A Microfoundation of the Term Structure*

MASTER'S THESIS

AUTUMN 2009

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Submitted on: 17 December 2009

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*I would like to thank Aleksander Berentsen, Sylvia Kaufmann and Alessandro Marchesiani for their very helpful comments. The paper has also benefited from comments and research support by the CFP team of F. Hoffmann-La Roche AG, Basel. Corresponding author: Samuel Huber <samuel_h@gmx.ch>.

Abstract

In this thesis we construct a general equilibrium monetary model for pricing government zero bonds when agents are cash constrained. The following results emerge from our analysis. For any positive inflation rate, bonds are essential to economies in improving the allocation. The efficiency improvement results from the possibility that some agents can deposit their idle money and earn positive interest. The main driving forces of the price development of the bonds are the relative number of consumers and producers in the economy and the efficiency of trades. We offer the results of different techniques used to check our approach and observe that the price development explained by our model proves competent in providing some forecasting capacity. Furthermore, we offer advice on the practical implementation of our results.

1 Introduction

Lagos and Wright [24] develop a divisible money model that provides a microfoundation for money demand and enables the introduction of heterogeneous preferences for consumption and production while still keeping the distribution of money balances analytically tractable. Berentsen, Camera and Waller [3] show that credit (or *inside* bonds) can improve the monetary allocation in the Lagos and Wright [24] framework since it allows agents to borrow or lend money depending on their liquidity needs. In a later paper Berentsen and Waller [7] show that under the best policies the allocation with outside bonds strictly dominates the allocation with inside bonds since a government can always print money to repay its loans, implying that there is no counterparty default risk. In both models the source of the efficiency improvement results from the possibility that some agents can deposit their idle money and earn positive interest.

We use these results and construct a general equilibrium model where we extend the basic framework in Berentsen and Waller [7] with zero bonds. Zero bonds are offered by the government and have different maturities. This allows us to model the term structure of interest rates when agents are budget constrained. As in Kocherlakota [21], we assume that the outside bonds are illiquid in the sense that they cannot be used as a medium of exchange in the goods market. We assume additionally that the government has restricted collection power so that it cannot impose taxes or run a deflation since this requires the lump-sum taxation of money balances.

We show that for any positive inflation rate, bonds are essential and improve the allocation by providing the possibility that some agents can deposit their idle money and earn positive interest. We analyze two possible cases: one where consumers and producers carry both money and bonds into the second market; the other where consumers carry only money and producers carry only bonds. In the first case trades are efficient and the optimal policy is the Friedman rule, implying that bonds are useless. In the second case consumers are budget-constrained. Here the price of bonds is a decreasing function of maturity and the real interest rate. Further the price of bonds depends on the efficiency of trades and the relative number of consumers and producers in the economy. The economics underlying this finding is quite intuitive. First, the less efficient an economy is, causing marginal utility to be greater than marginal cost, the more expensive bonds are. This follows from the higher value for the producer of depositing a marginal unit of an idle balance. That is because the inflation protecting role of bonds increases and so the price rises. Second, the more consumers there are in the economy, the more the bond price increases. This is due to the greater production of producers which implies more idle money balances for investment, in turn, resulting in a higher demand for bonds.

This approach allows us to explain a flat yield curve of the term structure of interest rates and the shifts which occur. Further we include no-arbitrage conditions in the pricing of bonds with different maturities and show how to express the bond price as a function of all endogenous as well as exogenous variables.

The literature on the term structure of interest rates is vast and dates back at least as far as Lutz [26]. He describes the no-arbitrage conditions that have to hold under the assumption of forecasting and no investment costs. We include these no-arbitrage conditions in our approach. Lutz further explains how the term structure behaves in different environments and describes graphically the equilibrium of demand for long and short-term bonds under the assumption that different market participants have different expectations. He finds that the more elastic the demand curves for these bonds are, the smaller the term spread is. As a consequence of our environment, we are not able to explain the term spread since we can only model a flat yield curve of the term structure.

Cox, Ingersoll and Ross [12] develop an intertemporal general equilibrium asset pricing model to study the term structure of interest rates (CIR-model, hereafter). In their CIR-model they include many traditionally mentioned factors influencing the term structure of interest rates. In the CIR-model time is continuous, while in our model it is discrete. With their single factor model they gain similar results to ours; namely, the description of the bond price as a decreasing function of the interest rate and of maturity. The less efficient an economy is, the higher risk-averse investors value the guaranteed redemption of a bond and the higher its price is driven. While we are only able to construct a flat-shaped term structure, they can also construct rising, falling and hump-shaped term structures. The factors they use are based on the current yield curve, since they assume that all the information that is currently known about future movements of interest rates is already embodied in the actual term structure. As a consequence the current bond price is derived endogenously; however, we are able to derive it exogenously as well.

In a later paper Duffie and Kan [14] extend the CIR-model to a multivariate approach and use an arbitrage-free multifactor model to price zero bonds. They still derive the bond price endogenously.

Nelson and Siegel [28] use a parsimonious model that is flexible enough to represent the range of shapes generally associated with yield curves. In their approach they fit observed yield curves using a second-order model with factors composed of the slope and the maturity of the examined yield curve. They are not able to deliver a cause-and-effect relationship.

Campbell and Shiller [10] find that a high term spread forecasts rising short-term interest rates over the long term, but declining long-term interest rates over the short term. They find that the variations in the term spread are due primarily to sudden movements in short rates and that long rates react too slowly. Hence, the movements of the term spread are too large to accord with the expectations theory of the term structure. Their results are too complex to introduce in our framework.

Our approach differs from all previous literature in that we provide a microfoundation of the term structure of interest rate in a general equilitbrium monetary model.

The remainder of this thesis is organized as follows. In Section 2 we describe the environment. We report and discuss the empirical results in Section 3 and Section 4 concludes.

2 The Environment

Our environment is based on the framework of the divisible money model in Lagos and Wright [24]. There is a $[0, 1]$ continuum of infinitely lived agents. Time is discrete and the model is basically extended as in Berentsen and Waller [7]. In each period there are three perfectly competitive markets which open sequentially.

The first market is an asset market where agents trade money for bonds as in Berentsen, Camera and Waller [3]. The second market is a goods market where agents trade money for market 2 goods. In the third market all agents consume and produce and readjust their portfolios.

At the beginning of the first market agents receive a preference shock that determines whether they can produce or consume in the second market. With probability $(1 - n)$ an agent can consume and cannot produce. With probability n an agent can produce and cannot consume. We refer to consumers as buyers and to producers as sellers. Buyers learn that they will get utility $u(q)$ from q consumption in the second market, where $u'(q) > 0$, $u''(q) < 0$, $u'(0) = \infty$, and $u'(\infty) = 0$. Sellers in the second market incur a utility cost $c(q) = q$ from producing q units of output. To motivate a role for fiat money, we assume that all goods trades in market 2 and 3 are subject to anonymity which means that agents cannot identify their trading partners. Consequently, trading histories of agents are private information and sellers require immediate compensation so buyers must pay with money.

In the third market all agents produce and consume in a centralized market, getting $U(x)$ from x consumption of a general good, with $U'(x) > 0$, $U'(0) = \infty$, $U'(+\infty) = 0$ and $U''(x) \leq 0$. The difference in preferences over the good sold in the last market allows us to impose technical conditions such that the distribution of money holdings is degenerate at the beginning of a period. Agents can produce one unit of the consumption good x with one unit of labor h which generates one unit of disutility. This implies that all agents will choose to carry the same amount of money out of market 3, independent of their trading history. The discount factor across periods is $\beta = 1/(1+r)$, where $\beta \in (0,1)$ and r represents the real interest rate across periods.

At the beginning of period t, agents learn whether they will be sellers or buyers in market 2. Sellers might want to buy bonds since they have idle money, while buyers might want to sell bonds since they need money. There are k-types of bonds in the economy, where k denotes the maturity. These bonds are nominal government debt obligations that are sold at a price discount $\rho_{k,t} \leq 1$ in market 3 and mature after k periods. We assume that the government has a record-keeping technology over bond trades, and acts as the intermediary in the bond market. Bond holdings are book-keeping entries, so no physical object exists. This makes these instruments incapable of being used as media of exchange in market 2: They are illiquid. The government has no record-keeping technology over good trades. Since agents are anonymous in market 2, a buyer's promise to deliver outside bonds to a seller in market 3 is not credible. Consequently, money is essential for trade in market 2.

In what follows we study a model for discount bonds with a maturity of one and two periods, that is $k = 1, 2$. As in Berentsen and Waller [7], we assume that a government exists that controls the supply of fiat currency and issues zero bonds. These bonds are perfectly divisible, payable to the bearer and default free since the government can always repay its bonds by printing money at no cost. One bond pays off one unit of currency at maturity. Denote M_t as the end-of-period stock of money supply in period t, and $B_{k,t}$ the end-of-period stock of bonds with maturity k issued at time t. Hence the change in the money supply in period t is described as follows:

$$M_t - M_{t-1} = \tau_t M_{t-1} + B_{1,t-1} + B_{2,t-2} - \rho_{1,t} B_{1,t} - \rho_{2,t} B_{2,t} + P_t G_t,$$

where $P_t G_t$ is the period-t nominal amount of government spending in market 3, and P_t is the price of goods in market 3. The total change in the money supply is given by three components: first, a lump-sum transfer of cash $\tau_t M_{t-1}$; second, the net difference between the cash created to redeem bonds $B_{k,t-k}$, and the net cash withdrawal from selling $B_{k,t}$ units of bonds at the price $\rho_{k,t}$ for any k; and thirdly, the cash printed to pay for government goods. We assume that there are positive initial stocks of money and outside bonds M_0, $B_{1,0}$ and $B_{2,0}$. For $\tau_t < 0$ the government must be able to extract money via lump-sum taxes from the economy. Throughout the paper we assume limited enforcement so that $\tau_t < 0$ is not feasible.

