The Role of Liquidity and Financial Intermediation for Investment *

by

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Master's Thesis

Spring 2010

Submitted July 17th, 2010

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*I would like to thank Prof. Dr. Aleksander Berentsen for his very helpful comments.

1 Introduction

The feature of this thesis is to investigate the behavior of a representative agent regarding an investment subject to a productivity shock. Also, we analyze how the behavior of agents changes with various forms of liquidity. Finally, we perform this investigation within a widely used framework in monetary theory.

Investment is essential for economic growth. Its process is facilitated by financial intermediation. Financial intermediaries borrow from savers and lend to investors. In our model, the need for financial intermediaries occurs due to a productivity shock. This shock creates a shortage of liquidity which can be filled with financial intermediation. Our goal is to find the optimal level of investment in such an environment. We start with computing the first-best solution in autarky, without money. Then, we focus on a monetary economy where agents must trade with each other. The optimal level of investment changes in the sense that it takes inflation into consideration. Finally, we see how financial intermediation affects the investment behavior of agents.

Rochet and Tirole (1996) developed a model where a productivity shock can induce the closure of banks because of their credit relations. Holmström and Tirole (1998) adapted this model for the productive sector. Our model is based on the former. However, it is simplified. We choose this particular model because unlike others, it can easily be adapted to the framework studied here.

A growing literature focuses on financial intermediation. Berentsen, Camera and Waller (2007) examine a monetary economy with trading shocks, which lead to trading inefficiencies because agents hold suboptimal money balances. This justifies intermediation, since it creates a possibility for agents with idle balances to lend to cash constrained agents. We deviate from their model in the sense that the main focus here is on a productivity shock. Furthermore, we do not impose a preference shock to agents. Kobayashi (2009) extends the model of Berentsen, Camera and Waller (2007). He focuses on bank crises and allows banks to perform credit creation. We allow inside bonds to circulate, but we do not consider a banking sector.

Liquidity consists of a class of assets that are useful in trades or available for investment. We consider two assets: money and inside bonds. Berentsen and Waller (2009) analyze two different possibilities for liquidity constrained agents: selling outside bonds or issuing inside bonds. In addition to money, we opt for an economy with inside bonds as it is able to replicate the economy with outside bonds. Our definition of inside bonds is similar as in Kocherlakota (2003), where they transfer money from agents with a low marginal value of money to agents with a high one. Our model provides unforeseen results regarding financial intermediation.

Search and matching is an approach used by many monetary theories. Williamson and Wright (2010; a,b) redefine the Lagos and Wright model and describe a unique framework for monetary economics, useful to discuss a variety of issues in monetary theory. We build our framework on Lagos and Wright (2005) to stay in such a common setup.

The paper is organized as follows. Section 2 describes the benchmark environment of the model. In Section 3, we construct the basic model and focus on autarky. Further, in Section 4, we introduce money into the model. Section 5 adds financial intermediation. Finally, we conclude in Section 6.

2 The benchmark environment

The framework is a variant of the divisible money model developed in Lagos and Wright (2005). Time is discrete and each period t is divided into two perfectly competitive markets that open sequentially. First, the agents go through a *settlement market* (SM) and then enter a *decentralized market* (DM). There is only one type of agents which are households. They live forever and discount future utilities at rate β, where $0 < \beta < 1$. There is one perishable good. Agents produce and consume that general good in the settlement market, and work in the decentralized market.

In any period including one SM and one DM, preferences are described by a standard utility function $\mathcal{U} = (x, h_1, h_2)$ linear in h_1 and h_2. That is, $\mathcal{U}$ is quasi-linear. Consumption and production in the first market are denoted by x and h_1 respectively, and h_2 is the number of hours worked in the second market. Moreover, we assume $u(x) > 0$, $u'(x) > 0$, $u'(0) = +\infty$, $u'(+\infty) = 0$, and $u''(x) < 0$. Furthermore, one unit of labor is rewarded with one unit of the good, and generates one unit of disutility.

The agents have a technology which allows them to store the general good across periods. More precisely, this technology allows to transform an investment I in the SM at date t into $R(I)$ one period later. We call this sequence a *project*. We assume that $R(I)$ is monotonically increasing and strictly differentiable, with $R(0) = 0$, $R'(0) = \infty$, $R'(\cdot) > 0$ and $R''(\cdot) < 0$.

Agents face a productivity shock in the DM. This shock is in the spirit of Rochet and Tirole (1996) and influences the project. An initial investment is made in the first market. In the second market, an amount ρI of supplementary hours of work is needed to continue with the project. The amplitude of the shock is given by $\rho \epsilon [0, \infty)$, so that ρI is proportional to the size of the investment I. The agent can also decide to let the project fail. He can either work this additional amount of hours, in which case the project is carried on and yields its return, or decide not to, in which case the project is abandoned and yields nothing.

In a first stage, an agent enters the settlement market. He decides how much to consume, produce, and invest in the technology. Then, he moves on to the decentralized market where he faces a productivity shock. He decides whether to continue or not with the project. If he continues, he works ρI hours. If he lets the project fail, he doesn't work. Finally, he enters the next period, where the outcome of his previous period investment depends on the decision made in the DM.

An agent must work h_2 hours in the DM if he decides to continue with the project. This work can be interpreted as some repairs or maintenance on a "machine" called *project*. That is, work in the second market is only done if the agent decides to continue with the project. Because quasi-linearity is assumed, we have $h_2 = \rho I$. Moreover, ρ is distributed according to the cumulative distribution function $F(\rho)$ with support $[0, 1]$ and density $f(\rho)$ on $[0, \infty)$. Thus, the project consists of an initial investment, a productivity shock, and a return. In the middle stage, i.e., when the shock hits, the agent can decide whether to continue or not with the project. This in turn influences the return, which is either $R(I)$ or 0.

The timing of events is represented in Figure 1. In what follows, we analyze a representative period t and focus on an economy in equilibrium.

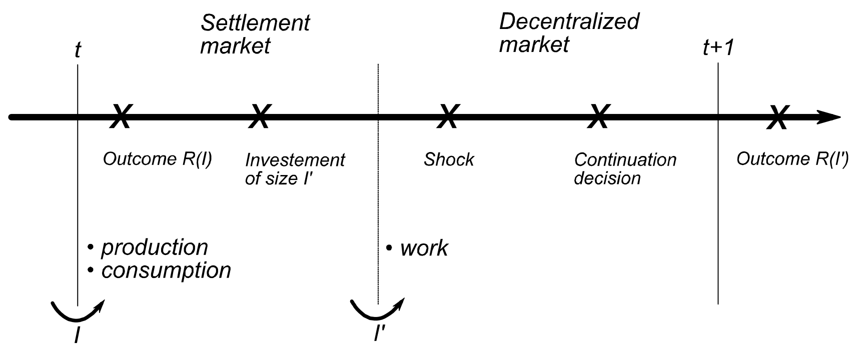

Figure 1: The timing of events

3 Autarky

In this Section, we look at a simple version of the model. We consider only one agent, living in autarky à la Robinson Crusoe. Our first step is to describe an optimal, non-monetary setup.

The good can be pictured as apples. In the first market, he can work in order to gather the fruits, eat them, or decide to plant an apple which turns into an apple tree. If he takes care of it in the second market, the tree gives R apples in the next period. In the SM, the agent makes an investment I and chooses the amount of goods to produce and to consume. Then, conditional on the level of ρ, he decides to work ρI hours, and earns the return $R(I)$ in the next period. On the other hand, if he decides not to work, the project is abandoned and becomes worthless.

At the beginning of the DM, the agent has to decide whether to continue or not with the project. He takes the initial investment I as given. Thus, I is ignored for the decision. Continuation yields $R(I)$ in the following period. The additional amount of hours incurred is ρI. Therefore, the project is continued only if the discounted return is greater than or equal to the shock. The continuation constraint for the project is

$$
\begin{aligned}
0 &\leqslant -\rho I + \beta R(I) \\
\rho &\leqslant \frac{\beta R(I)}{I}.
\end{aligned}
$$

This inequality implies that an agent can decide to work ρI hours also when the expected value of the entire project is negative. The cut-off value for $\overline{\rho}$ is

$$
\overline{\rho} = \frac{\beta R(I)}{I}. \tag{1}
$$

That is, the project is continued if and only if $\rho \leqslant \overline{\rho}$. In the case of $\rho > \overline{\rho}$, the project is abandoned. Note that $\bar{\rho}$ is decreasing in I. This implies a wedge between a large initial investment with the possibility to withstand a low shock and a small initial investment with the possibility to withstand a high shock. The number of hours worked in the DM is

$$
h_2 = \begin{cases} \rho I & \\ 0 & \end{cases} \text{if} \quad \begin{matrix} \rho \leqslant \overline{\rho} \\ \rho > \overline{\rho} \end{matrix}
$$

and this in turn implies that the return for the project is

$$
R(I) = \begin{cases} R(I) & \\ 0 & \end{cases} \text{if} \quad \begin{matrix} \rho \leqslant \overline{\rho} \\ \rho > \overline{\rho} \end{matrix}
$$

The agent earns the return $R(I)$ in the SM at date t on his investment I made at date $t-1$. In the SM at date t, he makes a new investment I', which expires at $t+1$. Moreover, in the DM, he chooses whether to carry on or not with the project given a certain shock. The lifetime maximization problem of a representative agent thus is

$$
\begin{aligned}
&\max_{x,h_1,I'} u(x) - h_1 + \int_0^{\infty} \left[\beta R(I') - h_2\right] f(\rho) d\rho. \\
&s.t. \qquad x + I' = R(I) + h_1
\end{aligned} \tag{2}
$$

An agent maximizes his surplus $u(x) - h_1$ and the continuation value of his project. The budget constraint shows that the consumption added to the amount of goods invested must be equal to the return earned from the previous period investment plus the hours worked (in terms of goods). In what follows, we replace h_2 with ρI. As noted before, if $\rho > \bar{\rho}$, the project is abandoned. Therefore, $\int_{\bar{\rho}}^{\infty} [\beta R(I) - \rho I] f(\rho) d\rho = 0$. Isolating h_1 from the constraint and substituting into Equation (2) yields

$$\max_{x,I'} u(x) - x + R(I) - I' + \int_0^{\bar{\rho}} [\beta R(I') - \rho I'] f(\rho) d\rho. \tag{3}$$

The first-order condition with regard to x is $u'(x) = 1$, which means that the optimal consumption must be equal to the marginal production cost, 1. To compute the first-order condition for I', let us first rewrite Problem (3) and replace $\bar{\rho}$ with the cut-off value computed in Equation (1). We have

$$\max_{I'} -I' + \int_0^{\frac{\beta R(I')}{I'}} [\beta R(I') - \rho I'] f(\rho) d\rho.$$

Using the Leibniz integral rule, the derivative is

$$\begin{aligned} -1 + \frac{\partial \frac{\beta R(I')}{I'}}{\partial I'} [\beta R(I') - \beta R(I')] F \left[\frac{\beta R(I')}{I'}\right] \\ + \int_0^{\frac{\beta R(I')}{I'}} \frac{\partial}{\partial I'} [\beta R(I') - \rho I'] f(\rho) d\rho = 0. \end{aligned}$$

This can be reduced as follows

$$\begin{aligned} -1 + \int_0^{\frac{\beta R(I')}{I'}} [\beta R'(I') - \rho] f(\rho) d\rho = 0 \\ -1 + \beta R'(I') F \left[\frac{\beta R(I')}{I'}\right] - \int_0^{\frac{\beta R(I')}{I'}} \rho f(\rho) d\rho = 0 \end{aligned}$$

Therefore, the optimal level of investment I'^{*} in autarky satisfies the following equality

$$\beta R'(I')F\left[\frac{\beta R(I')}{I'}\right] = 1 + \int_0^{\frac{\beta R(I')}{I'}} \rho f(\rho)d\rho. \qquad (4)$$

The left-hand side (LHS) is the discounted expected marginal return of the project and the right-hand side (RHS) is the expected marginal cost of the project. In other words, the discounted conditional marginal return on investment[1] must be equal to the marginal costs of the initial investment made in the SM, namely 1, plus the conditional marginal cost of the additional investment made in the DM. Note that the probability to continue with the project $F\left[\frac{\beta R(I')}{I'}\right]$ is decreasing in I'. Figure 2 represents Equation (4).

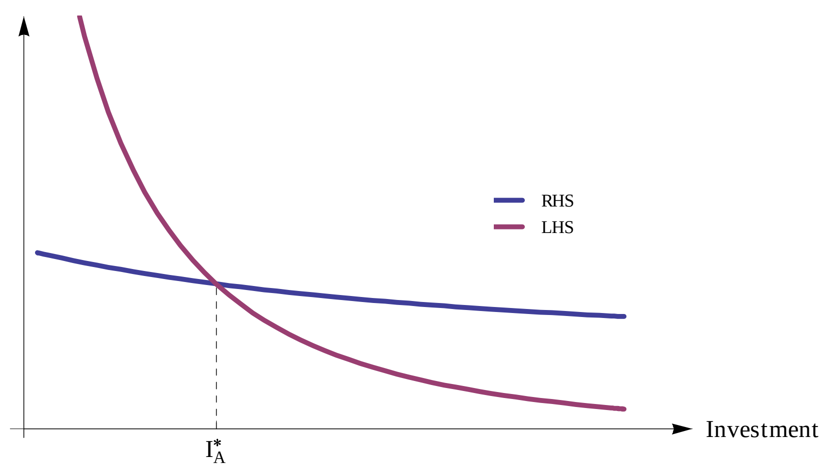

Figure 2: Optimal investment under autarky

4 A monetary economy

In this Section, we add some assumptions to analyze how agents behave when they must interact and how the introduction of money affects the results. We assume now that if agent i invests an amount I_i in the SM, ρI_i hours must be done by agent $j \neq i$ in order to continue with the project. Also,

[1]The conditionality is on the realization of the return, given by the probability $F\left[\frac{\beta R(I)}{I}\right]$.

we assume anonymity in the decentralized market. Those two assumptions create a problem of double coincidence of wants, which can be solved by introducing fiat money. Therefore, the productivity shock can be considered as a liquidity shock since the agents need to pay with cash. They continue to behave as before in the settlement market.

Let ϕ be the real price of money at time t and ϕ_+ the price in $t+1$. We allow the stock of money to change over time at a constant rate of $\gamma = \frac{\phi}{\phi_+}$. New money is injected in the economy through lump-sum transfers T from the government to each agent. We focus on a symmetric and stationary equilibrium.

Let $V_1(m, I)$ denote the expected value of an agent in the first market. He enters at time t with I units of general good invested at $t-1$, as well as with m units of money. He consumes, produces, and makes a new investment I' which eventually yields a return in the next period. Similarly, $V_2(m', I')$ denotes the expected value of an agent in the second market, carrying m' units of money and having invested I' units of the good in the previous submarket of this period. In the first market, an agent consumes x and produces h_1 units of goods, receives $R(I)$ for his previous period investment and makes a new investment I'. He enters the SM with m and leaves it with m' units of money. The representative agent's lifetime program is[2]

$$\begin{aligned} V_1(m, I) &= \max_{x, h_1, m', I'} u(x) - h_1 + V_2(m', I') \\ s.t. \qquad x + I' + \phi m' &= h_1 + R(I) + \phi m + \phi T \end{aligned}$$

Assuming an interior solution, we isolate h_1 from the constraint and insert it in the value function:

$$V_1(m, I) = \max_{x, m', I'} u(x) - x - I' - \phi m' + R(I) + \phi m + \phi T + V_2(m', I'). \quad (5)$$

[2] Although I can either be I or 0, we always write the value function in the form $V(m, I)$, because it depends on a choice made in the previous period.

The first-order conditions for x, m' and I' are

$$\begin{aligned} u'(x) &= 1 \\ V_2^{m'}(m', I') &= \phi \\ V_2^{I'}(m', I') &= 1, \end{aligned}$$

where $V_2^{m'}(m', I')$ is the marginal value of an additional unit of money taken into next period, and $V_2^{I'}(m', I')$ is the marginal value of an additional unit of good invested.

According to the first-order conditions, it is clear that the optimal choice of x is time-invariant. The choice of m' and I' is independent of m and I, respectively. Thanks to quasi-linear preferences, the distributions of m' and I' are degenerate. That is, agents reset their positions regarding their money balance and their investment after every period.

The envelope conditions are

$$V_1^m(m, I) = \phi \tag{6}$$

$$V_1^I(m, I) = R'(I). \tag{7}$$

Let w denote the nominal hourly wage in the DM. Since we focus on symmetric equilibrium, we assume that the same wage is given to every agent. Because there is no trade in the DM, there is also no price. Therefore, agents value their wage at price ϕ_+ and discount it. The discounted hourly real wage is equal to the disutility created by working one hour, namely 1. That is, we have

$$\beta\phi_+ w = 1. \tag{8}$$

The agent can either disburse the total real amount $\beta\phi_+ w\rho I$ in order to continue with the project, or decide not to, in which case the project is abandoned. Following the same reasoning as in Section 3, it must hold that

$$\beta\phi_+ w\rho I \leqslant \beta R(I).$$

Using Equation (8), we can write this expression as follows

$$\rho \leqslant \frac{\beta R(I)}{I}.$$

The cut-off value $\bar{\rho}$ is therefore

$$\bar{\rho} = \frac{\beta R(I)}{I}. \tag{9}$$

Since it is optimal for an agent to continue with his project when $\rho \leqslant \bar{\rho}$, he must hold enough money to be able to pay such amounts. That is, an agent who enters the DM with an amount m' of money and an investment I' must satisfy this condition

$$\phi_+ m' \geqslant \phi_+ w\bar{\rho}I'.$$

Using Equations (8) and (9), this yields

$$\phi_+ m' \geqslant R(I')$$

which states that the real amount of money at the beginning of the period is at least the return on investment.

At the opening of the decentralized market, the expected lifetime utility for an agent holding m' units of money and made an investment of size I' is

$$\begin{aligned} V_2(m', I') = & \int_0^{\bar{\rho}} \beta V_1\left(m' - w\rho I', I'\right) f(\rho) d\rho \\ & + \int_{\bar{\rho}}^{\infty} \beta V_1\left(m', 0\right) f(\rho) d\rho \qquad (10) \\ s.t. \qquad & \phi_+ m' \geqslant R(I') \qquad (11) \end{aligned}$$

In the case where $0 < \rho \leqslant \bar{\rho}$, the project is continued onto the next period until maturity. Otherwise, an agent stays with the same amount of money, but no longer has a project. The value function in the DM is the discounted value function in the next market, which depends on the amount of money left given the decision made in the DM whether to continue or not with the project, and the amount of the initial investment, if carried on.

We could solve the value functions backwards as in Lagos and Wright (2005) in order to compute the equilibrium values for m' and I'. But without changing the results, we opted for a Lagrangian. Inserting Equation (10) in (5) yields

$$\begin{aligned} V_1(m, I) = &\max_{x,m',I'} u(x) - x - I' - \phi m' + R(I) + \phi m + \phi T \\ &+ \int_0^{\bar{\rho}} \beta V_1\left(m' - w\rho I', I'\right) f(\rho) d\rho \\ &+ \int_{\bar{\rho}}^{\infty} \beta V_1\left(m', 0\right) f(\rho) d\rho \end{aligned} \tag{12}$$

$$s.t. \qquad \phi_+ m' \geqslant R(I') \qquad [\lambda] \tag{13}$$

The Lagrange function is

$$\begin{aligned} \mathcal{L} = &u(x) - x - I' - \phi m' + R(I) + \phi m + \phi T \\ &+ \int_0^{\overline{\rho}} \beta V_1\left(m' - w\rho I', I'\right) f(\rho) d\rho \\ &+ \int_{\overline{\rho}}^{\infty} \beta V_1\left(m', 0\right) f(\rho) d\rho \\ &+ \lambda\left[\phi_+ m' - R(I')\right] \end{aligned}$$

which can be rewritten as

$$\begin{aligned} \mathcal{L} = &u(x) - x - I' - \phi m' + R(I) + \phi m + \phi T \\ &+ \int_0^{\frac{\beta R(I')}{I'}} \beta V_1\left(m' - w\rho I', I'\right) f(\rho) d\rho \\ &+ \int_{\frac{\beta R(I')}{I'}}^{\infty} \beta V_1\left(m', 0\right) f(\rho) d\rho \\ &+ \lambda\left[\phi_+ m' - R(I')\right] \end{aligned}$$

The first-order condition for λ is

$$\phi_+ m' = R(I'). \tag{14}$$

Using the envelope condition for $V_1^m(m,I)$ given by Equation (6), lagged one period, we get $V_1^{m'}(m',I') = \phi_+$. Thus, the first-order condition for m' is

$$\begin{aligned} -\phi + \int_0^{\frac{\beta R(I')}{I'}} \beta V_1^{m'} f(\rho)d\rho + \int_{\frac{\beta R(I')}{I'}}^{\infty} \beta V_1^{m'} f(\rho)d\rho + \lambda\phi_+ &= 0 \\ -\phi + \beta\phi_+ + \lambda\phi_+ &= 0 \end{aligned}$$

$$\begin{aligned} \lambda &= \frac{\phi}{\phi_+} - \beta \\ \lambda &= \gamma - \beta \end{aligned} \tag{15}$$

where $\gamma > \beta$ in order to have $\lambda > 0$. The first-order condition for I' is

$$\begin{aligned} &-1 + \frac{\partial \frac{\beta R(I')}{I'}}{\partial I'} \beta V_1(0,I') F\left[\frac{\beta R(I')}{I'}\right] \\ &\quad - \frac{\partial \frac{\beta R(I')}{I'}}{\partial I'} \beta V_1(m',0) F\left[\frac{\beta R(I')}{I'}\right] \\ &+ \int_0^{\frac{\beta R(I')}{I'}} \frac{\partial}{\partial I'} \beta V_1(m' - w\rho I', I') f(\rho)d\rho \\ &+ \int_{\frac{\beta R(I')}{I'}}^{\infty} \frac{\partial}{\partial I'} \beta V_1(m',0) f(\rho)d\rho - \lambda R'(I') = 0 \end{aligned} \tag{16}$$

In equilibrium, we have $V_1(0,I') = V_1(m',0)$: by construction, we look at this exact point[3] where the agent is indifferent between investing but holding no money or holding money and not investing.

Proof. We have

$$\begin{aligned} V_1(0,I) &= u(x) - x - I' + R(I) + 0 + \phi T + V_2(m',I') \\ V_1(m,0) &= u(x) - x - I' + 0 + \phi m + \phi T + V_2(m',I') \end{aligned}$$

[3] This point is when $\rho = \bar{\rho}$.

Computing the difference yields

$$V_1(0, I') - V_1(m', 0) = R(I) - \phi m$$

The condition under Equation (14) tells us that

$$V_1(0, I') = V_1(m', 0).$$

■

Moreover, it must hold that $\frac{\partial}{\partial I'}\beta V_1(m', 0) = 0$ because $\beta V_1(m', 0)$ is independent of I'. Therefore, Equation (16) can be simplified as

$$-1 + \int_0^{\frac{\beta R(I')}{I'}} \frac{\partial}{\partial I'}\beta V_1(m' - w\rho I', I') f(\rho) d\rho - \lambda R'(I') = 0 \tag{17}$$

Using the envelope condition computed in (7) lagged one period, we get $V_1^{I'}(m', I') = R'(I')$. Consequently, Equation (17) can be written as

$$\begin{aligned} -1 + \int_0^{\frac{\beta R(I')}{I'}} \left[-\beta w \rho V_1^{m'} + \beta V_1^{I'}\right] f(\rho) d\rho - \lambda R'(I') &= 0 \\ -1 + \int_0^{\frac{\beta R(I')}{I'}} \left[-\rho + \beta R'(I')\right] f(\rho) d\rho - \lambda R'(I') &= 0 \\ -1 - \int_0^{\frac{\beta R(I')}{I'}} \rho f(\rho) d\rho + \beta R'(I') F\left[\frac{\beta R(I')}{I'}\right] - \lambda R'(I') &= 0 \end{aligned} \tag{18}$$

Inserting Equation (15) in (18) yields

$$\beta R'(I') F\left[\frac{\beta R(I')}{I'}\right] = 1 + \int_0^{\frac{\beta R(I')}{I'}} \rho f(\rho) d\rho + \left(\frac{\gamma - \beta}{\beta}\right) \beta R'(I'). \tag{19}$$

Equation (19) gives the condition which must hold in order to attain the optimal I' in a monetary economy. We distinguish two cases. In the first case, the representative agent has enough liquidity. The cash constraint is not binding, i.e. $\lambda = 0$, and the first-best solution can always be reached.

The optimal monetary policy is the Friedman rule, $\gamma = \beta$. Then, Equation (19) can be reduced to

$$\beta R'(I')F\left[\frac{\beta R(I')}{I'}\right] = 1 + \int_0^{\frac{\beta R(I')}{I'}} \rho f(\rho) d\rho$$

which is the same as Equation (4) in autarky.

If the agent is cash constrained, Equation (19) reveals an additional term, $\left(\frac{\gamma-\beta}{\beta}\right)\beta R'(I')$, which is the effect of inflation on investment. In autarky, the optimal level of investment is bigger. The difference between Equations (19) and (4) is depicted in Figure 3. With inflation, the value of money drops

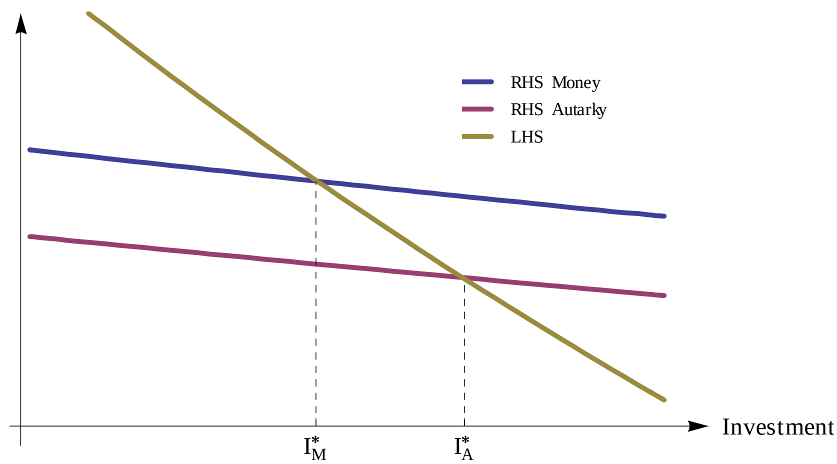

Figure 3: The optimal investment levels in autarky and in a monetary economy

across periods. Hence, the real value of the return on their investment is smaller than without inflation. Thus, agents have an incentive to spend more in immediate consumption rather than investing. From the Fisher equation, $(1 + i) = (1 + r)(1 + \pi)$. In our case, $\pi = \gamma$. Thus, the real interest rate is smaller in an inflationary economy, and so is investment. Figure 4 shows how the optimal investment level changes with a change in inflation γ. It is straightforward that if γ increases, the optimal level of investment decreases. Therefore, there is a negative relation between γ

and I^*. If inflation increases, so does uncertainty and investment planning becomes more difficult.

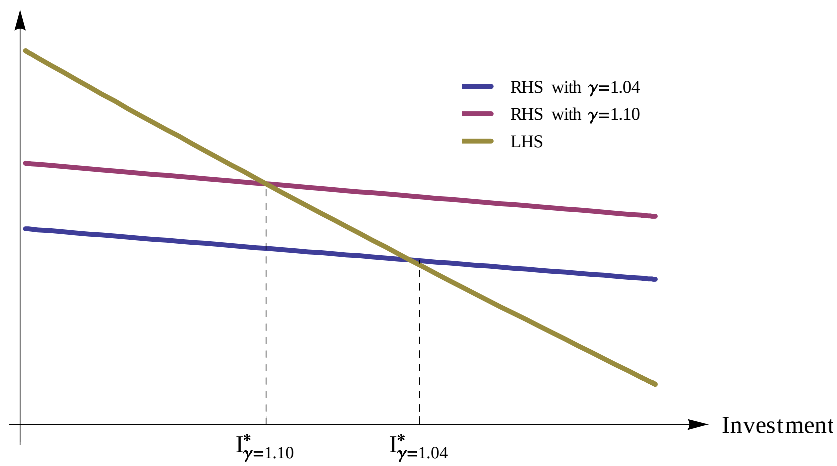

Figure 4: Optimal investments in a monetary economy

5 Financial intermediation

In this Section, we allow agents to make financial transactions among them in the DM. We assume that they learn the shock at the beginning of the decentralized market. After they learn the intensity of the shock, agents can lend or borrow to each other. For simplicity, we do not add a supplementary market, because it would not change the results. We reserve a small period of time at the beginning of the DM where agents can deal financial contracts. During this period, they can lend or borrow money by trading inside bonds. The purpose is that agents with a low liquidity shock is able to lend to those with a higher shock. The inside bonds serve as an insurance against the shortage in money holdings when the decision is made to continue with the project. Without this possibility, agents have to insure themselves, for example in holding idle balances. But with financial intermediation, risks are pooled amongst agents.

We expect that financial intermediation changes the possibility for the agents to meet the liquidity constraint. The threshold $\bar{\rho}$ remains unaffected, but the amount needed to continue with the project is easier to attain. We allow financial transactions in the DM exclusively. Financial contracts are redeemed in the SM. Financial intermediaries operate record keeping over inside bonds and thus keep track of financial histories. Agents are still anonymous in trades. We assume that the government acts as the only financial intermediary by keeping track of bonds and redistributing money. In the DM, agents learn their shock. Afterwards, they are offered the possibility to make financial transactions.

Default would have the consequence that the agent is banned from the financial market. We assume that the lifetime value function of a defaulter is strictly smaller than the value function of a paying agent. Given this restriction, we assume voluntary redemption. We define the financial contract by b. If an agent takes out a loan, b is positive, and negative if he gives out a loan. The price of this financial contract is given by $\varphi = \frac{1}{1+i}$ with i being the nominal interest rate. An agent takes a loan, hence receiving φb units of money. He redeems the amount b after one subperiod. The difference $b - \varphi b$ is the interest.

A representative agent with a bond b, which has a real value of ϕb, has the following value function

$$\begin{aligned} V_1(m, b, I) &= \max_{x, h_1, m', I'} u(x) - h_1 + V_2(m', I') \\ s.t. \qquad x + I' + \phi m' &= h_1 + R(I) + \phi m - \phi b + \phi T \end{aligned}$$

An agent finances his consumption, investment, money holdings and the reimbursement of the bond with his production, the return on the previous-period investment, the money he brought in, the transfer from the government. Therefore, an agent takes out a loan in the previous DM, and reimburse b in the SM of this period. This explains the minus sign before ϕb.

Substituting h_1 from the constraint yields

$$V_1(m,b,I) = \max_{x,m',I'} u(x) - x - I' - \phi m' + R(I) + \phi m - \phi b + \phi T \\ + V_2(m',I'). \tag{20}$$

The first-order conditions for x, m' and I' are

$$\begin{aligned} u'(x) &= 1 \\ V_2^I(m',I') &= 1 \\ V_2^m(m',I') &= \phi. \end{aligned}$$

The envelope conditions are

$$\begin{aligned} V_1^m(m,b,I) &= \phi \\ V_1^b(m,b,I) &= -\phi \\ V_1^I(m,b,I) &= R'(I). \end{aligned}$$

Following the same reasoning as in the previous Sections, the cut-off value for ρ remains unchanged at

$$\bar{\rho} = \frac{\beta R(I)}{I}.$$

An agent must hold enough money to be able to pay every amount $w\rho I$ up to $\rho \leqslant \bar{\rho}$. That is, an agent who enters the DM with an amount of money m', an investment I' and makes a financial contract b' must satisfy the following liquidity constraint

$$\phi_+ m' + \phi_+ \varphi b' \geqslant \phi_+ w \bar{\rho} I', \tag{21}$$

which means that in order to be able to continue with the project for a range of shocks up to $\bar{\rho}$, an agent must hold a sufficient total amount of money and bonds superior to the highest liquidity shock. Replacing $\bar{\rho} = \frac{\beta R(I')}{I'}$ in Equation (21) yields

$$\phi_+ m' + \phi_+ \varphi b' \geqslant R(I')$$

The value function of an agent in the second market is

$$
\begin{aligned}
V_2(m', I') = \max_{b'} & \left[\int_0^{\bar{\rho}} \beta V_1(m' + \varphi b' - w\rho I', b', I') f(\rho) d\rho \right. \\
& \left. + \int_{\bar{\rho}}^{\infty} \beta V_1(m' + \varphi b', b', 0) f(\rho) d\rho \right] \\
& s.t. \qquad \phi_+ m' + \phi_+ \varphi b' - R(I') \geqslant 0
\end{aligned} \tag{22}
$$

Inserting (22) in Equation (20) yields

$$
\begin{aligned}
V_1(m, b, I) = \max_{x, m', I'} & u(x) - x - I' - \phi m' + R(I) + \phi m - \phi b + \phi T \\
& + \max_{b'} \left[\int_0^{\bar{\rho}} \beta V_1(m' + \varphi b' - w\rho I', b', I') f(\rho) d\rho \right. \\
& \left. + \int_{\bar{\rho}}^{\infty} \beta V_1(m' + \varphi b', b', 0) f(\rho) d\rho \right] \\
& s.t. \qquad \phi_+ m' + \phi_+ \varphi b' - R(I') \geqslant 0 \qquad [\beta\lambda]
\end{aligned}
$$

The Lagrangian is therefore

$$
\begin{aligned}
\mathcal{L} = & u(x) - x - I' - \phi m' + R(I) + \phi m - \phi b + \phi T \\
& + \int_0^{\bar{\rho}} \beta V_1(m' + \varphi b' - w\rho I', b', I') f(\rho) d\rho \\
& + \int_{\bar{\rho}}^{\infty} \beta V_1(m' + \varphi b', b', 0) f(\rho) d\rho \\
& + \beta\lambda \left[\phi_+ m' + \varphi b' \phi_+ - R(I') \right].
\end{aligned}
$$

The first-order condition for $\beta\lambda$ yields

$$
\phi_+ m' + \phi_+ \varphi b' = R(I').
$$

The first-order condition for b' is

$$\begin{aligned}
\beta\varphi V_1^{m'} + \beta V_1^{b'} + \beta\lambda\varphi\phi_+ &= 0 \\
\beta\varphi\phi_+ - \beta\phi_+ + \beta\lambda\varphi\phi_+ &= 0 \\
\varphi - 1 + \lambda\varphi &= 0 \\
\lambda &= \frac{1}{\varphi} - 1
\end{aligned}$$

inserting $\varphi = \frac{1}{1+r}$, we have

$$\lambda = r.$$

The first-order condition for m' is

$$\begin{aligned}
-\phi + \beta V_1^{m'} + \beta\lambda\phi_+ &= 0 \\
-\phi + \beta\phi_+ + \beta\lambda\phi_+ &= 0 \\
-\phi + \beta\phi_+(1+r) &= 0 \\
r &= \frac{\gamma - \beta}{\beta},
\end{aligned}$$

and the first-order condition for I' is

$$\begin{aligned}
-1 + \int_0^{\bar{\rho}} \beta\left(-\rho w\right) V_1^{m'} f(\rho) d\rho + \int_0^{\bar{\rho}} \beta V_1^{I'} f(\rho) d\rho - \beta\lambda R'(I') &= 0 \\
-1 - \int_0^{\bar{\rho}} \beta\phi_+ \rho w f(\rho) d\rho + \int_0^{\bar{\rho}} \beta R'(I') f(\rho) d\rho - \beta\lambda R'(I') &= 0
\end{aligned}$$

Which can be rewritten in a familiar way:

$$\beta R'(I') dF\left[\frac{\beta R(I')}{I'}\right] = 1 + \int_0^{\bar{\rho}} \rho f(\rho) d\rho + \left(\frac{\gamma - \beta}{\beta}\right)\beta R'(I') \qquad (23)$$

Equation (23) is the same as Equation (19), i.e., without financial intermediation. In our setup, financial intermediation does not meet our expectations. A reason can be the equivalence that prevails sometimes between money and bonds. Kocherlakota (2007) provides a deeper analysis for this similarity. One solution worth considering is changing the liquidity constraint to make it state-contingent.

6 Conclusion

We have presented a unified model for monetary theory where a productivity shock hits a project. This shock is interpreted as a liquidity shock when money is present in the economy. We found that the optimal level of investment, computed in autarky, can be achieved in a monetary economy under the Friedman rule. For every other values of $\gamma > \beta$, the optimal level of investment is smaller than in autarky. This result is coherent since inflation is prohibitive for investment.

Agents must also be careful regarding their liquidity constraint. They can face the situation where continuing the project would be optimal, but they don't have enough money holdings to afford it. Financial intermediation was expected to solve this problem in pooling the risk among the agents and redistributing idle money balances of agents with a low liquidity shock to agents with a high shock. Unlike what we expected, it had no influence on the investment behavior of the agents.

Our model is open to many extensions. We could extend it to the banking sector and investigate problems like systemic risk. We could also analyze other types of shocks, for example information shocks.

Appendix

We present the codes used to draw Figures 2, 3 and 4 respectively. We used the software Mathematica 7 distributed by Wolfram.

We plot the optimal investment level in autarky with

```
In[431]:= alpha = 0.2; beta = 0.9; mu = 0; sigma = 1;

Needs["PlotLegends`"]

Plot[
 {1 + Integrate[(rho * PDF[LogNormalDistribution[mu, sigma], rho]),
    {rho, 0, (beta * i ^ alpha / i)}],
  beta * alpha * i ^ (alpha - 1) / i * CDF[LogNormalDistribution[mu, sigma], beta * i ^ alpha / i]},
 {i, 0.12, 0.55}, PlotRange → {{0.1, 0.6}, {0, 5}},
 AxesOrigin → {0.11, 0.1}, AxesLabel → {"Investment", None},
 Epilog → {
   {Dashed, Line[{{0.251, 0.1}, {0.251, 1.85}}]},
   {Arrowheads[0.03], Arrow[{{0.11, 0}, {0.11, 5}}]},
   {Arrowheads[0.03], Arrow[{{0.11, 0.1}, {0.6, 0.1}}]}
  },
 Ticks → {{{0.251, Subsuperscript["I", "A", "*"]}}, None},
 PlotStyle → AbsoluteThickness[3], BaseStyle → {FontSize → 15}, LegendTextOffset → Automatic,
 PlotLegend → {Style["RHS", 11], Style["LHS", 11]},
 LegendPosition → {0.2, -0.1}, LegendShadow → None, LegendSpacing → -0.4,
 LegendBorder → None, ImageSize → 500
]
```

The difference in optimal investment between autarky and a monetary economy is depicted using

```
In[428]:= alpha = 0.2; beta = 0.9; gamma = 1.04; mu = 0; sigma = 1;

Needs["PlotLegends`"]

Plot[
 {1 + Integrate[(rho * PDF[LogNormalDistribution[mu, sigma], rho]),
    {rho, 0, (beta * i ^ alpha / i)}] + (gamma - beta) * alpha * i ^ (alpha - 1),
  1 + Integrate[(rho * PDF[LogNormalDistribution[mu, sigma], rho]),
    {rho, 0, (beta * i ^ alpha / i)}],
  beta * alpha * i ^ (alpha - 1) / i * CDF[LogNormalDistribution[mu, sigma], beta * i ^ alpha / i]},
 {i, 0.2314, 0.26}, PlotRange → {{0.2305, 0.262}, {1.65, 2.1}},
 AxesOrigin → {0.231, 1.67}, AxesLabel → {"Investment", None},
 Epilog → {
   {Dashed, Line[{{0.2443, 1.67}, {0.2443, 1.925}}]},
   {Dashed, Line[{{0.251, 1.67}, {0.251, 1.8229}}]},
   {Arrowheads[0.03], Arrow[{{0.231, 1.65}, {0.231, 2.1}}]},
   {Arrowheads[0.03], Arrow[{{0.2305, 1.67}, {0.262, 1.67}}]}
  },
 Ticks → {{{0.2443, Subsuperscript["I", "M", "*"]},
    {0.251, Subsuperscript["I", "A", "*"]}}, None},
 PlotStyle → AbsoluteThickness[3], BaseStyle → {FontSize → 15}, LegendTextOffset → Automatic,
 PlotLegend → {Style["RHS Money", 11], Style["RHS Autarky", 11], Style["LHS", 11]},
 LegendPosition → {0.1, 0.1}, LegendShadow → None, LegendSpacing → -0.4,
 LegendBorder → None, ImageSize → 500
]
```

The effect of inflation on optimal investment is showed applying

```
In[1]:= alpha = 0.2; beta = 0.9; gamma1 = 1.04; gamma2 = 1.10; mu = 0; sigma = 1;

Needs["PlotLegends`"]

Plot[
 {1 + Integrate[(rho * PDF[LogNormalDistribution[mu, sigma], rho]),
    {rho, 0, (beta * i ^ alpha / i)}] + (gamma1 - beta) * alpha * i ^ (alpha - 1),
  1 + Integrate[(rho * PDF[LogNormalDistribution[mu, sigma], rho]),
    {rho, 0, (beta * i ^ alpha / i)}] + (gamma2 - beta) * alpha * i ^ (alpha - 1),
  beta * alpha * i ^ (alpha - 1) / i * CDF[LogNormalDistribution[mu, sigma], beta * i ^ alpha / i]},
 {i, 0.2371, 0.249}, PlotRange → {{0.2368, 0.25}, {1.82, 2.07}},
 AxesOrigin → {0.237, 1.83}, AxesLabel → {"Investment", None},
 Epilog → {
   {Dashed, Line[{{0.24165, 1.83}, {0.24165, 1.969}}]},
   {Dashed, Line[{{0.24455, 1.83}, {0.24455, 1.925}}]},
   {Arrowheads[0.03], Arrow[{{0.237, 1.815}, {0.237, 2.07}}]},
   {Arrowheads[0.03], Arrow[{{0.2345, 1.83}, {0.25, 1.83}}]}
  },
 Ticks → {{{0.24165, Subsuperscript["I", "γ=1.10", "*"]},
    {0.24455, Subsuperscript["I", "γ=1.04", "*"]}}, None},
 PlotStyle → AbsoluteThickness[3], BaseStyle → {FontSize → 15}, LegendTextOffset → Automatic,
 PlotLegend → {Style["RHS with γ=1.04", 11], Style["RHS with γ=1.10", 11], Style["LHS", 11]},
 LegendPosition → {0.1, 0.1}, LegendShadow → None, LegendSpacing → -0.4,
 LegendBorder → None, ImageSize → 500
]
```

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Declaration of Honor

Ich bezeuge mit meiner Unterschrift, dass meine Angaben über die bei der Abfassung meiner Arbeit benützten Hilfsmittel sowie über die mir zuteil gewordene Hilfe in jeder Hinsicht der Wahrheit entsprechen und vollständig sind. Ich habe das Merkblatt zu Plagiat und Betrug vom 23.11.05 gelesen und bin mir den Konsequenzen eines solchen Handelns bewusst.

Ort, Datum: Unterschrift: