than sticky price firms, even after the imposition of the tax/subsidy scheme. This way, the planner can ensure that energy price shocks affect sticky price firms before they affect flexible price firms.[23] Simplifying condition (39) under which after-tax/subsidy profits are larger for flexible price firms, the rank preservation condition on firm profits under the optimal policy simplifies as:

$$\sigma + \tau \frac{\mu - m_t^*(\sigma)}{\mu - 1} \left[m_t^*(\sigma)\right]^{\frac{1}{\mu-1}} \leq 1 - \frac{\mu - m_t^*(\sigma)}{\mu - 1} \left[m_t^*(\sigma)\right]^{\frac{1}{\mu-1}} \tag{47}$$

When the marginal cost of production $m_t^*(\sigma)$ is larger than one, then condition (47) defines a negative relationship between the tax rate τ and the subsidy rate σ. Ensuring that flexible price firms earn larger profits despite the tax/subsidy scheme therefore requires either large subsidies to sticky price firms, but low taxes on flexible price firms or, large taxes on flexible price firms but low subsidies to sticky price firms. Second, the social planner needs to ensure a balanced budget. The total amount of subsidies paid to sticky price firms $n_{st}\sigma$ should not exceed the total tax revenues $n_f\tau$. Using the expression for the equilibrium number of sticky price firms under optimal monetary policy, the social planner's budget is balanced if and only if the tax rate applied to flexible price firms is sufficiently large:

$$\tau \geq \frac{\sigma}{\theta - m_t(\sigma)} \left[\frac{1}{(1-\sigma)n_f} \frac{\mu - m_t(\sigma)}{\mu - 1} \frac{Z(\varepsilon_t)}{1 + \varepsilon_t} - \frac{\theta - 1}{\left[m_t(\sigma)\right]^{\frac{1}{\mu-1}}} \right] \tag{48}$$

The term on the right-hand side of condition (48) being increasing in the subsidy rate σ, the balanced budget condition defines a positive relationship by which larger expenditures, i.e. a higher subsidy rate σ, naturally require larger revenues, i.e. a higher tax rate τ.

Combining these two constraints, there is a tax/subsidy scheme which minimises the set of energy price shocks for which monetary policy looses effectiveness. This tax/subsidy scheme is such that condition (47) and condition (48) both hold with equality and both sticky and flexible price firms make zero profits.

By setting taxes and subsidies such that sticky and flexible price firms earn the same level of profits, the social planner effectively taxes away the rent that flexible price firms

[23] It also precludes the possibility that flexible price firms find it more profitable to keep prices unchanged and prefer to behave as sticky price firms.

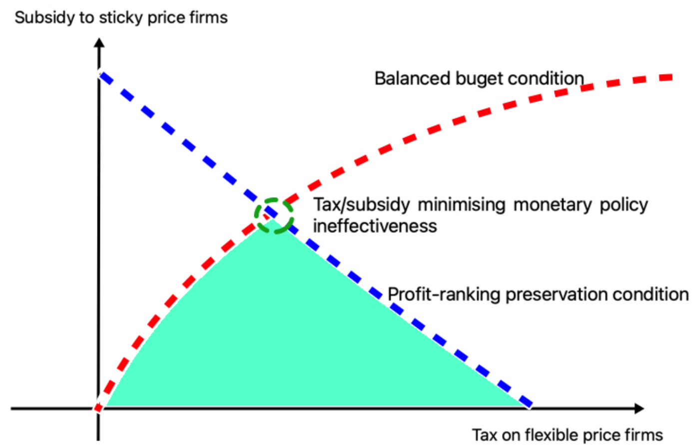


Figure 8: Feasible and optimal tax/subsidy schemes.

enjoy following the energy price shock, which suppresses ex post heterogeneity in profits. In practise this means that the social planner is using all available resources in the economy to keep as many sticky price firms in operation as possible. As a result, the economy functions with a larger degree of price rigidity, which in turn gives more leeway for monetary policy to respond to energy price shocks. This is why redistribution from profitable to unprofitable firms minimises the set of shocks for which monetary policy is ineffective.

7 Conclusions

In this paper, we investigate the issue of ”profit-driven” inflation in the context of a New Keynesian model which we enrich with reservation profits on the supply side. With this framework, we investigate the positive and normative implications of cost push shocks, taking the example of energy price shocks and focusing on monetary policy. We first show that energy price shocks lead to inefficiently large supply contractions and thereby inefficiently large (profit-driven) inflation, as firms which retrench do not internalise the social costs of doing so. Second, we show that optimal monetary policy follows a pecking order. It first aims at shielding the supply side from the fallout of the shock, thereby undoing the negative retrenchment externality. It then splits the burden of the shock between supply and demand, when insulating the supply side is too costly. Finally, when the energy price shock is very large, monetary policy looses traction. Budget-neutral fiscal interventions, e.g. redistribution from high- to low-income households and/or from high- to low-profit firms, can then restore monetary policy effectiveness.

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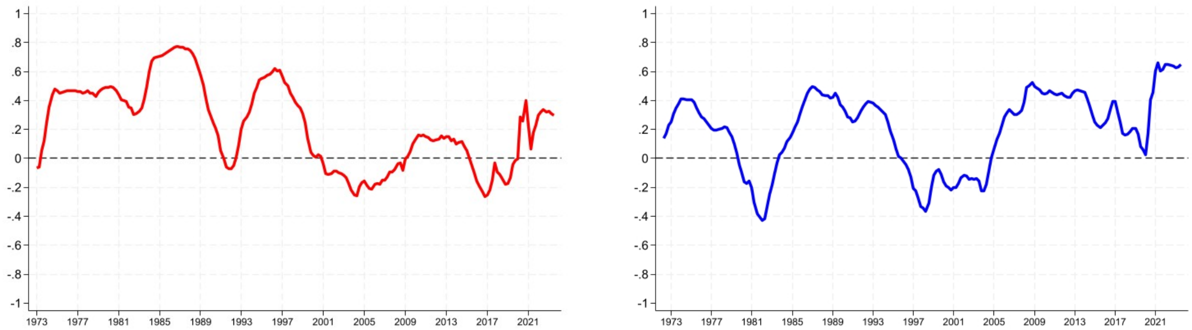
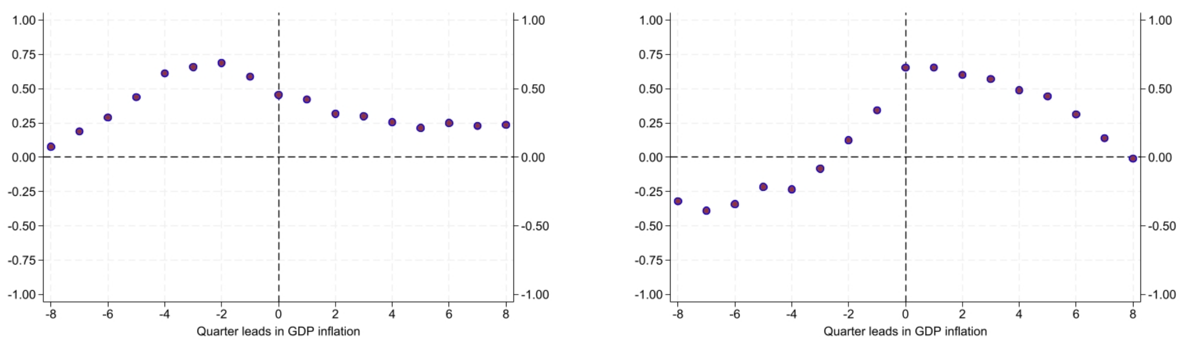
Appendix A.1: Unit Profits lead GDP inflation since Covid.



(a) : Unit Labour Costs (b) : Unit Profits

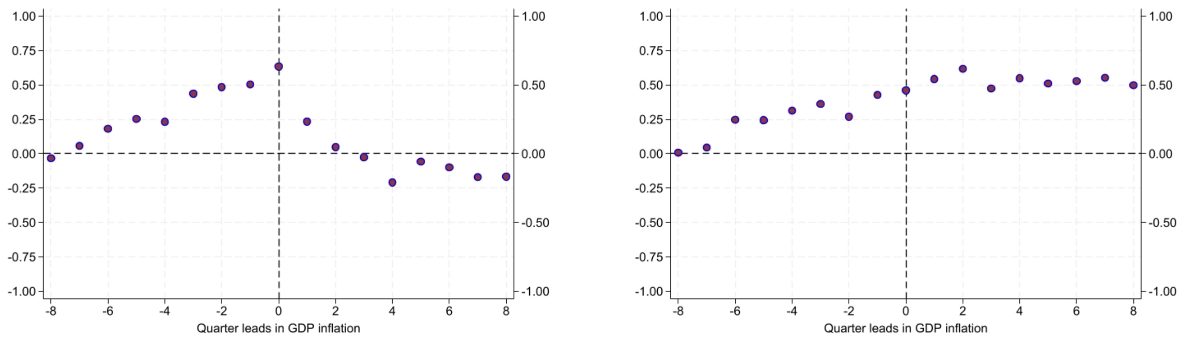
Figure 9: Correlation between UP and subsequent inflation at highest level ever since Covid. Figure 9 plots the correlation between ULC inflation and 1-year ahead inflation in Panel (a) (between UP inflation and 1-year ahead inflation in Panel (b)) considering a 10-year backward looking rolling window. Lines display correlations smoothed using a 1-year moving average. **Source**: OECD Quarterly National Accounts.

Appendix A.2: Unit Profits as a leading indicator of GDP inflation.



(a) : Unit Labour Costs (b) : Unit Profits

Figure 10: In Canada, ULC lag inflation, while UP lead inflation. Figure 10 plots the lead/lag correlations between GDP inflation and ULC inflation in Panel (a) (GDP inflation and UP inflation in Panel (b)) for Canada for the period 2014q1-2023q3. Dots to the left (to the right) of the vertical dashed line display correlations with lags (leads) of GDP inflation. **Source**: OECD Quarterly National Accounts.



(a) : Unit Labour Costs

(b) : Unit Profits

Figure 11: In Germany, ULC lag inflation, while UP lead inflation. Figure 11 plots the lead/lag correlations between GDP inflation and ULC inflation in Panel (a) (GDP inflation and UP inflation in Panel (b)) for Germany for the period 2014q1-2023q3. Dots to the left (to the right) of the vertical dashed line display correlations with lags (leads) of GDP inflation. **Source**: OECD Quarterly National Accounts.

Appendix A.3: Proof of Proposition 3. Optimal monetary policy under full price rigidity

Let us assume that the measure the measure of sticky price firms n_{st} is positive but less than one, i.e. $0 \leq n_{st} \leq 1$. Then household aggregate real income writes as

$$\frac{d_t}{p_t} = d_t \left[\frac{1+\varepsilon_t}{\beta\left(1+i_t\right)}\right]^{-\delta_h} \left[\frac{\mu-1}{\mu-m_t} \frac{1+\varepsilon_t}{\beta\left(1+i_t\right)} \frac{1}{d_t}\right]^{-\left(\mu-1\right)\left(1-\delta_h\right)}$$

Then using expression (23) for aggregate demand, the expression for household aggregate real income simplifies as

$$\ln \frac{d_t}{p_t} = \left[\delta_h + \left(1-\delta_h\right)\mu\right] \ln\left[\frac{1}{\theta}\left[\frac{Z}{1+\varepsilon_t} + \left[\frac{\beta\left(1+i_t\right)}{1+\varepsilon_t} m_t\right]^{\frac{1}{1-\delta_f}}\right]\right] + \left(1-\delta_h\right) \ln\left[\frac{1+\varepsilon_t}{\beta\left(1+i_t\right)}\left[\frac{\mu-m_t}{\mu-1}\right]^{\mu-1}\right]$$

Taking the first derivative relative to the interest rate and using the aggregate supply equation (24), the marginal cost under the optimal policy should satisfy

$$\left[\frac{\delta_h}{1-\delta_h} + \mu\right]\left[\frac{1}{1+i_t} + \frac{1}{m_t}\frac{\partial m_t}{\partial i_t}\right] = \frac{\left(1-\delta_f\right)\theta}{m_t}\left[\frac{1}{1+i_t} + \frac{\mu-1}{\mu-m_t}\frac{\partial m_t}{\partial i_t}\right]$$

Finally deriving the aggregate supply equation (24), the marginal cost of production m_t

should satisfy

$$\frac{1+i_t}{m_t}\frac{\partial m_t}{\partial i_t} = -\frac{\theta - m_t}{\theta\delta_f - m_t}$$

Equating the right-hand side term of the last two equations, we end up with the following expression for the marginal cost m_t under the optimal policy:

$$\frac{(\mu-1)(\theta-\mu)}{\mu - m_t} = \frac{\theta\delta_f}{m_t} + \frac{\delta_h}{1-\delta_h}$$

Let us denote m_0^* the marginal cost which solves this equation. Then the optimal policy consists is setting the interest rate such that the marginal cost satisfies $m_t = m_0^*$ if and only if the energy price shock ε_t and the marginal cost m_0^* are such that $n_{st} \leq 1$, which simplifies as

$$\frac{\mu - m_0^*}{\theta - m_0^*}\frac{Z(\varepsilon_t)}{1+\varepsilon_t} \leq \mu - 1$$

The policy which targets a constant marginal cost $m_t = m_0^*$ is therefore optimal if and only if the energy price shock satisfies $\varepsilon_t \geq \varepsilon(e_c)$ with $\varepsilon'(e_c) > 0$. Conversely, when the energy price shock ε_t such that $\varepsilon_t \leq \varepsilon(e_c)$, then the optimal policy keeps all firms in operation, i.e. $n_{st} = 1$ and the marginal cost under the optimal policy $m_0(\varepsilon_t)$ satisfies

$$\frac{\mu - m_0(\varepsilon_t)}{\theta - m_0(\varepsilon_t)}\frac{Z}{1+\varepsilon_t} = \mu - 1$$

Appendix A.4: Proof of Proposition 4. Optimal monetary policy under partial price rigidity

Let the measure of sticky price firms n_{st} be positive but less than $1-n_f$. Then, household aggregate real income writes as

$$\frac{d_t}{p_t} = d_t\left[\frac{1+\varepsilon_t}{\beta(1+i_t)}\right]^{-\delta_h}\left[\frac{\mu-1}{\mu-m_t}\frac{1+\varepsilon_t}{\beta(1+i_t)}\frac{1}{d_t}\right]^{-(\mu-1)(1-\delta_h)}$$

Using expression (31) for aggregate demand, the expression for household aggregate real income simplifies as

$$\ln\frac{d_t}{p_t} = [\delta_h + (1-\delta_h)\,\mu]\ln\left[\frac{1}{\theta}\left[\frac{Z}{1+\varepsilon_t} + \left[\frac{\beta\,(1+i_t)}{1+\varepsilon_t}m_t\right]^{\frac{1}{1-\delta_f}}\right]\right] + (1-\delta_h)\ln\left[\frac{1+\varepsilon_t}{\beta\,(1+i_t)}\left[\frac{\mu - m_t}{\mu - 1}\right]^{\mu-1}\right]$$

Taking the first derivative relative to the interest rate and using equation (30) for aggregate supply, the optimal policy is such that

$$\frac{1+i_t}{di_t}\frac{dm_t}{m_t} = -\frac{\left[1 - \frac{\frac{\delta_h}{1-\delta_h}+\mu}{1-\delta_f}\frac{m_t}{\theta}\right]\frac{Z(\varepsilon_t)}{1+\varepsilon_t} - n_f\,[\mu-1]\,\frac{m_t-1}{\mu-m_t}m_t^{-\frac{\mu}{\mu-1}}\left[1 - \frac{\frac{\delta_h}{1-\delta_h}+\mu}{1-\delta_f}\right]m_t}{\left[\frac{(\mu-1)m_t}{\mu-m_t} - \frac{\frac{\delta_h}{1-\delta_h}+\mu}{1-\delta_f}\frac{m_t}{\theta}\right]\frac{Z(\varepsilon_t)}{1+\varepsilon_t} - n_f\,[\mu-1]\,\frac{m_t-1}{\mu-m_t}m_t^{-\frac{\mu}{\mu-1}}\left[\frac{(\mu-1)m_t}{\mu-m_t} - \frac{\frac{\delta_h}{1-\delta_h}+\mu}{1-\delta_f}\right]m_t}$$

Finally deriving the aggregate supply equation (30), the marginal cost of production m_t should satisfy

$$\frac{1+i_t}{m_t}\frac{\partial m_t}{\partial i_t} = -\left[\frac{\delta_f\theta - m_t}{\theta - m_t} + \frac{\left(1-\delta_f\right)\theta n_f\,[\mu-1]\,\frac{m_t-1}{\mu-m_t}m_t^{-\frac{\mu}{\mu-1}}}{\frac{Z(\varepsilon_t)}{1+\varepsilon_t} - \theta n_f\,[\mu-1]\,\frac{m_t-1}{\mu-m_t}m_t^{-\frac{\mu}{\mu-1}}}\left[\frac{(\mu-1)\,m_t}{(m_t-1)\,(\mu-m_t)} - \frac{\mu}{\mu-1}\right]\right]^{-1}$$

Equating the right-hand side of these two last equations and taking a first-order approximation w.r.t. n_f, and denoting $\lambda = \mu + \delta_h/(1-\delta_h)$ the marginal cost under the optimal policy satisfies

$$m_t = m_0 + n_f F(m_0)\frac{1+\varepsilon_t}{Z\,(\varepsilon_t)}$$

where the function $F(.)$ satisfies

$$F(m) = m^{-\frac{\mu}{\mu-1}}\frac{\mu + [\mu-1]^2\,\frac{m}{\mu-m} + \mu\,(\mu-1)\,\lambda\frac{(\theta-m)m}{\mu-m}\left[\frac{1-m}{(1-\delta_f)\theta-\lambda m}\right]^2}{\frac{\mu-m}{(\theta-m)^2} + \frac{\mu(\mu-1)}{(1-\delta_f)\theta-\lambda m} - \frac{\lambda\mu(1-m)}{\left[(1-\delta_f)\theta-\lambda m\right]^2}}$$