To simplify the analysis, we assume $G_t = 0$ for all t. This implies that all money creation comes from paying off net nominal bond obligations and the lump-sum gifts of money $\tau_t M_{t-1}$. Consequently the government budget constraint reduces to

$$M_t - M_{t-1} = \tau_t M_{t-1} + B_{1,t-1} + B_{2,t-2} - \rho_{1,t} B_{1,t} - \rho_{2,t} B_{2,t}. \quad (1)$$

Divide (1) by M_{t-1} and get

$$\gamma_t - 1 = \tau_t + \frac{1}{M_{t-1}} \left(B_{1,t-1} + B_{2,t-2} - \rho_{1,t} B_{1,t} - \rho_{2,t} B_{2,t} \right).$$

Let $\eta_{k,t} = B_{k,t}/B_{k,t-k}$ be the gross growth rate of bonds with maturity k and $\gamma_t = M_t/M_{t-1}$ the gross growth rate of the money supply in period t. Replacing the last two terms, $B_{1,t}$ and $B_{2,t}$, and rearranging, the last equation can be rewritten as

$$\gamma_t - 1 - \tau_t = \frac{B_{1,t-1}}{M_{t-1}} \left[\left(1 - \rho_{1,t}\eta_{1,t}\right) + \frac{B_{2,t-2}}{B_{1,t-1}} \left(1 - \rho_{2,t}\eta_{2,t}\right) \right]$$

This equation relates the gross growth rate of money γ_t to the lump-sum gifts of money τ_t and the gross growth rate of bonds $\eta_{k,t}$.

In period t, let $\phi_t = 1/P_t$ be the real price of money in market 3. For notational ease, variables corresponding to the next period are indexed by $+1$, and variables corresponding to the previous period are indexed by -1. We focus on symmetric and stationary monetary equilibria where all agents follow identical strategies and where real allocations are constant over time. In a stationary equilibrium, end-of-period real money balances are time-invariant

$$\phi M = \phi_{+1} M_{+1}. \quad (2)$$

Moreover, we restrict our attention to equilibria where γ is time invariant which implies that $\gamma = \phi/\phi_{+1} = M_{+1}/M$.

As mentioned before, we analyze an economy with zero bonds with a maturity of one and two periods. Let $V(m, b_k)$ denote the expected value from entering a market with m units of money and b_k units of bonds, where k denotes the maturity. For notational simplicity, we suppress the dependence of the value function on the time index t. In what follows, we look at a representative period t and work backwards, from the third to the first market.

The settlement market

In the third market agents produce h units of goods using h hours of labor, consume x units of goods, receive repayment of the maturing zero bonds, adjust their money balances by trading money for bonds and receive the lump-sum gifts of money τ from the government. Since in this market the government issues new zero bonds, the agents have the possibility to buy new bonds or to trade earlier issued bonds between themselves. For arbitrage opportunities the price of a bond with a maturity of two periods, issued one period ago, has to be equal to the price of a newly issued bond with a maturity of one period. An agent entering market 3 with a portfolio (m, b_0, b_1) solves the following optimization problem

$$V_3(m, b_0, b_1) = \max_{x,h,m_{+1},b_{1,+1},b_{2,+1}} [U(x) - h + \beta V_1(m_{+1}, b_{1,+1}, b_{2,+1})] \tag{3}$$

s.t.

$$x + \phi m_{+1} + \phi\rho_1 b_{1,+1} + \phi\rho_2 b_{2,+1} = h + \phi m + \phi b_0 + \phi\rho_1 b_1 + \tau, \tag{4}$$

where ρ_k is the third-market money price of bonds with maturity k, m_{+1} the units of money taken into the next period, and $b_{k,+1}$ the units of type-k bonds taken into the next period.

Using (4) to eliminate h in (3), one obtains

$$\begin{aligned} V_3(m, b_0, b_1) &= \phi\,[m + b_0 + \rho_1 b_1 + \tau] \\ &\quad + \max_{x,m_{+1},b_{1,+1},b_{2,+1}} \begin{bmatrix} U(x) - x - \phi m_{+1} - \phi\rho_1 b_{1,+1} \\ -\phi\rho_2 b_{2,+1} + \beta V_1(m_{+1}, b_{1,+1}, b_{2,+1}) \end{bmatrix}. \end{aligned}$$

The first-order conditions with respect to $x, m_{+1}, b_{1,+1}$,and $b_{2,+1}$ are

$$U'(x) = 1, \tag{5}$$

$$\frac{\beta\partial V_1}{\partial m_{+1}} = \phi, \tag{6}$$

$$\frac{\beta\partial V_1}{\partial b_{1,+1}} = \phi\rho_1, \text{ and} \tag{7}$$

$$\frac{\beta\partial V_1}{\partial b_{2,+1}} = \phi\rho_2 \tag{8}$$

where the term $\beta\partial V_1(m_{+1}, b_{1,+1}, b_{2,+1})/\partial m_{+1}$ in (6) is the marginal benefit of taking money out of market 3 in the next period, and ϕ is its marginal cost. In competitive markets, the uniqueness of m_{+1} is a direct consequence of $u''(q) < 0$, therefore all agents in the third market choose the same m_{+1}. Note that due to the quasi-linearity of the consumption function $b_{1,+1}$, $b_{2,+1}$ and m_{+1} are independent of b_0, b_1 and m. As a result, the distribution of money holdings is degenerate at the beginning of the following period. Agents who bring too much cash into the third market spend some by buying goods, while those with too little cash sell goods. From (5) we see that the quantity of goods x consumed by every agent is equal to the efficient level x^*, where x^* is such that $U'(x^*) = 1$. From (7) and (8) we see that the marginal value of taking a type-k bond into the next period equals its real price, $\phi\rho_k$. Envelope conditions in market 3 are

$$\frac{\partial V_3}{\partial m} = \phi, \ \frac{\partial V_3}{\partial b_0} = \phi, \text{ and } \frac{\partial V_3}{\partial b_1} = \phi\rho_1. \tag{9}$$

The goods market

Let q_B and q_S respectively denote the quantities consumed by a buyer and produced by a seller in market 2. Let p be the nominal price of goods in market 1.

A seller entering market 2 with a portfolio (m, b_0, b_1) has the expected lifetime utility

$$V_2^S(m, b_0, b_1) = \max_{q_S} \left[-q_S + V_3(m + pq_S, b_0, b_1) \right]. \tag{10}$$

Using (9), the first order condition is

$$p\phi = 1. \tag{11}$$

If (11) holds, sellers are indifferent on how much they produce in market 2. Since we focus on symmetric equilibria, we assume that they all produce the same quantity q_S.

A buyer has expected lifetime utility

$$V_2^B(m, b_0, b_1) = \max_{q_B} \left[u(q_B) + V_3(m - pq_B, b_0, b_1) \right] \tag{12}$$

s.t.

$$pq_B \leq m. \tag{13}$$

Using (9) and (11), the buyer's first order condition in market 2 is

$$u'(q_B) = 1 + \frac{\lambda_q}{\phi}, \tag{14}$$

where λ_q is the multiplier of the buyer's cash constraint. If the cash constraint is not binding, trade is efficient ($\lambda_q = 0$). If it is binding, then $u'(q_B) > 1$, which means that trades are inefficient. In this case, the buyer spends all his money.

Using the envelope theorem, (9), and (14), the marginal values of bonds and the marginal values of money for buyers and sellers at the beginning of the second market are

$$\frac{\partial V_2^B}{\partial b_0} = \frac{\partial V_2^S}{\partial b_0} = \phi, \tag{15}$$

$$\frac{\partial V_2^B}{\partial b_1} = \frac{\partial V_2^S}{\partial b_1} = \rho_1\phi \tag{16}$$

$$\frac{\partial V_2^B}{\partial m} = \phi u'(q_B), \text{ and } \frac{\partial V_2^S}{\partial m} = \phi. \tag{17}$$

Finally, market clearing satisfies

$$(1-n)q_B = nq_S. \tag{18}$$

The asset market

Let φ_k be the price of a type-k bond in market 1. Note that there are three short-selling constraints that must be satisfied in market 1; that is, agents cannot sell more bonds or spend more money than the amount they carry with them from the previous period.

An agent with a portfolio (m, b_0, b_1) at the opening of the first market has expected the lifetime utility

$$V_1(m, b_0, b_1) = (1-n)\left[\max_{\widehat{m},\widehat{b_0},\widehat{b_1}} V_2^B(\widehat{m}, \widehat{b_0}, \widehat{b_1})\right] + n\left[\max_{\widehat{m},\widehat{b_0},\widehat{b_1}} V_2^S(\widehat{m}, \widehat{b_0}, \widehat{b_1})\right] \tag{19}$$

subject to the budget constraint

$$\phi m + \varphi_0 \phi b_0 + \varphi_1 \phi b_1 \geq \phi \widehat{m} + \varphi_0 \phi \widehat{b_0} + \varphi_1 \phi \widehat{b_1} \tag{20}$$

and subject to the short-selling constraints

$$\widehat{m} \geq 0,\ \widehat{b_0} \geq 0, \text{ and } \widehat{b_1} \geq 0. \tag{21}$$

where "^" denotes market 2 variables, λ^j the Lagrange multiplier on (20), and λ_m^j, λ_0^j and λ_1^j the Lagrange multipliers on (21) for $j = B, S$. The first order conditions in market 1 are

$$\begin{aligned} \frac{\partial V_2^j}{\partial \widehat{m}} - \phi \lambda^j + \lambda_m^j &= 0, \\ \frac{\partial V_2^j}{\partial \widehat{b_0}} - \varphi_0 \phi \lambda^j + \lambda_0^j &= 0, \text{ and} \\ \frac{\partial V_2^j}{\partial \widehat{b_1}} - \varphi_1 \phi \lambda^j + \lambda_1^j &= 0 \end{aligned} \tag{22}$$

where $j = B, S$ indicates the agent's type.

Now, apply the envelope theorem to (19), and get

$$\frac{\partial V_1}{\partial m} = (1-n)\,\phi \lambda^B + n\phi \lambda^S \tag{23}$$

$$\frac{\partial V_1}{\partial b_0} = (1-n)\,\phi \varphi_0 \lambda^B + n\phi \varphi_0 \lambda^S, \text{ and} \tag{24}$$

$$\frac{\partial V_1}{\partial b_1} = (1-n)\,\phi \varphi_1 \lambda^B + n\phi \varphi_1 \lambda^S. \tag{25}$$

Consider a seller first. If $i < 0$, then $\widehat{b_k} = -b_k$, and he wants to sell all his bonds for money in market 1. A buyer also wants to sell all his bonds for money since he needs cash for consumption. This obviously cannot be an equilibrium. Hence a seller always carries bonds into market 2, that is $\lambda_k^S = 0$ for any k when $i \geq 0$. Consider now a buyer. Since the Inada conditions are assumed on $u(q)$, he will always carry some money into market 2, that is $\lambda_m^B = 0$.

Now we can have two possible cases: one where buyers and sellers carry both money and bonds into the second market; the other where buyers carry only money and sellers carry only bonds. Let us now analyze the two cases separately.

The asset market. Unconstrained case

Sellers carry a strictly positive amount of money into market 2, that is $\hat{m} > 0$ which implies $\lambda_m^S = 0$. Buyers carry strictly positive amounts of bonds into market 2, that is $\hat{b}_0 > 0$ and $\hat{b}_1 > 0$ which implies $\lambda_0^B = \lambda_1^B = 0$.

Using (15)-(17), for a seller (22) becomes

$$\lambda^S = 1, \; \varphi_0 \lambda^S = 1, \text{ and } \varphi_1 \lambda^S = \rho_1. \tag{26}$$

From the first and second condition we have $\varphi_0 = 1$, while from the second and third condition we have $\varphi_1 = \rho_1$.

Analogously, for a buyer (22) becomes

$$\lambda^B = u'(q_B), \; \varphi_0 \lambda^B = 1, \text{ and } \varphi_1 \lambda^B = \rho_1. \tag{27}$$

Note from the second equation in (26) and (27) that $\lambda^B = \lambda^S$. From the first equation in (26) and (27) this implies $u'(q_B) = 1$.

Using $\lambda^B = \lambda^S = 1$ into (23), one gets $\partial V_1 / \partial m = \phi$. Now, replace $\partial V_1 / \partial m$ using (6) lagged one period to get the Friedman rule

$$\frac{\gamma}{\beta} = 1 \tag{28}$$

where we have also used γ to eliminate ϕ_{-1} and ϕ. Similarly, use $\lambda^B = \lambda^S = 1$ in (24), to get $\partial V_1 / \partial b_0 = \phi \varphi_0$. Now, replace $\partial V_1 / \partial b_0$ using (7) lagged one period to get

$$\frac{\gamma}{\beta} = \frac{\varphi_0}{\rho_{1,-1}} = \frac{1}{\rho_{1,-1}} \tag{29}$$

where we have used (26) to replace φ_0. From comparison of (28) and (29), we have $\rho_{1,-1} = 1$. Similarly, use $\lambda^B = \lambda^S = 1$ in (25), to get $\partial V_1 / \partial b_1 = \phi \varphi_1$. Now, replace $\partial V_1 / \partial b_1$ using (8) lagged one period to get

$$\frac{\gamma}{\beta} = \frac{\varphi_1}{\rho_{2,-1}} = \frac{\rho_1}{\rho_{1,-1}} \tag{30}$$

where we have used (26) to replace φ_1. From comparison of (28) and (30), we have $\rho_1 = \rho_{1,-1} = 1$. These results are straightforward. With the Friedman rule[1] ($\gamma = \beta$) agents do

[1] "According to the logic of the Friedman rule, the opportunity cost of holding money faced by private agents should equal the social cost of creating additional fiat money. Therefore, nominal rates of interest should be zero. In practice, this means that the central bank should seek a rate of deflation equal to the real interest rate on government bonds and other safe assets, in order to make the nominal interest rate zero."(http://en.wikipedia.org/wiki/Friedman_rule [01.12.2009])

not need to protect themselves against the inflation tax, hence bonds are inessential. If $\gamma = \beta$, bonds and money are perfect substitutes, and therefore the price of bonds is 1. With the Friedman rule, efficiency is achieved and buyers consume the efficient quantity q_B^* where q_B^* satisfies $u'(q_B^*) = 1$.

The asset market. Constrained case

We now analyze the case where sellers spend all their money acquiring bonds and buyers are constrained in the asset market. This means that buyers sell all their bonds, that is $\hat{b}_0 = 0$ and $\hat{b}_1 = 0$, which implies $\lambda_0^B > 0$ and $\lambda_1^B > 0$, respectively.

Sellers invest all their money in bonds, that is $\hat{m} = 0$ which implies $\lambda_m^S > 0$. Now, using (15)-(17) to replace $\partial V_2^S/\partial \widehat{m}$, $\partial V_2^S/\partial \widehat{b_0}$ and $\partial V_2^S/\partial \widehat{b_1}$ we can rewrite (22) as

$$1 = \lambda^S - \frac{\lambda_m^S}{\phi},\ 1 = \varphi_0 \lambda^S, \text{ and } \rho_1 = \varphi_1 \lambda^S. \tag{31}$$

From the second and third equation in (31) we get the no-arbitrage condition for one-period and two-period bonds

$$\varphi_0 \rho_1 = \varphi_1. \tag{32}$$

Consider now, an agent who will be a buyer in market 2. Using (15)-(17) to replace $\partial V_2^B/\partial \widehat{m}$, $\partial V_2^B/\partial \widehat{b_0}$ and $\partial V_2^B/\partial \widehat{b_1}$ we can rewrite (22) as

$$\lambda^B = 1 + \frac{\lambda_q}{\phi},\ 1 = \varphi_0 \lambda^B - \frac{\lambda_0^B}{\phi}, \text{ and } \rho_1 = \varphi_1 \lambda^B - \frac{\lambda_1^B}{\phi}. \tag{33}$$

Replace the right-hand side of the first equation in (33) using (14) to get

$$\lambda^B = u'(q_B) \tag{34}$$

then replace λ^B using the second equation in (33) to get

$$\phi = \frac{\lambda_0^B}{\varphi_0 u'(q_B) - 1}. \tag{35}$$

Finally, clearing conditions in market 1 are

$$\begin{aligned} m &= (1-n)\,\hat{m}^B + n\hat{m}^S \\ b_1 &= (1-n)\,\hat{b}_1^B + n\hat{b}_1^S, \text{ and} \\ b_2 &= (1-n)\,\hat{b}_2^B + n\hat{b}_2^S. \end{aligned} \tag{36}$$

A symmetric stationary equilibrium consists of the agents' decisions which meet the following requirements: (*i*) The decisions solve the maximization problems specified above; (*ii*) The decisions are symmetric across all agents; (*iii*) The bond market and the goods market clear.

We now derive the symmetric stationary equilibrium allocation. In any symmetric equilibrium, where money and bonds have positive values $m = M_{-1}$ and $b = B_{-1}$.

Use (6) lagged one-period to replace $\partial V_1/\partial m$ in (23) and eliminate ϕ_{-1} and ϕ using $\gamma = \phi_{-1}/\phi$ to get

$$\frac{\gamma}{\beta} = (1-n)\,\lambda^B + n\lambda^S.$$

Replacing λ^B using (34), and eliminating λ^S using the second equation in (31), we can rewrite the last expression as

$$\frac{\gamma}{\beta} = (1-n)\,u'(q_B) + \frac{n}{\varphi_0} \tag{37}$$

The right-hand side measures the value of bringing one extra unit of money into the first market. The first term reflects the marginal utility of spending one unit of money in the goods market for the buyer, and the second term is the marginal utility of investing an extra unit of idle balances for the seller. The effect of zero bonds on the marginal value of money is positive since sellers can deposit their idle money and earn positive interest.

Now, replace λ^B from (34) and λ^S from the second equation in (31) to rewrite (24) as

$$\frac{\partial V_1}{\partial b_0} = (1-n)\,\phi\varphi_0 u'(q_B) + n\phi$$

then replace $\partial V_1/\partial b_0$ using (7) lagged one period to get

$$\frac{\phi_{-1}\rho_{1,-1}}{\beta} = (1-n)\,\phi\varphi_0 u'(q_B) + n\phi$$

then, eliminate ϕ_{-1} and ϕ using $\gamma = \phi_{-1}/\phi$ and divide by φ_0 to get

$$\frac{\gamma}{\beta}\frac{\rho_{1,-1}}{\varphi_0} = (1-n)\,u'(q_B) + \frac{n}{\varphi_0}. \tag{38}$$

Comparing (37) and (38) implies the following no arbitrage condition for one-period bonds,

$$\rho_{1,-1} = \varphi_0. \tag{39}$$

The economics underlying (39) is quite intuitive. (39) means that the price of one-period bonds in market 1, φ_0, must be equal to the price of one-period bonds in the previous-period market 3, $\rho_{1,-1}$. This is because there is no aggregate uncertainty on how many buyers and sellers there are in market 1. Using (32) to replace φ_0, we can rewrite (39) as

$$\rho_1\rho_{1,-1} = \varphi_1. \tag{40}$$

Next, replace λ^B from (34) and λ^S from the third equation in (31) to rewrite (25) as

$$\frac{\partial V_1}{\partial b_1} = (1-n)\,\phi\varphi_1 u'(q_B) + n\phi\rho_1.$$

Using (32) to replace ρ_1, and (7) lagged one period to replace $\partial V_1/\partial b_1$, we can rewrite the last equation as

$$\frac{\phi_{-1}\rho_{2,-1}}{\beta} = (1-n)\,\phi\varphi_1 u'(q_B) + n\phi\frac{\varphi_1}{\varphi_0}.$$

Then eliminate ϕ_{-1} and ϕ using $\gamma = \phi_{-1}/\phi$ and divide by φ_1 to get

$$\frac{\gamma}{\beta}\frac{\rho_{2,-1}}{\varphi_1} = (1-n)\,u'(q_B) + \frac{n}{\varphi_0}. \tag{41}$$

Comparing (41) and (38) yields the no-arbitrage condition for two-period bonds

$$\rho_{2,-1} = \varphi_1 \tag{42}$$

Using (32) to eliminate φ_1, the last expression can be rewritten recursively as

$$\rho_{2,-1} = \rho_1 \rho_{1,-1}.$$

The meaning of (42) is similar to the one underlying (39), that is the current-period price of two-period bonds in market 1, φ_1, must be equal to the previous-period market 3 price of two period bonds, $\rho_{2,-1}$.

Lemma 1 *In a steady state, φ_0, φ_1, ρ_1 and ρ_2 are constant.*

Proof. Solve (37) for φ_0 and get

$$\varphi_0 = \frac{n\beta}{\gamma - \beta\,(1-n)\,u'(q_B)} \tag{43}$$

Since γ, q_B, β, and n are constant in a steady state, then φ_0 must also be constant in a steady state. Now, solve (38) for $\rho_{1,-1}$ and get

$$\rho_{1,-1} = \frac{\beta}{\gamma}\left[\varphi_0\,(1-n)\,u'(q_B) + n\right]$$

Since φ_0, γ, q_B, β, and n are constant in a steady state, it follows that $\rho_{1,-1}$ is also constant, which implies $\rho_{1,-1} = \rho_1$. Since ρ_1 and φ_0 are constant in a steady state, (32) implies that φ_1 is also constant. Since φ_1 is constant it follows from (42) that $\rho_{2,-1}$ must also be constant in a steady state, which implies $\rho_{2,-1} = \rho_2$. ■

Since bond prices are constant in a steady state, (39) can be rewritten as $\rho_1 = \varphi_0$. Then replacing φ_0 with ρ_1 in (43), we derive the first equilibrium equation

$$\rho_1 = \frac{n\beta}{\gamma - \beta\,(1-n)\,u'(q_B)}. \tag{44}$$

In a steady state, (42) can be rewritten as $\rho_2 = \varphi_1$. Replacing φ_1 from (32), this implies $\rho_2 = \varphi_0\rho_1 = \rho_1^2$. Finally, replacing ρ_1 from (44) we have

$$\rho_2 = \left[\frac{n\beta}{\gamma - \beta\,(1-n)\,u'(q_B)}\right]^2. \tag{45}$$

[2]

[2] We also extended the model to k-types of bonds in the economy, where k denotes the maturity. These bonds are, as before, nominal government debt obligations that are sold at a price discount $\rho_{k,t} \leq 1$ in market 3 and mature after $k = 1, 2, ..., K$ periods.

We can derive the price of a k-period bond as $\rho_k = \varphi_0\rho_{k-1} = \rho_1^k$. Using (44) we have

$$\rho_k = \left[\frac{n\beta}{\gamma - \beta\,(1-n)\,u'(q_B)}\right]^k.$$

The economics underlying (44) are straightforward. First, the less efficient an economy is, such that that $u'(q_B)$ becomes bigger, the more expensive bonds are. This follows from the higher value to a seller of depositing an extra unit of idle balances. The explanation for this result is that higher inflation reduces the efficiency of an economy, so that the inflation protecting role of bonds increases and their price rises. Second, the more buyers $(1-n)$ there are in the economy, the more the bond price increases. This is due to the higher production of sellers which implies more idle money balances being available for investment. The higher the demand for bonds, the higher their price is.

Since (44) is restricted to the interval between zero and one, (45) implies that the price of a zero bond with a maturity of two periods has to be strictly lower than the price of a one-period zero bond. That is, the bond price is a decreasing function of maturity as well as of the real interest rate. Equation (45) implies a flat-shaped curve for the term structure of interest rates as well. As a consequence, we are not able to explain the term spread between bonds with different maturities. Shocks in the relative quantity of buyers and sellers and/or the efficiency of trades in the economy cause shifts in the current term structure.

For bonds to be essential $\rho_{k,t} < 1$ has to hold. Using this constraint in (44) and rearranging for γ to get

$$\gamma > n\beta + \beta(1-n)\,u'(q_B) > \beta \tag{46}$$

For any $\rho_{k,t} < 1$ is $\gamma > \beta$.

We now derive the second equilibrium condition from the government budget constraint which we rewrite here for convenience

$$\gamma - 1 - \tau = \frac{B_{1,-1}}{M_{-1}}\left[(1-\rho_1\eta_1) + \frac{B_{2,-2}}{B_{1,-1}}(1-\rho_2\eta_2)\right] \tag{47}$$

where $\gamma = M/M_{-1}$, $\eta_1 = B_1/B_{1,-1}$, and $\eta_2 = B_2/B_{2,-2}$. In a steady state, real one-period-bond holdings are constant; i.e., $\phi\rho_1 B_1 = \phi_{-1}\rho_{1,-1}B_{1,-1}$ or

$$\phi B_1 = \phi_{-1}B_{1,-1} \tag{48}$$

Similarly, real two-period bond holdings are constant; i.e., $\phi\rho_2 B_2 = \phi_{-1}\rho_{2,-1}B_{2,-1} = \phi_{-2}\rho_{2,-2}B_{2,-2}$ or

$$\phi B_2 = \phi_{-1}B_{2,-1} = \phi_{-2}B_{2,-2} \tag{49}$$

Using (2) and (48), we have $\eta_1 = \gamma$ or $M/M_{-1} = B_1/B_{1,-1}$, which implies

$$\frac{M}{B_1} = \frac{M_{-1}}{B_{1,-1}} = \frac{M_{-2}}{B_{1,-2}} = ... = \frac{M_0}{B_{1,0}} \tag{50}$$

Using (2) and (49), we have $B_{2,-1}/B_{2,-2} = \eta_1 = \gamma$, which implies

$$\frac{B_{2,-1}}{B_{1,-1}} = \frac{B_{2,-2}}{B_{1,-2}} = \frac{B_{2,-3}}{B_{1,-3}} = ... = \frac{B_{2,0}}{B_{1,0}} \tag{51}$$

Replacing $B_{2,-1}$ with $\gamma B_{2,-2}$, the last expression can be rewritten as $B_{2,-2}/B_{1,-1} = B_{2,0}/\gamma B_{1,0}$. Note that (49) also implies $\eta_2 = \eta_1^2 = \gamma^2$, where we have replaced ϕ_{-1}/ϕ by γ. Replacing $B_{1,-1}/M_{-1}$ with $B_{1,0}/M_0$, $B_{2,-2}/B_{1,-1}$ with $B_{2,0}/\gamma B_{1,0}$, $\eta_1 = \gamma$ and $\eta_2 = \gamma^2$, we can rewrite (47) as

$$\gamma - 1 - \tau = \frac{B_{1,0}}{M_0}\left(1 - \rho_1\gamma\right) + \frac{B_{2,0}}{M_0\gamma}\left(1 - \rho_2\gamma^2\right) \tag{52}$$

Definition 2 *A steady-state symmetric monetary equilibrium is a triplet* (ρ_1, ρ_2, q_B) *satisfying* (44), (45), *and* (52), *given preference parameters* (n, β), *policy decisions* (γ, τ), *and initial values* $(M_0, B_{1,0}, B_{2,0})$.

2.1 Quantitative exercise

Replace ρ_2 with ρ_1^2 in (52) and rearrange terms to get

$$\rho_1^2 + \frac{B_{1,0}}{B_{2,0}}\rho_1 + \frac{M_0\left(\gamma - 1 - \tau\right) - B_{1,0}}{B_{2,0}\gamma} - \frac{1}{\gamma^2} = 0$$

Let $b_1 = B_{1,0}/M_0 = B_1/M$ and $b_2 = B_{2,0}/M_0 = B_2/M$, then the last expression can be rewritten as

$$\rho_1^2 + \frac{b_1}{b_2}\rho_1 + \frac{\gamma - 1 - \tau - b_1}{b_2\gamma} - \frac{1}{\gamma^2} = 0. \tag{53}$$

Solve for ρ_1 and get the positive solution

$$\rho_1 = \frac{-\frac{b_1}{b_2} + \sqrt{\left(\frac{b_1}{b_2}\right)^2 - 4\left[\frac{\gamma-1-\tau-b_1}{b_2\gamma} - \frac{1}{\gamma^2}\right]}}{2}. \tag{54}$$ [3]

We use the following functional form for the utility function $u\left(q\right) = \ln q$. Using this functional form to replace $u'\left(q_B\right)$ into (44) we get

$$\rho_1 = \frac{n\beta q_B}{\gamma q_B - \beta\left(1 - n\right)}$$

Solve the last equation for q_B and we get

$$q_B = \frac{\rho_1\beta\left(1 - n\right)}{\rho_1\gamma - n\beta} \tag{55}$$

Once we know ρ_1, which is a function of all exogenous parameters, we can derive q_B using this last equation. As in Berentsen and Waller [7] there is an extensive margin inefficiency. Due to the time-cost of holding money, the quantities consumed by all buyers

[3] Extending the model to k-types of bonds, where $k = 1, 2, ..., K$ denotes the maturity, we can rearrange (52) to get

$$\sum_{k=1}^{K} B_{k,0}\gamma\rho_1^k = \sum_{k=1}^{K}\frac{B_{k,0}}{\gamma^{k-1}} - M_0\left(\gamma - 1 - \tau\right)$$

It is well known that there exists a general analytical solution for ρ_1 for $K \leq 4$ otherwise one cannot expect an expression for its zeros by radicals to exist.

are inefficiently low if $\gamma > \beta$. When efficiency is achieved $u'(q_B^*) = 1$, $q_B^* = 1$ and the following equation has to hold

$$\gamma - 1 - \tau = b_1 \left(1 - \frac{\gamma n \beta}{\gamma - \beta(1-n)}\right) + \frac{b_2}{\gamma}\left(1 - \left[\frac{\gamma n \beta}{\gamma - \beta(1-n)}\right]^2\right)$$

where we replaced ρ_1 with (44) in (52). Set $u'(q_B^*) = 1$, $b_1 = B_{1,0}/M_0 = B_1/M$ and $b_2 = B_{2,0}/M_0 = B_2/M$. The only way this equation can hold is to let $b_k \longrightarrow 0$, so that $\gamma = 1 + \tau$. As a consequence, efficiency can never be achieved for given positive values of b_1 and b_2 and $q_B < 1 < \gamma$. Using this result in (46) to get $\gamma > 1 \geq \beta$. Only when $\gamma > 1 \geq \beta$ holds are bonds essential and thus improve the allocation.

3 Empirical results

The data set used here has been downloaded from Bloomberg and the U.S. Department of Commerce, the statistical program used is R[4]. All R-outputs and the used data set can be found in the Appendix.

3.1 Testing equation (45),

Here we use quarterly data from the second quarter of 1991 to the second quarter of 2009, implying 73 data points. We use the result of (45), $\rho_2 = \rho_1^2$ and take the logarithms to get

$$\ln \rho_2 = 2 \ln \rho_1. \tag{56}$$

Where ρ_1 represents a government zero bond with a maturity of three months. For $\ln \rho_1$ we use the approximation

$$\ln \rho_1 = \ln\left(\frac{1}{(1+i)^{1/4}}\right) = -\frac{1}{4}\ln(1+i) \approx -i/4,$$

since i is small enough. For i we use the Bloomberg index C0793M which consists of U.S. Treasury STRIPS[5] with a maturity of three months. The yield i at each maturity point represents the composite yearly return of securities around this maturity. For the return on ρ_2 we use the Bloomberg index C0796M and the approximation

[4]Source: http://www.r-project.org

[5]"STRIPS stands for **S**eparate **T**rading of **R**egistered **I**nterest and **P**rincipal of **S**ecurities. With these securities, interest and principal payments from U.S. Treasury securities are registered separately through the Federal Reserve. Each interest payment and the principal amount can then be sold to investors as a zero coupon bond maturing on the date of the scheduled payment." (http://dictionary.reference.com/browse/separate+trading+of+registered+interest+and+principal+of +securities+%28strips%29 [26.11.2009])

$$\ln \rho_2 = \ln \left(\frac{1}{(1+i)^{1/2}} \right) = -\frac{1}{2} \ln(1+i) \approx -i/2.$$

We first have a look at the time series of $\ln \rho_1$ and $\ln \rho_2$ which are shown in Figure 1.

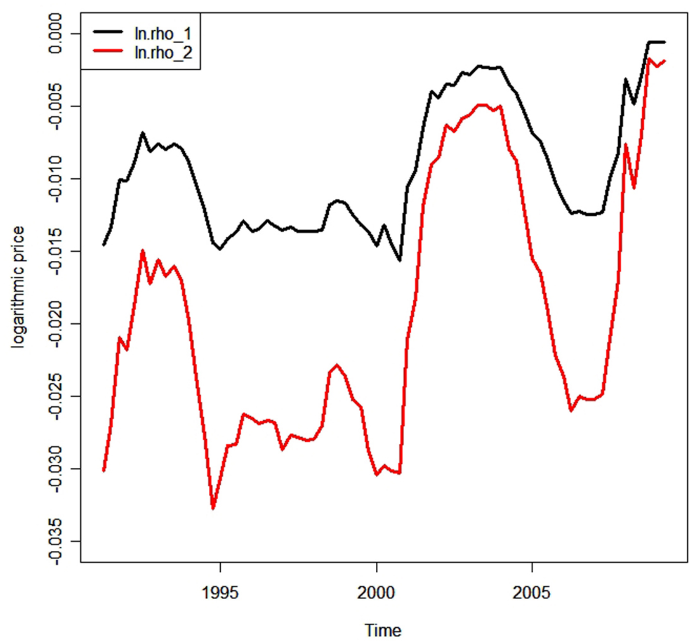

Figure 1: Time series of $\ln \rho_1$ and $\ln \rho_2$

Before we estimate equation (56) we first have to find out whether $\ln \rho_1$ and $\ln \rho_2$ are stationary. For this purpose we use the Augmented-Dicky-Fuller-Test and the KPSS-Test out of the package *tseries* of R. Both tests come to the result that the two time series are not stationary. Taking this result into consideration, we use the Phillips-Ouliaris Cointegration Test to check whether the two time series are cointegrated. The null hypothesis, which states that there is no cointegration between the two variables, is rejected. We test the relation explained in (56) with

$$\ln \rho_{2,t} = \beta_1 + \beta_2 \ln \rho_{1,t} \text{ and } H_0 : \beta_1 = 0,\ \beta_2 = 2. \tag{57}$$

We estimate (57) with the package *dynlm* of R and get the results shown in Table 1:

$\ln \rho_{2,t} \sim \beta_1 + \beta_2 \ln \rho_{1,t}$				
Coefficients	Estimate	Std. Error	t value	$\Pr(> \|t\|)$
β_1	-0.000575	0.000259	-2.22	0.0296
β_2	2.017670	0.025162	80.19	0.0000
Adjusted R-squared		0.9889		

Table 1: Estimation of (57), sample period: 1991:Q2 to 2009:Q2

Table 1 shows that the null hypothesis in (57) cannot be rejected at the 99%-significance level since $\beta_1 = 0$ and $\beta_2 = 2$ are within the 99%-confidence interval (99% CI, hereafter)

$$\begin{aligned} 99\%\ CI\ for\ \beta_1 &: [-0.000575 \pm 2.36 * 0.000259] = [-0.001, 0.000] \text{ and} \\ 99\%\ CI\ for\ \beta_2 &: [2.017670 \pm 2.36 * 0.025162] = [1.958, 2.077]\,. \end{aligned}$$

To see whether (57) is legitimated, we further analyze the residuals out of the regression shown in Table 1. The result of the Phillips-Ouliaris Cointegration Test already showed that the time series of the residuals is stationary. We test the null hypothesis that the first 24 autocorrelations of the residuals are zero using the Box-Pierce Test out of the package *tseries*. We test them as well for heteroscedasticity with the Breusch-Pagan Test out of the package *lmtest*. It appears that we are confronted with homoscedastic and autocorrelated residuals. Homoscedasticity and stationarity of the residuals mean that our specified cointegration relation is a good way to explain the long-run relation between $\ln(\rho_1)$ and $\ln(\rho_2)$, autocorrelation indicates that it has shortcomings in the short run.

3.2 Testing equation (44)

Here we use quarterly data from the second quarter of 1991 to the third quarter of 2008, implying 70 data points. We rewrite (44) for convenience

$$\rho_1 = \frac{n\beta}{\gamma - \beta\,(1-n)\,u'(q_B)}.$$

We use the following data set: For $\gamma - 1 = \pi_t$, we use the quarterly growth rate of the "U.S. Consumer Price Index for all urban consumers: all items"[6]. We assume that the discount factor across periods, β, is constant and set it to $\beta = 1/(1+r) = 0.997$, representing a real quarterly interest rate of $r = 0.003$. Out of the theoretic part, we

[6] Source: CPI index in Bloomberg

know that q_B is the quantity consumed by buyers in the decentralized market. We set $q_B = (1+g_t)/(1+g_{\max})$ where g_t represents the calculated quarterly changes in the U.S. Personal Consumption Expenditures[7]. Since $q_B < 1$ has to hold, we divide all values of $(1+g_t)$ by $(1+g_{\max})$, where $g_{\max}$ is the biggest growth rate observed in the data set rounded up to the second decimal point. We observe $g_{\max} = 0.0251 \approx 0.03$ in the first quarter of 1992. Therefore we set $g_{\max} = 0.03$ so that every value of $q_B = (1+g_t)/(1+g_{\max}) < 1$. As before we first take the logarithms in (44) to get

$$\ln \rho_1 = \ln n\beta - \ln\left(\gamma - \beta\,(1-n)\,u'(q_B)\right)$$

and use the simplifying approximation

$$\ln \rho_1 \approx -n\beta - \pi_t + \beta\,(1-n)\,u'(q_B). \tag{58}$$

How can we justify the above approximation? First, we assume that trades in the economy are maximized so that $n \sim 0.5$. Since $\ln x \approx x - 1$ for x close to 1, it follows for $\ln(0.5) \approx 0.5 - 1 \approx -0.5$ and

$$\ln n\beta \approx n\beta - 1 \approx -n\beta. \tag{59}$$

Second, we have to ensure that $u'(q_B)$ is small enough so that

$$\begin{aligned}\ln\left(\gamma - \beta\,(1-n)\,u'(q_B)\right) &= \ln\left(1+\pi_t - \beta\,(1-n)\,u'(q_B)\right)\\ &\approx \pi_t - \beta\,(1-n)\,u'(q_B).\end{aligned}$$

Due to the fact that in our data sample $q_B \sim 1$ we use a utility function where $u'(1) \sim 0$. Hence, we take the following functional form for the utility function

$$u\,(q) = q\ln(1/q) + q \text{ and } u'\,(q) = \ln(1/q), \text{ for } q \in\,]0,1[\,.$$

This utility function implies decreasing relative risk aversion and is not part of the HARA class[8]. For $u'\,(q_B) = \ln(1/q_B)$ we use the approximation $\ln(1/q_B) = \ln((1+g_{\max})/\,(1+g_t)) = \ln(1+g_{\max}) - \ln(1+g_t) \approx g_{\max} - g_t$. So (58) reduces to

$$\ln \rho_{1,t} \approx -n\beta - \pi_t + \beta\,(1-n)\,(g_{\max} - g_t). \tag{60}$$

[7]Source: U.S. Department of Commerce: Bureau of Economic Analysis, Series ID: PCEC, http://research.stlouisfed.org/fred2/series/PCEC?cid=100 [16.10.2009]

[8]A HARA utility function implies linear risk tolerance. For further details see Lengwiler [25], page 84ff.

As explained in Hyndman [17], we smooth the data of π_t and g_t using a two-sided moving average of degree q,

$$\{y_t\} \to \left\{ \frac{1}{2q+1} \sum_{j=-q}^{q} y_{t-j} \right\},$$

where we use $q = 1, 2$. We make this adjustment since we observe too great variation in the unfiltered data of π_t and g_t to provide useful results. Using a moving average transformation reduces our sample period by q periods at the beginning and the end. We finally test the relation explained in (60) with

$$\begin{aligned} \ln \rho_{1,t} &= \beta_1 + \beta_2 \pi_t + \beta_3 (g_{\max} - g_t) \text{ and} \\ H_0 &: \beta_1 = -n\beta,\ \beta_2 = -1,\ \beta_3 = \beta(1-n). \end{aligned} \tag{61}$$

We first show the results for $q = 2$ in the moving average transformation of π_t and g_t since we get the best results with this adjustment. Later we show the results for $q = 1$ as well.

Testing (61) with q = 2

Please keep in mind that we do not smooth the data of $\ln \rho_{1,t}$. The moving average transformation of π_t and g_t reduces our sample period from the fourth quarter of 1991 to the first quarter of 2008. We estimate (61) with the package *dynlm* of R and get the results shown in Table 2:

$\ln \rho_{1,t} \sim \beta_1 + \beta_2 \pi_t + \beta_3 (g_{\max} - g_t)$				
Coefficients	Estimate	Std. Error	t value	Pr(> $\|t\|$)
β_1	-0.014978	0.003366	-4.450	0.0000
β_2	-0.163492	0.276283	-0.592	0.5561
β_2	0.394691	0.184572	2.138	0.0364
Adjusted R-squared		0.03985		

Table 2: Estimation of (61), sample period: 1991:Q4 to 2008:Q1

To get the optimal value of n, we rearrange the conditions for β_1 and β_3 under the null hypothesis in (61) for n and so gain $n = \beta_1/\beta$ and $n = 1 - \beta_3/\beta$. Hence, we get a 95%-confidence interval (95% CI, hereafter) for n with

$$\begin{aligned} 95\%\ CI_{\beta_1}\ for\ n &: [(-\beta_1 \pm 1.96 * 0.003366)/\beta] = [0.008, 0.022] \text{ and} \\ 95\%\ CI_{\beta_3}\ for\ n &: [1 - (\beta_3 \pm 1.96 * 0.184572)/\beta] = [0.241, 0.967]. \end{aligned}$$

In the two above confidence intervals we do not get an intersection for the optimal value of n. That is why we show the time series of $\ln \rho_{1,t}$ and $(-n\beta - \pi_t + \beta(1-n)(g_{\max} - g_t))$ in Figure 2, where we use the value of $n = 0.018$[9]. We get a correlation of 0.25 between the two time series.

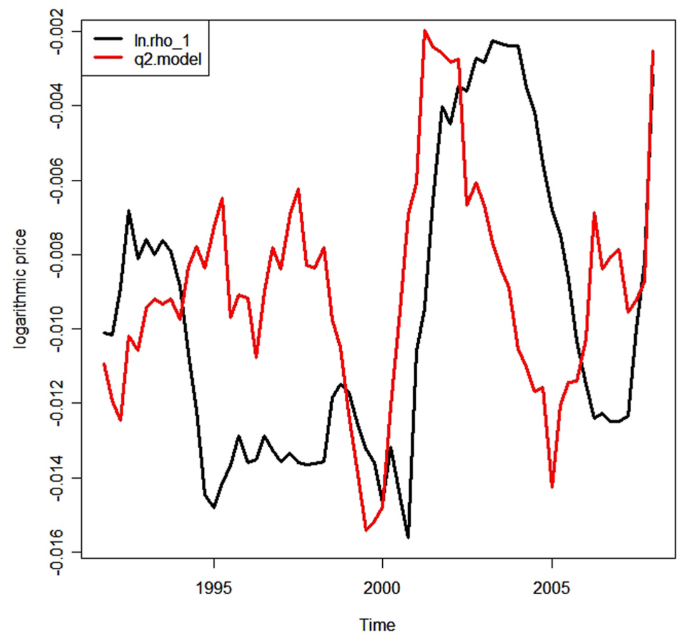

Figure 2: Time series of $\ln \rho_{1,t}$ and the model $(-n\beta - \pi_t + \beta(1-n)(g_{\max} - g_t))$, where $n = 0.018$

In reality people do not act perfectly rationally. That is why the time series of $\ln \rho_{1,t}$ in Figure 2 seems to be delayed compared to the rational behaviour of the agents in our model. To further investigate this result, and to get the best fit in the augmentation periods of the two time series, we try different time lags of π_t and g_t. With a time lag of three quarters we get the most appropriate results. This adjustment shifts our data set of π_t and g_t three quarters into the future, and it is now confined from the third quarter of 1992 to the fourth quarter of 2008.

Figure 2 shows as well that the main problem of the indicated model price lies in the time period from the first quarter of 1994 to the fourth quarter of 1999. An explanation for this result could be the beginning boom in the stock market and the implied danger of inflation. That is why in February 1994 the U.S. Federal Reserve System (US-Fed, here-

[9]Since we do not get an intersection for the optimal value of n, we only consider the condition under the null hypothesis for β_1 in (61). So we choose a value of $n \in (0.008, 0.022)$ that best fits our data set.

after) unexpectedly increased the federal funds rate by 25 basis points. Within the twelve following months the US-Fed intervened six times and increased the federal funds rate from three to six percent. In hindsight this reaction was far too extreme and cannot be explained with our approach. After 1999, the US-Fed changed its course of action from a tightened monetary policy to a neutral one which is in line with our theory. That is why we observe the period from the first quarter of 2000 to the fourth quarter of 2008 in more detail, where we lag the time series of π_t and g_t by three quarters. We estimate equation (61) again with $\ln \rho_{1,t}$, π_{t-3} and g_{t-3} and get the results shown in Table 3:

$\ln \rho_{1,t} \sim \beta_1 + \beta_2 \pi_{t-3} + \beta_3 (g_{\max} - g_{t-3})$				
Coefficients	Estimate	Std. Error	t value	$\Pr(> \lvert t \rvert)$
β_1	-0.01870	0.00328	-5.703	0.0000
β_2	-0.63649	0.25044	-2.541	0.0159
β_2	0.95313	0.15705	6.069	0.0000
Adjusted R-squared		0.5488		

Table 3: Estimation of (61), sample period: 2000:Q1 to 2008:Q4

Again we rearrange the conditions for β_1 and β_3 under the null hypothesis in (61) to get a 95%-confidence interval for the optimal value of n with

$$\begin{aligned} 95\%\ CI_{\beta_1}\ for\ n &: \quad [(-\beta_1 \pm 1.96 * 0.00328)\ / \beta] = [0.012, 0.025] \text{ and} \\ 95\%\ CI_{\beta_3}\ for\ n &: \quad [1 - (\beta_3 \pm 1.96 * 0.15705)\ / \beta] = [-0.265, 0.353]\,. \end{aligned}$$

We have an intersection for the optimal value of n, $n \in (0.012, 0.025)$ and use the value in between $n = (0.012 + 0.025)/2 = 0.0185$. The null hypothesis in (61),

$$H_0 : \beta_1 = -n\beta = -0.0184,\ \beta_2 = -1 \text{ and } \beta_3 = \beta (1 - n) = 0.979,$$

cannot be rejected at the 95% significance level since the specified values are within the 95% confidence interval

$$\begin{aligned} 95\%\ CI\ for\ \beta_1 &: \quad [-0.01870 \pm 1.96 * 0.00328] = [-0.251, -0.012]\,, \\ 95\%\ CI\ for\ \beta_2 &: \quad [-0.63649 \pm 1.96 * 0.25044] = [-1.127, -0.146] \text{ and} \\ 95\%\ CI\ for\ \beta_3 &: \quad [0.95313 \pm 1.96 * 0.15705] = [1.261, 0.645]\,. \end{aligned}$$

We test $\ln \rho_{1,t}$, π_{t-3} and $(g_{\max} - g_{t-3})$ with the Augmented-Dicky-Fuller-Test and the KPSS-Test for stationarity. We get a clear result for stationarity of $\ln \rho_{1,t}$, non-stationarity of π_{t-3} and an ambiguous result for $(g_{\max} - g_{t-3})$. Due to the non-stationarity of π_{t-3}

our results in Table 3 cannot be interpreted directly, since normal test statistics are only correct under the assumption of stationarity. The adjusted R-Squared has a value of 0.55, indicating that our proposed relation between $\ln \rho_{1,t}$, π_{t-3} and $(g_{\max} - g_{t-3})$ is adequate in the long run. We further analyze the residuals out of the regression shown in Table 3. We see that we are confronted with homoscedastic and autocorrelated residuals, whilst we get an ambiguous result for stationarity. The observed autocorrelation in the residuals tells us that our approach has shortcomings in the short run.

One might ask if we can use these results to forecast the price development of $\ln \rho_{1,t}$. In the above analysis we used the value $q = 2$ for the moving average transformation of π_t and g_t and afterwards lagged these two time series by three quarters. Using $q = 2$ reduces our time series of π_t and g_t by two quarters at the beginning and the end of the sample period, whilst the lagging shifts the two time series by three quarters into the future. It follows that the last quarter is the forecasted value of $\ln \rho_{1,t}$. When we consider that the actual values of π_t and g_t are usually available with a delay of about one quarter, we do not have any forecasting power anymore. In Figure 3 we show the time series of $\ln \rho_{1,t}$ and $(-n\beta - \pi_{t-3} + \beta(1-n)(g_{\max} - g_{t-3}))$ where the value of $n = 0.0185$. We highlight the last quarter of our model price in green and the realization of $\ln \rho_{1,t}$ in blue.

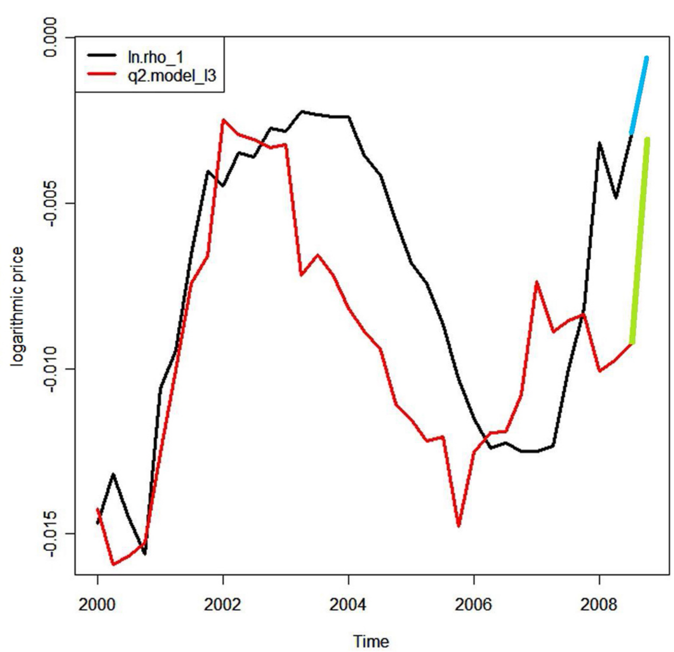

Figure 3: Time series of $\ln \rho_{1,t}$ and the model $(-n\beta - \pi_t + \beta(1-n)(g_{\max} - g_t))$, where $n = 0.0185$

We observe a correlation of 0.75 between the two time series in Figure 3. The standard deviaton of $\ln \rho_{1,t}$ is 4.5% and of $(-n\beta - \pi_{t-3} + \beta (1-n)(g_{\max} - g_{t-3}))$ it is 3.8%. According to the lower standard deviation of $(-n\beta - \pi_{t-3} + \beta (1-n)(g_{\max} - g_{t-3}))$ compared to $\ln \rho_{1,t}$ a moving average transformation with $q < 2$ of π_t and g_t seems to provide accurate results.

Testing (61) with $q = 1$:

Again we do not smooth the data of $\ln \rho_{1,t}$. After the moving average transformation of π_t and g_t with $q = 1$, we directly lag these two time series by three quarters. We analyze the sample period from the first quarter of 2000 to the first quarter of 2009. We estimate (61) and get the results shown in Table 4:

$\ln \rho_{1,t} \sim \beta_1 + \beta_2 \pi_{t-3} + \beta_3 (g_{\max} - g_{t-3})$				
Coefficients	Estimate	Std. Error	t value	Pr(> |t|)
β_1	-0.01716	0.00261	-6.575	0.0000
β_2	-0.43798	0.20694	-2.116	0.0417
β_2	0.77970	0.13160	5.925	0.0000
Adjusted R-squared		0.4945		

Table 4: Estimation of (61), sample period: 2000:Q1 to 2009:Q1

As before we get a 95%-confidence interval for the optimal value of n with

$$\begin{aligned} 95\%\ CI_{\beta_1}\ for\ n &: \left[\left(-\beta_1 \pm 1.96 * 0.00261\right)/\beta\right] = [0.012, 0.022], \\ 95\%\ CI_{\beta_3}\ for\ n &: \left[1 - \left(\beta_3 \pm 1.96 * 0.13160\right)/\beta\right] = [-0.041, 0.477]. \end{aligned}$$

We have an intersection for the optimal value of n, $n \in (0.012, 0.022)$ and use the value in between $n = (0.012 + 0.022)/2 = 0.017$. In contrast to the estimation with $q = 2$, the null hypothesis in (61),

$$H_0 : \beta_1 = -n\beta = -0.017,\ \beta_2 = -1 \text{ and } \beta_3 = \beta(1-n) = 0.98,$$

is still rejected at the 95% significance level since $\beta_2 = -1$ is not within the 95% confidence interval

$$95\%\ CI\ for\ \beta_2 : [-0.43798 \pm 1.96 * 0.20694] = [-0.844, -0.032].$$

In Figure 4 we show the time series of $\ln \rho_{1,t}$ and $(-n\beta - \pi_{t-3} + \beta (1-n)(g_{\max} - g_{t-3}))$ for $q = 1$, where the value of $n = 0.017$. Out of the adjustment with $q = 1$ and the lagging of π_t and g_t by three quarters, it follows that the last two quarters are the forecasted values of $\ln \rho_{1,t}$. When we consider that the actual values of π_t and g_t are usually available with a delay of about one quarter, only the last quarter is the feasible forecast. In Figure 4 we highlight the last two quarters of our model price in green and the realization of $\ln \rho_{1,t}$ in blue.

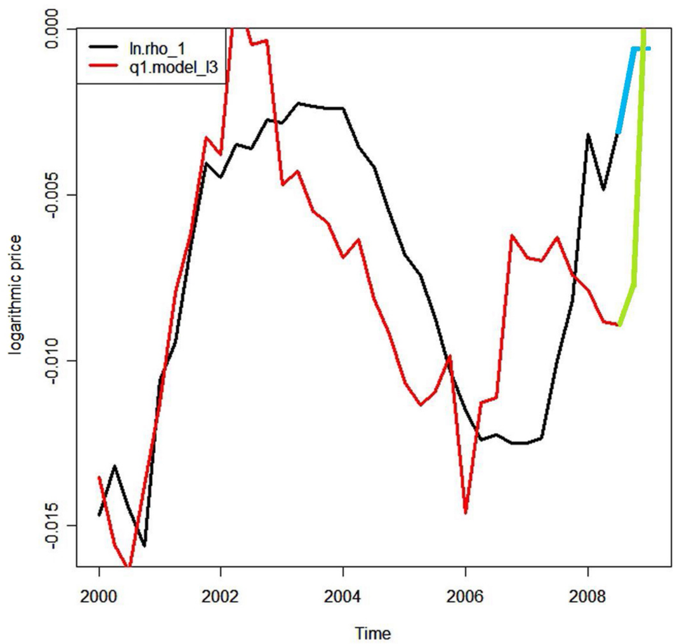

Figure 4: Time series of $\ln \rho_{1,t}$ and the model $(-n\beta - \pi_t + \beta(1-n)(g_{\max} - g_t))$, where $n = 0.017$.

In Figure 4 we see that our forecasted price path is useful when sharp price movements are observed. We still have a correlation of 0.70 between the two time series. The standard deviaton of $\ln \rho_{1,t}$ and $(-n\beta - \pi_{t-3} + \beta(1-n)(g_{\max} - g_{t-3}))$ is in each case 4.5%. Using a moving average transformation with $q = 1$ of π_t and g_t seems to be appropriate when our theory is used to forecast $\ln \rho_{1,t}$.

The main problem of our empirical approach is that assumption (59) is critical for such small values of n. The fact that the optimized values of n are so small indicates that the price would be much lower if trades in the economy were maximized and $n \sim 0.5$. As a consequence of these small values of n, the main driving forces in the price development of $\ln \rho_{1,t}$ are caused by g_t and π_t.

4 Conclusion

In this thesis we analyzed the factors that determine the prices of government zero bonds when agents are cash constrained. The following results emerged from our analysis. For any positive inflation rate, bonds are essential and thus improve the allocation. The efficiency improvement results from the possibility that some agents can deposit their idle money and earn positive interest. The main driving forces in the price development of the zero bonds are the relative quantity of consumers and producers in the economy and the efficiency of trades. With our approach we showed how to derive the bond price as a function of all endogenous as well as exogenous variables. We were able to explain a flat yield curve of the term structure of interest rates and the shifts which occur. As a consequence of our environment, we were not able to deliver an explanation for the term spread between zero bonds of different maturities.

We offered the results of different techniques used to check our approach and observed that the price development explained by our model proves competent in providing some forecasting capacity. This is due to the fact that in reality people do not act perfectly rationally. Furthermore, we offered advice on the practical implementation of our results in forecasting the price development of government zero bonds with a maturity of three months.

Appendix

R-outputs

3.1 Testing equation (45)

Sample period: From the the second quarter of 1991 to the second quarter of 2009.

Augmented-Dicky-Fuller-Test and KPSS-Test for $\ln \rho_{1,t}$

```
R> adf.test(ln.rho_1)

        Augmented Dickey-Fuller Test

data:  ln.rho_1
Dickey-Fuller = -3.0554, Lag order = 4, p-value = 0.1454
alternative hypothesis: stationary

R> kpss.test(ln.rho_1)

        KPSS Test for Level Stationarity

data:  ln.rho_1
KPSS Level = 1.1448, Truncation lag parameter = 1, p-value = 0.01

Warning message:
p-value smaller than printed p-value in: kpss.test(ln.rho_1)
```

Augmented-Dicky-Fuller-Test and KPSS-Test for $\ln \rho_{2,t}$

```
R> adf.test(ln.rho_2)

        Augmented Dickey-Fuller Test

data:  ln.rho_2
Dickey-Fuller = -3.152, Lag order = 4, p-value = 0.1060
alternative hypothesis: stationary

R> kpss.test(ln.rho_2)

        KPSS Test for Level Stationarity

data:  ln.rho_2
KPSS Level = 1.1593, Truncation lag parameter = 1, p-value = 0.01

Warning message:
p-value smaller than printed p-value in: kpss.test(ln.rho_2)
```

Phillips-Ouliaris Cointegration Test for $\ln \rho_{1,t}$ *and* $\ln \rho_{2,t}$

```
R> x <- cbind(ln.rho_1, ln.rho_2)
R> po.test(x)

        Phillips-Ouliaris Cointegration Test

data:  x
Phillips-Ouliaris demeaned = -32.8035, Truncation lag parameter = 0,
p-value = 0.01

Warning message:
p-value smaller than printed p-value in: po.test(x)
```

Estimation of (57), source of Table 1

```
R> lm1 <- dynlm(ln.rho_2~ln.rho_1)
R> summary(lm1)

Call:
dynlm(formula = ln.rho_2 ~ ln.rho_1)

Residuals:
       Min         1Q     Median         3Q        Max
-2.970e-03 -4.087e-04  3.451e-06  6.018e-04  1.801e-03

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.000575   0.000259   -2.22   0.0296 *
ln.rho_1     2.017670   0.025162   80.19   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.0009425 on 71 degrees of freedom
Multiple R-Squared: 0.9891,     Adjusted R-squared: 0.9889
F-statistic:  6430 on 1 and 71 DF,  p-value: < 2.2e-16
```

Box-Pierce Test and Breusch-Pagan Test for the residuals out of the regression shown in Table 1

```
R> residuals <- residuals(lm1)
R> Box.test(residuals, lag=24, type="Box-Pierce")

        Box-Pierce test

data:  residuals
X-squared = 50.9067, df = 24, p-value = 0.001084

R> bptest(lm1)

        studentized Breusch-Pagan test

data:  lm1
BP = 3.7552, df = 1, p-value = 0.05264
```

3.2 Testing equation (44)

Testing (61) with q = 2

Sample period: From the the fourth quarter of 1991 to the first quarter of 2008.

Estimation of (61), source of Table 2

```
R> lmq2 <- dynlm(ln.rho_1~q2.Pi+q2.ln.q_B)
R> summary(lmq2)

Call:
dynlm(formula = ln.rho_1 ~ q2.Pi + q2.ln.q_B)

Residuals:
       Min         1Q     Median         3Q        Max
-0.0069421 -0.0032113 -0.0008496  0.0027940  0.0078937

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.014978   0.003366  -4.450 3.57e-05 ***
q2.Pi       -0.163492   0.276283  -0.592   0.5561
q2.ln.q_B    0.394691   0.184572   2.138   0.0364 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.003955 on 63 degrees of freedom
Multiple R-Squared: 0.0694,     Adjusted R-squared: 0.03985
F-statistic: 2.349 on 2 and 63 DF,  p-value: 0.1038
```

Sample period: From the the first quarter of 2000 to the fourth quarter of 2008.

Augmented-Dicky-Fuller-Test and KPSS-Test for $\ln \rho_{1,t}$

```
R> adf.test(ln.rho_1)

        Augmented Dickey-Fuller Test

data:  ln.rho_1
Dickey-Fuller = -3.9752, Lag order = 3, p-value = 0.02179
alternative hypothesis: stationary

R> kpss.test(ln.rho_1)

        KPSS Test for Level Stationarity

data:  ln.rho_1
KPSS Level = 0.243, Truncation lag parameter = 1, p-value = 0.1

Warning message:
p-value greater than printed p-value in: kpss.test(ln.rho_1)
```

Augmented-Dicky-Fuller-Test and KPSS-Test for π_{t-3} *with* $q = 2$ *in the moving average transformation*

```
R> adf.test(q2.Pi_l3)

        Augmented Dickey-Fuller Test

data:  q2.Pi_l3
Dickey-Fuller = -2.3739, Lag order = 3, p-value = 0.4277
alternative hypothesis: stationary

R> kpss.test(q2.Pi_l3)

        KPSS Test for Level Stationarity

data:  q2.Pi_l3
KPSS Level = 0.5163, Truncation lag parameter = 1, p-value = 0.038
```

Augmented-Dicky-Fuller-Test and KPSS-Test for $(g_{\max} - g_{t-3})$ *with* $q = 2$ *in the moving average transformation*

```
R> adf.test(q2.ln.q_B_l3)

        Augmented Dickey-Fuller Test

data:  q2.ln.q_B_l3
Dickey-Fuller = -0.9697, Lag order = 3, p-value = 0.9287
alternative hypothesis: stationary

R> kpss.test(q2.ln.q_B_l3)

        KPSS Test for Level Stationarity

data:  q2.ln.q_B_l3
KPSS Level = 0.3068, Truncation lag parameter = 1, p-value = 0.1

Warning message:
p-value greater than printed p-value in: kpss.test(q2.ln.q_B_l3)
```

Estimation of (61), source of Table 3

```
R> lmq2_l3 <- dynlm(ln.rho_1~q2.Pi_l3+q2.ln.q_B_l3)
R> summary(lmq2_l3)

Call:
dynlm(formula = ln.rho_1 ~ q2.Pi_l3 + q2.ln.q_B_l3)

Residuals:
       Min         1Q     Median         3Q        Max
-0.0061240 -0.0019105 -0.0003156  0.0028507  0.0045727

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  -0.01870    0.00328  -5.703 2.31e-06 ***
q2.Pi_l3     -0.63649    0.25044  -2.541   0.0159 *
q2.ln.q_B_l3  0.95313    0.15705   6.069 7.87e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.002993 on 33 degrees of freedom
Multiple R-Squared: 0.5746,     Adjusted R-squared: 0.5488
F-statistic: 22.28 on 2 and 33 DF,  p-value: 7.513e-07
```

Augmented-Dicky-Fuller-Test and KPSS-Test for the residuals out of the regression shown in Table 3

```
R> residuals <- residuals(lmq2_l3)
R> adf.test(residuals)

        Augmented Dickey-Fuller Test

data:  residuals
Dickey-Fuller = -3.0585, Lag order = 3, p-value = 0.1613
alternative hypothesis: stationary

R> kpss.test(residuals)

        KPSS Test for Level Stationarity

data:  residuals
KPSS Level = 0.1974, Truncation lag parameter = 1, p-value = 0.1

Warning message:
p-value greater than printed p-value in: kpss.test(residuals)
```

Box-Pierce Test and Breusch-Pagan Test for the residuals out of the regression shown in Table 3

```
R> Box.test(residuals, lag=24, type="Box-Pierce")

        Box-Pierce test

data:  residuals
X-squared = 82.3937, df = 24, p-value = 2.517e-08

R> bptest(lmq2_l3)

        studentized Breusch-Pagan test

data:  lmq2_l3
BP = 0.9904, df = 2, p-value = 0.6094
```

Testing (61) with $q = 1$:

Sample period: From the the first quarter of 2000 to the first quarter of 2009

Estimation of (61), source of Table 4

```
R> lmq1_l3 <- dynlm(ln.rho_1~q1.Pi_l3+q1.ln.q_B_l3)
R> summary(lmq1_l3)

Call:
dynlm(formula = ln.rho_1 ~ q1.Pi_l3 + q1.ln.q_B_l3)

Residuals:
       Min         1Q     Median         3Q        Max
-0.0060079 -0.0019377 -0.0001585  0.0026784  0.0046440

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  -0.01716    0.00261  -6.575 1.56e-07 ***
q1.Pi_l3     -0.43798    0.20694  -2.116   0.0417 *
q1.ln.q_B_l3  0.77970    0.13160   5.925 1.08e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.003222 on 34 degrees of freedom
Multiple R-Squared: 0.5226,     Adjusted R-squared: 0.4945
F-statistic: 18.61 on 2 and 34 DF,  p-value: 3.472e-06
```

Data set

Date	C0793M Index	C0796M Index	CPI Index	PCEC
29.03.1991			134.8	3963.3
28.06.1991	0.0583	0.0603	136.0	4008.7
30.09.1991	0.0535	0.0537	137.0	4038.6
31.12.1991	0.0404	0.0419	138.2	4140.1
31.03.1992	0.0407	0.0436	139.1	4193.5
30.06.1992	0.0356	0.0376	140.1	4267.7
30.09.1992	0.0272	0.0298	141.1	4346.2
31.12.1992	0.0324	0.0345	142.3	4384.9
31.03.1993	0.0304	0.0312	143.3	4452.1
30.06.1993	0.0320	0.0335	144.3	4516.3
30.09.1993	0.0305	0.0321	145.0	4581.1
31.12.1993	0.0317	0.0341	146.3	4650.4
31.03.1994	0.0354	0.0395	147.1	4709.8
30.06.1994	0.0425	0.0484	147.9	4786.3
30.09.1994	0.0487	0.0553	149.3	4856.7
30.12.1994	0.0578	0.0654	150.1	4888.7
31.03.1995	0.0593	0.0613	151.2	4957.5
30.06.1995	0.0566	0.0568	152.4	5022.9
29.09.1995	0.0547	0.0566	153.1	5080.1
29.12.1995	0.0515	0.0524	153.9	5156.5
29.03.1996	0.0544	0.0530	155.5	5248.8
28.06.1996	0.0540	0.0538	156.7	5304.4
30.09.1996	0.0515	0.0533	157.7	5384.7
31.12.1996	0.0531	0.0536	159.1	5467.1
31.03.1997	0.0543	0.0573	159.8	5504.0
30.06.1997	0.0534	0.0554	160.2	5613.3
30.09.1997	0.0544	0.0556	161.2	5698.1
31.12.1997	0.0546	0.0561	161.8	5757.5
31.03.1998	0.0545	0.0559	162.0	5870.2
30.06.1998	0.0543	0.0540	162.8	5968.0
30.09.1998	0.0474	0.0468	163.5	6078.2
31.12.1998	0.0460	0.0456	164.4	6157.4
31.03.1999	0.0467	0.0472	164.8	6290.0
30.06.1999	0.0500	0.0504	166.0	6398.9
30.09.1999	0.0529	0.0514	167.8	6524.9
31.12.1999	0.0544	0.0575	168.8	6683.0

Date	C0793M Index	C0796M Index	CPI Index	PCEC
31.03.2000	0.0587	0.0609	171.0	6775.7
30.06.2000	0.0527	0.0596	172.2	6881.7
29.09.2000	0.0580	0.0603	173.6	6981.1
29.12.2000	0.0625	0.0606	174.6	7058.1
30.03.2001	0.0424	0.0421	176.1	7118.7
29.06.2001	0.0378	0.0365	177.7	7151.2
28.09.2001	0.0262	0.0240	178.1	7267.2
31.12.2001	0.0161	0.0181	177.4	7309.0
29.03.2002	0.0179	0.0171	178.5	7403.4
28.06.2002	0.0139	0.0127	179.6	7491.2
30.09.2002	0.0144	0.0135	180.8	7553.2
31.12.2002	0.0109	0.0117	181.8	7646.9
31.03.2003	0.0113	0.0112	183.9	7723.8
30.06.2003	0.0089	0.0099	183.1	7882.5
30.09.2003	0.0093	0.0099	185.1	7962.8
31.12.2003	0.0095	0.0106	185.5	8105.3
31.03.2004	0.0095	0.0100	187.1	8209.4
30.06.2004	0.0141	0.0159	188.9	8330.7
30.09.2004	0.0166	0.0176	189.8	8494.9
31.12.2004	0.0221	0.0247	191.7	8609.6
31.03.2005	0.0272	0.0310	193.1	8747.2
30.06.2005	0.0298	0.0331	193.6	8908.8
30.09.2005	0.0347	0.0384	198.7	9010.3
30.12.2005	0.0413	0.0443	198.3	9148.2
31.03.2006	0.0460	0.0473	199.8	9266.6
30.06.2006	0.0497	0.0521	201.7	9391.8
29.09.2006	0.0491	0.0500	202.8	9484.1
29.12.2006	0.0500	0.0504	203.3	9658.5
30.03.2007	0.0500	0.0504	205.3	9762.5
29.06.2007	0.0494	0.0497	207.0	9865.6
28.09.2007	0.0400	0.0420	208.4	10019.2
31.12.2007	0.0327	0.0341	211.7	10095.1
31.03.2008	0.0126	0.0153	213.7	10194.7
30.06.2008	0.0194	0.0214	217.0	10220.1
30.09.2008	0.0118	0.0139	218.7	10009.8
31.12.2008	0.0023	0.0035	211.6	9987.7
31.03.2009	0.0024	0.0047		
30.06.2009	0.0024	0.0038		

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"Ich bezeuge mit meiner Unterschrift, dass meine Angaben über die bei der Abfassung meiner Arbeit benützten Hilfsmittel sowie über die mir zuteil gewordene Hilfe in jeder Hinsicht der Wahrheit entsprechen und vollständig sind. Ich habe das Merkblatt zu Plagiat und Betrug vom 23.11.05 gelesen und bin mir den Konsequenzen eines solchen Handelns bewusst."

Ort, Datum: Unterschrift: