Master Thesis

Central Bank Policy in the European Sovereign Debt Crisis

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An Analysis of Martingale Gambling Strategies and Self-Fulfilling Prophecies

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1. Introduction

After the eruption of the global financial crisis, the European economy found itself confronted with a severe recession, from which the troubled GIIPS countries did not recover up until this point in time. Low productivity, weakened competitiveness and a limitation of policy instruments forced the governments to bridge the occurred deficits through the accumulation of public debt, which in turn exposed them to the financial institutions' willingness to roll over these contracts. However, instead of taking actions to escape this vulnerability, debt positions soared and the initially national problems started to spill over to the European banking sector. Optimism evaporated and the threat of a country's default through a self-fulfilling crisis became probable.

As a reaction on these happenings, the European Central Bank and other authorities were forced to intervene. Several policy measures were implemented, following the objective of calming the market's distrust, by facilitating the countries' task towards restructured public finances. However, the contrary case occurred and turbulences within the monetary union continued, expressed through an increase in public debt. Within this paper, the motives for these resulting government policy decisions are being analyzed.

The basic framework used for this analysis was developed by Harold L. Cole and Timothy J. Kehoe in 1996. Through the application of a dynamic stochastic general equilibrium model, its purpose is to provide optimal policy decisions for a strategic benevolent government, which is vulnerable to speculative attacks on the part of non-strategic financial institutions. Juan Carlos Conesa and Timothy J. Kehoe then established a modified version in 2011, introducing the applicability of martingale gambling strategies in recessionary times. Through this extension, a game theoretic investigation of the government policy decisions during the European sovereign debt crisis is enabled.

As soon as a country is depending on the ability to roll over its debt contracts in order to remain capable of acting, it becomes vulnerable to uncoordinated investment stops on behalf of financial institutions. Therefore, in order to avoid the high costs of a default, the government has an incentive to run down its public debt positions in order to escape the danger of being hit by suchlike. These incentives can change in times of an economic downturn. Is the country suffering from a long lasting recession, martingale gambling strategies may dominate, inducing the government to increase public borrowing in order to avoid sharp cuts in expenditures. The idea behind this behaviour is to gamble on a recovery of the private sector and the subsequent recovery of tax revenues in the foreseeable future, enabling a repayment of the debt contracts under more advantageous conditions. However, as the country thereby builds up high levels of borrowing, this strategy bears as well the risk of being forced to default, if the end of the recession remains absent. The decision about which policy is being followed is thereby highly depending on the individual situation of the country.

Using this theoretical framework, the characteristics of the EU authorities' policy interventions are being applied, enabling an analysis of their influence on the optimal debt policy of a troubled country. Overall, the results show that policy measures, which increase the costs of a default, i.e. the risk premium and the default penalty, provide a government with the incentive to run down the current debt positions in order to avoid the costs following a self-fulfilling crisis. However, are the costs of default lowered through the intervention of a third party, the dominant strategy consists of running up the future borrowing level and gamble for redemption. While this outcome applies to the calibrated results of the European Economic and Monetary Union as a whole, this unambiguousness does not necessarily persist in a country specific analysis.

Related literature has been provided by different economists. The papers elaborated by Krugman (1979), Obstfeld (1986, 1996) and Calvo (1988) focus on self-fulfilling currency crises. Araujo et al. (2011) in turn used the basic framework, established by Cole and Kehoe (1996), to extend it for the investigation of self-fulfilling crises in various monetary and currency regimes. The work of Cohen and Villemot (2008) deals with the introduction of panglossian borrowers, acting in an environment where the government is more optimistic than the markets. Another related paper has been established by Morris and Shin (1998), assigning the reason for the occurrence of a self-fulfilling crisis to a lack of knowledge within a global game framework.

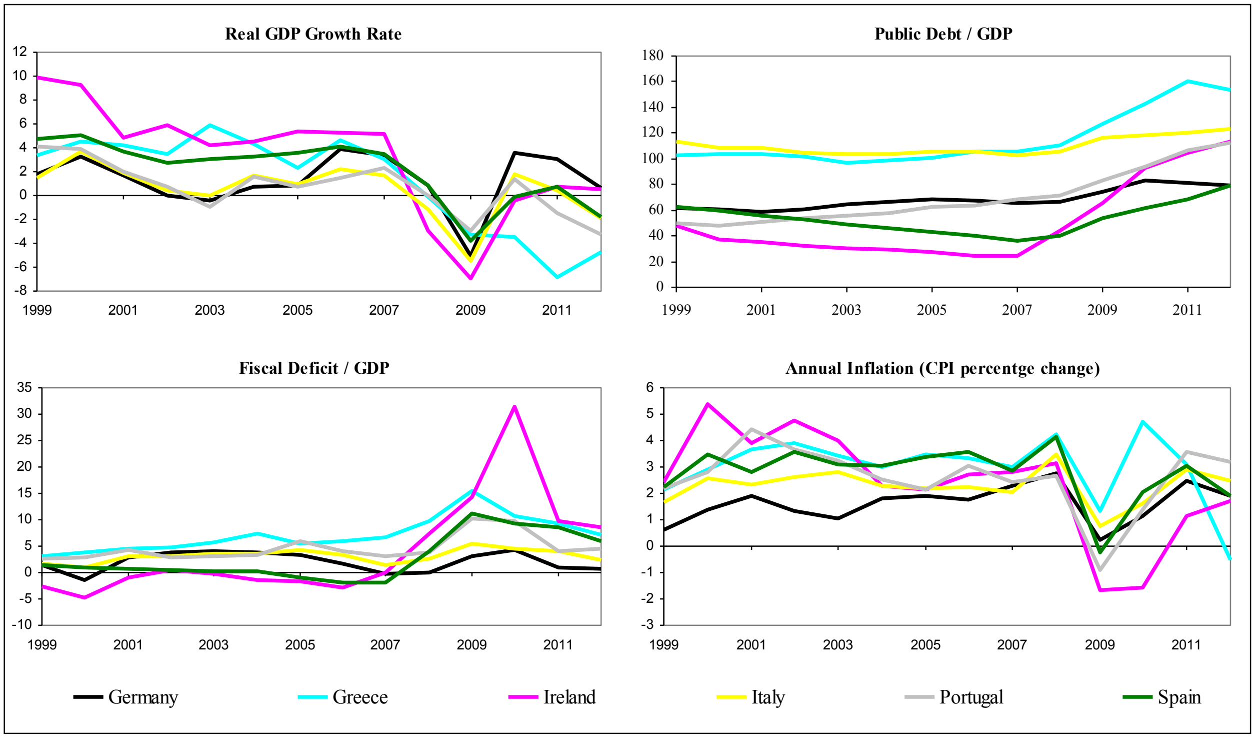
Concerning the organisation of the paper, section 2 will provide an insight into the European sovereign debt crisis, explaining its emergence and the resulting central bank policy measures. Section 3 then presents the theoretical framework and the consequential determination of government policy, while in section 4, the developments in Europe will be analyzed, using the calibrated results of the model. The conclusion can be found in section 5.

2. Historical Perspective

Since the beginning of 2010, the European Economic and Monetary Union (EMU) is struggling with certain member countries in severe economic distress. Most affected of this downturn are Greece, Ireland, Italy, Portugal and Spain, commonly referred to as the GIIPS countries. The purpose of this section is to examine the cause for this crisis, the applied policy measures and the consequential reactions of the respective governments up until now. In a first section (2.1), the crucial factors leading to this sinister situation are being explained, in order to get an understanding for the facts that drove those countries into a state, where they had little chance to overcome their respective problems on their own. Being aware of those facts, the attention will then be turned to the policy interventions applied by different authorities, i.e. the European Central Bank, the International Monetary Fund and the European Commission (section 2.2). The focus thereby lies on the understanding of those measures' respective mechanics as well as on the incentives and reactions they can evoke in the GIIPS countries' governments and on the financial markets. By taking a look at the resulting current economic situation in section 2.3 then, a first confirmation for these incentives can be given, building the basis for the investigations within the theoretical framework in chapter 3 and 4.

The graphs in Figure 1 provide an overview over the stylized facts of the European Economy, or more specifically the GIIPS countries, since the inception of the EMU. They include the real GDP growth rate, the fiscal deficit, the debt to GDP ratio and last but not least an overview over the annual change of the GIIPS' countries consumer price index. As these variables will play an important role during the lecture of this thesis, they will serve as a reference for the forthcoming argumentations.

Figure 1: The Economy of the GIIPS countries vs. Germany (1999 – 2012)



Source Data: IMF (2012b)

2.1 Emergence of the European Sovereign Debt Crisis

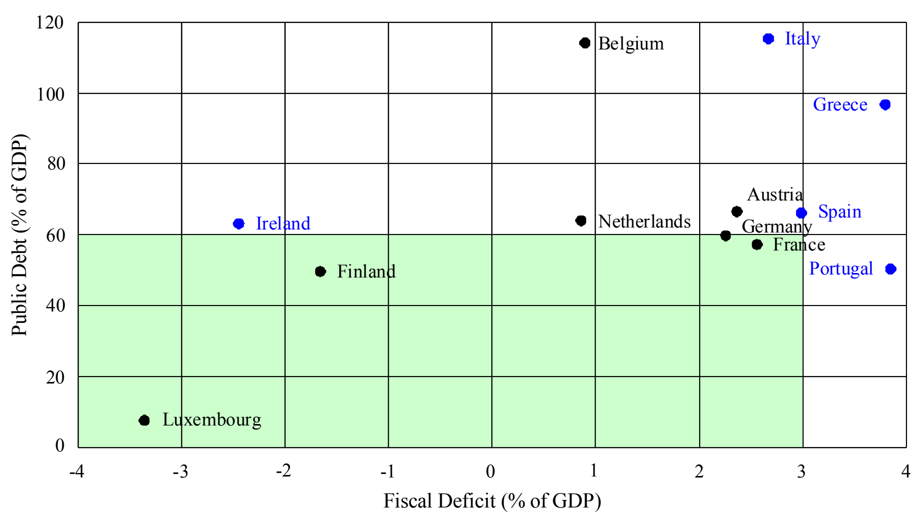
The purpose of this chapter is to outline the determining factors, having contributed to the emergence of the European Sovereign Debt Crisis. The respective development of the GIIPS countries is thereby in the foreground and builds the basis for understanding the policy measures applied, among others, by the European Central Bank (ECB). Additional to that, it delivers an understanding for the behaviour of the troubled countries from the recession onwards. The chapter is subdivided into three sections, chronologically arranged. While section 2.1.1 deals with the inception of the European Economic and Monetary Union (EMU) in 1999, section 2.1.2 focuses on the chances, as well as on the costs of this framework, enlightening the origins of the crisis currently prevailing. In the last section (2.1.3), the outcome of these proceedings is then considered, by presenting the actual starting point of the crisis.

2.1.1 The European Economic and Monetary Union

Economists around the world are providing reasons for the occurrence of the sovereign debt crisis in Europe. Different views are being advanced, leading to a mixture of possible explanations. However, more or less commonly agreed is the fact that part of the crisis' origins need to be searched back in time, around the end of the 20th century, the inception of the European Economic and Monetary Union (EMU).

The cornerstone for this project was set in June 1988, the date at which the European Council agreed for the realization of a unified currency area within the European Community (ECB, 2012a). In order to clarify the eligibility of the potential countries, the so called Maastricht Treaty was established, defining, besides many other, the economic requirements for a monetary unification (Arellano/ Conesa/ Kehoe 2012, p.2) (De Grauwe 2007, p.143). Five major criteria were elaborated, guiding the countries through a process of gradualism and convergence towards the qualification of their economies to join. The first three convergence criteria contain requirements about inflation, long-term interest rates and currency exchange rate mechanisms. The fourth criterion dealt with the government's budget deficit, stating that a maximum of 3% of GDP shall not be exceeded at the point of the entry, except the deviation can be considered as exceptional and temporary, or a positive trend towards this number can be anticipated within a short timeframe. The fifth and last criterion concerns the countries' past debt policy. It states that a country, exceeding a boundary of 60% debt to GDP, will not be entitled to participate in the EMU, unless respective actions have been taken by the government, allowing for the anticipation of meeting this target in the near future.[1] As a result of these targets, in May 1998 11 countries (Austria, Belgium, Finland, France, Germany, Ireland, Italy, Luxembourg, the Netherlands, Portugal and Spain) were tested and proven to be adequate with Greece following the 1st January 2001, completing the first 12 EMU countries. Nearly simultaneously with the countries' admission to the union, the introduction of the Euro as common currency (1st of January 1999) became reality and the European Central Bank took over control from the former national central banks (De Grauwe 2007, p.144). Then, within the time between January 2007 and 2009, Cyprus, Malta, Slovakia and Slovenia joined the EMU with Estonia as the last country in 2011 (Arellano/ Conesa/ Kehoe 2012, p.3). However, the focus of this thesis lies on the five GIIPS countries, as their individual development until the year 2007 is of great importance for the situation currently prevailing. Figure 2 reviews again the initial position of the first 12 countries and shows that merely four of them have actually met targets four and five while joining the Euro area in 1999 and 2001 respectively.

Figure 2: Starting point of the European Economic and Monetary Union [2]



Source Data: Eurostat (2012)

[1] The last two requirements were then later on, in 1997, outlined to build the basis for the European Stability and Growth Pact (SGP), an agreement of the 27 EU Member States to ensure the stability of the EMU through the introduction of penalties for violating the mentioned requirements (Schuknecht et al. 2011, p.8 f.). Nevertheless, no penalties were imposed when Germany and France violated the requirements in 2003, which clearly reduces its credibility (Arellano/ Conesa/ Kehoe 2012, p.4).

[2] The idea for this illustration was provided by Baldwin and Wyplosz (2009, p.494).

Turning the attention to the GIIPS countries, one can see that none of them was eligible, which allows for inferences that a couple of countries have applied certain "creative" accounting tricks in order to become suitable for the entry into the EMU, as shown in Koen and Van den Noord (2005) and in Von Hagen and Wolff (2004).[3,4]

Before moving on to the detailed analysis of the benefits and costs this entry involved, I quickly want to draw the attention to the European Central Bank, which got in charge with the introduction of the Euro in 1999. Its design was as well determined in the Maastricht Treaty and is based on the *German* central banking model, in which price stability is considered to be the primary objective. Although the pursuit of other goals e.g. stabilization of business cycles, ensurance of high employment and the maintenance of financial stability is followed as well, it remains always conditional on the premise that price stability is not endangered. To sustain this aim, it was clearly stated that the ECB enjoys complete political independence and therefore cannot be forced in any way to print money in order to finance governments' budget deficits. Additional to that, a "no-bailout" clause was introduced, justifying the debt and fiscal deficit requirements in the Maastricht Treaty. A country with high debt positions faces a higher default risk and therefore increases the pressure for a bailout within the union. To forestall such a case, the treaty states that neither national governments, nor the central bank will ever bailout other member countries' governments. By making this point, the ECB wanted to avoid that the semblance of a bailout guarantee (through the membership in a union) would have distorting effects, by lowering the risk premium of a country's debt (De Grauwe 2007, p.162 ff.). How credible this statement was to the governments of the respective countries remains to be seen and will be part of the theoretic analysis later on. However, the knowledge over the primary objective, as well as the room for manoeuvres of this institution is important, as it contributes to the understanding of the respective policy decisions.

2.1.2 Origins of the Crisis

Prior to the introduction of the Euro on 1st of January 1999, most countries in the euro area faced a history of high inflation rates, which lead to increased risk premiums on government bonds due to the investors' currency exposure (Gibson/ Hall/ Tavlas 2011, p.6) (Pessoa 2011, p.4). With the adoption of the Euro, the risk of such exchange rate fluctuations decreased sharply, lowering the countries' vulnerability to competitive devaluations (Gibson/ Hall/ Tavlas 2011, p.6). The reason for these effects laid in the credibility of the European Central Bank. Through the release of the anti-inflation credentials, financial markets trusted the ECB delivering price stability, removing the risk of inflation contingent losses in peripheral countries' debt investments (Gibson/ Hall/ Tavlas 2011, p.6) (Higgins/ Klitgaard 2011, p.3). The results of this phenomenon were bond spreads for the GIIPS countries constantly remaining between 0.1 and 0.4 percentage points during the years 2001-2007, with Germany as a benchmark. Taking a look at these numbers, it becomes obvious that a peripheral country's default risk was completely disregarded at this point in time. Rating agencies as well as investors assigned little probability to the occurrence of such

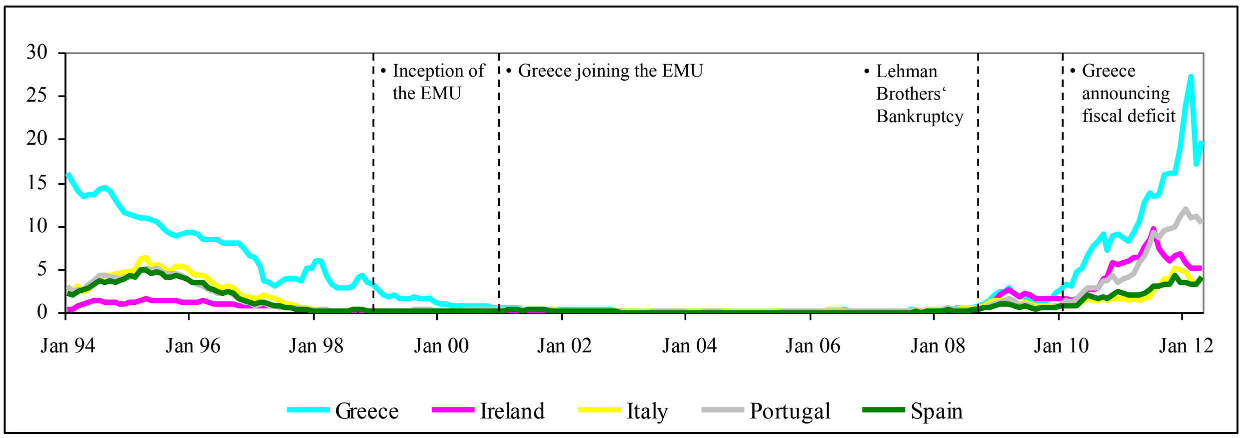
[3] So was for example Greece given the permission to enter the Euro zone even though their debt level was close to 100 percentage points of GDP and fiscal deficit at around 3.7 percent. The reason for this decision was on one hand that debt was assessed to be on a declining path, and on the other that Greece reported a fiscal deficit of 3 percent at the beginning, which was not corrected up to 3.7 until after the entry into the monetary union (Higgins/ Klitgaard 2011, p.8).

[4] A more detailed work on the violations of the Maastricht requirements is provided by Paul De Grauwe (2009).

a case, making it almost impossible to see a difference in the credit risk of e.g. Greek and German bonds. As a result, international investors evermore used those allegedly riskless assets to generate additional returns (Higgins/ Klitgaard 2011, p.2 f.).

The mentioned effects are described in Figure 3. The milestones, having contributed to the current situation in Europe, are thereby highlighted with black dotted lines. Those involve: The introduction of the Euro in 1999, the accession of Greece in 2001, Lehman Brothers filing bankruptcy on the 15th September 2008 and last but not least, Greece retrospectively announcing a fiscal deficit of 15.4 percent of GDP for 2009, representing double the value announced earlier the year (~6%). The respective impact of those events will be part of the explanations while moving on in the thesis.

Figure 3: 10-year government bond spreads – GIIPS vs. Germany (benchmark)



Source Data: ECB (2012e)

The low interest rates opened major chances for the peripheral countries' governments. First, improved confidence increased domestic demand and drove the incentive of consumers and investors to increase spending (Dadush/ Stancil 2010, p.8). In the same turn, due to the low debt servicing costs, the incentive for lending and borrowing at longer maturities increased, providing the government with the opportunity to stimulate private investment in order to advance economic growth (Gibson/ Hall/ Tavlas 2011, p.6). As a matter of fact, the GIIPS countries' nominal public debt positions increased constantly over the years. The only exception thereby was Ireland, with debt positions remaining flat until the eruption of the global financial crisis in 2007. However, the reason for being at the mercy of the actors in the financial markets will become clearer soon (IMF 2012b).

The extensive use of debt showed its impact and per capita GDP in the whole Euro area recorded an average annual growth of 2.3 percent during the years 1999-2007. Considering the peripheral GIIPS countries, one can see that annual growth was way above the European average with numbers for Ireland (6.0%), Greece (4.1%) and Spain (3.7%). Portugal (1.8%) and Italy's (1.5%) growth performance was a little weaker, while Germany only grew at 1.7% (Favaro et al. 2011, p.222) (OECD 2012). All in all, the Euro area found itself in an economic boom phase and realised continuous increases in tax revenues through the advantages of low interest rates on debt. However, the crucial point to identify is what drove this accelerating growth.

With the decreased costs of servicing the current debt positions, resources became free to be used for other, productivity enhancing purposes. The up until now weak performing peripheral countries were given the chance to build up business plants and execute investments, driving up their competitiveness in the international markets (Gibson/ Hall/ Tavlas 2011, p.6). However, the current situation in Europe serves as an indicator that this advantage was not used by all the members of the monetary union. With the adoption of the Euro and the consequential increase in domestic demand, prices and wages of services and non-tradable sectors (e.g. housing) rose in relation to prices of exportable and importable goods exposed to international competition (Dadush/ Stancil 2010, p.8). The reaction on this development came promptly. While Greece and Portugal used the additional resources for increased consumption, Ireland and Spain invested the funds to fuel the domestic housing market.[5] As a result, the gap between savings and investments, referred to a current account balance, steadily increased (Higgins/ Klitgaard 2011, p.4 f.).[6] The subsequent rapid growth in wages outpaced the countries productivity and lowered their competitiveness through an increase in unit labour costs of 25.7% between the years 1999 and 2007 (Ameco 2012). As the domestic demand within the Euro countries continuously increased, the GIIPS countries were not able to serve this additional demand and a loss in export market shares became unavoidable (Higgins/ Klitgaard 2011, p.5 f.).[7] But this trend didn't apply for all of the EMU countries. The increased domestic demand within the GIIPS opened a wider market for exports on behalf of the core countries, e.g. Germany and the Netherlands. Through the adoption of the common currency, which was valued on the overall competitiveness trends of the Euro-zone, the goods of these countries became more affordable, as the value of the currency was lower than that of the Deutsche Mark or the Guilder. Additional to that, those countries invested into productivity enhancing projects during the boom years, enabling them to serve the additional demand in the GIIPS countries (Dadush/ Stancil 2010, p.8 ff.).[8] An important factor, which needs to be considered within this context, is the countries' real exchange rates. It is known that the peripheral countries of the EMU were facing relatively high inflation rates prior to their entrance into the monetary union. Even though those rates declined rapidly after 1999 (and 2001 for Greece respectively), they were still relatively high for the Euro-area standards (See Figure 1).

Overall it can be seen, that the low interest rates indeed contributed to robust real growth rates, but from a stability perspective, the mix of savings and investments inducing this boom was anything but healthy. As a result of this expenditure driven expansionary fiscal policy, consumption and investment exceeded the country's production, which shot down the chance for an increased foreign income stream (exports) in the future (Gibson/ Hall/ Tavlas 2011, p.6 ff.) (Higgins/ Klitgaard 2011, p.1 ff.). The outcome was a low saving rate leading to a huge debt burden, needed to be rolled over in order to stay capable of acting.

The section up until now has shown the chances and potential benefits that came along with the entry into a monetary unified Europe. Nevertheless, the adoption of a common currency brought along as well high costs, especially in terms of a limitation of government policy instruments. Countries, entering the EMU, gave up the ability to set their own monetary policy, which foreclosed the possibility to devaluate the own currency in turbulent times (Gibson/ Hall/ Tavlas 2011, p.8). Against the background of regressive export

[5] According to Dadush and Stancil (2010), in the years 1999-2007, 4 percentage points of the GIIPS countries' GDP were shifted away form industry to financial services and real estate.

[6] An overview over the current account balances in the 12 EMU countries is provided in Appendix A.

[7] Higgins and Klitgaard (2011) ran a regression to this topic and found out that export market shares would have been 20% higher for Greece and Spain and 10% higher for Portugal, if labour costs would have held steady.

[8] These developments, as well as the transformation of the German economy after the countries unification in 1990, contributed to make Germany the world's largest export country (Dadush/ Eidelman 2010, p.17 ff.).

market shares, this instrument would have been useful to strengthen the countries' competitiveness in international trade. But the absence of this policy instrument was not only restricting the capability of generating revenue. It had as well an immediate effect on the accumulation of foreign debt, as the countries have lost the control over exactly this currency, in which their bonds are being issued (Pessoa 2011, p.11). Actors on the financial markets perceive this limitation, which makes them unconfident and susceptible to speculations.[9] According to De Grauwe (2011a), exactly this fact gives crises in a monetary union a fundamentally different character, as the vulnerability to self-fulfilling movements on behalf of the financial markets is way higher. Are creditors afraid of payment difficulties on the part of the countries' government, they suddenly stop rolling over the debt ("sudden stop"), which may bring the country into a liquidity squeeze, as there is no flexible exchange rate, nor a foreign exchange market to stop this. Once this situation has arrived, distrust installs itself and transforms the liquidity emergency into a solvency crisis, as other creditors as well stop rolling their debt positions due to the rapidly increasing interest rates. In the end, the prophecy made earlier is fulfilled and a country is forced to default due to the investors' fear of potential default in the future (De Grauwe 2011a, p.3 ff.).[10] The reason for the occurrence of such a case lies in the lack of coordination among creditors (Cohen/ Villemot 2011, p.2). However, whether this situation occurred and in which way this threat influences the determination of government policy will become clear, while moving on in the thesis.

The content of this section has shown how the wrong usage of chances can result in serious consequences, increasing the costs of a common currency many times over. Through the expansion of domestic demand and the associated increasing tax revenues, governments were given the impression of finding themselves in prospering times and therefore decided to run an expenditure driven fiscal policy. At this point in time it hasn't been realised that this additional revenue needed to be considered as temporary and therefore should have been saved for a possible upcoming recession (Dadush/ Stancil 2010, p.8).

2.1.3 From Boom to Bust

The prosperous times of the early 2000's came to an abrupt end with the breakdown of the United States mortgage market in 2007. Increased exposure in high risk products and excessive leverage ratios putted banks in the U.S, as well as in Europe, at danger of a potential default (Blundell-Wignall 2011, p.5). The result was a severe banking crisis, threatening the existence of the global financial system. Within a few days, all the benefits of the last decades' prosperous economy evaporated and banks ran into huge liquidity problems, due to the literal closure of the interbank market. With the bankruptcy of Lehman Brothers in September 2008, the threat of a complete meltdown of the global financial system generated panic on the stock markets. Increased risk awareness made investors look for the few remaining save havens and induced them to increase their exposure on less dangerous assets like government bonds. The consequential drop in asset markets soon reached the industrial firms. International trade plunged and sales around the world started to decline rapidly. As a result, Europe found itself in the deepest recession

[9] A comparison of the member country Spain to the United Kingdom, maintaining an independent currency, supports this theory. Even though the UK has a much higher debt to GDP ratio (89% in 2011) than Spain (72%), interest rates on Spanish government bonds are nearly 200 basis points higher than those of the UK (De Grauwe 2011a, p.1 f.).
[10] However, De Grauwe (2011) is not stating the current crisis in Europe necessarily needs to be a result of these characteristics.

since the 1930s (European Commission 2009, p.8).[11] The windfall gains through the continuous growth between 1999 and 2007 vanished and governments in Europe were forced to increase their foreign debt positions by roughly 15 percentage points, remarkably contributing to the current level of indebtedness (Schäfer 2012, p.3).[12] Nevertheless, the effect on the peripheral countries 10-year bond spreads kept within bounds and reached a maximum of around 250 basis points, after Lehman Brothers filed bankruptcy in September 2008. By the end of 2009, spreads declined again and hovered around 1 percent afterwards (See Figure 3) (Gibson/ Hall/ Tavlas 2011, p.9).

However, the second shock wasn't long in coming and convulsed the financial markets at the beginning of 2010, particularly affecting the economies of the European continent. The newly elected Governor of Greeks' national bank, George Provopoulos, announced that the initial government's projection of a 6 percent fiscal deficit for 2009 was a mistake and needed to be corrected up to an actual number of 15.4 percent of GDP. This announcement is commonly referred as the starting point of the European Sovereign Debt Crisis, as it galvanised investors around the world and arranged for a considerable increase in risk aversion (Pessoa 2011, p.4) (Gibson/ Hall/ Tavlas 2011, p.9 f.). Suddenly, concerns about the country's fiscal sustainability rose. The illusion that the extremely low spreads were justified only through the elimination of currency devaluation risks was damaged and financial markets reacted with increased interest rates to account for the probability of a default (Gibson/ Hall/ Tavlas 2011, p.10). This in turn increased the costs of servicing debt and again putted pressure on the country's fiscal position. A doom loop was created, making determination of policy decisions a lot more challenging.

However, Greece was not the only country suffering from the damages of the subprime crisis. Except for Italy, the fiscal deficits of the remaining GIIPS countries developed the same way, although there were clear differences prior to Greek's announcement. While Greece and Portugal were running huge fiscal deficits even before the crisis in 2008, Spain and Ireland had comparably strong fiscal positions. However, at the point in time (2008) when the Irish housing bubble burst, the country had to face large drops in tax revenue and the already sick banking sector got even more in trouble (Pessoa 2011, p.5). In order to prevent a collapse of this sector, the government of Ireland guaranteed the debts of the countries' six major private banks, leading to surge of public deficit and debt. By contrast, Spain's reactions on the recession of 2008 were massive Keynesian stimulus policies, pushing the country's fiscal deficits up. As a consequence of these deficits, the previously comparable low public debt level increased rapidly (Arellano/ Conesa/ Kehoe 2012, p.4). The subsequent collapse of the Spanish housing market in 2009 then even worsened the situation. In the end, fiscal deficits of these countries reached values for Ireland (14.0%), Spain (11.2%), Portugal (10.2%) and, to a lesser extend, Italy (5.4%) in the year 2009 (Eurostat 2012).

Even though countries in Europe already suffered from high costs to support their sick banking sectors, the ongoing recession and the large fiscal deficits even worsened the situation. Countries, facing a weak competitiveness, lacked the ability to increase exports in order to generate additional revenue, as they gave up their own monetary policy to counteract this circumstance. Therefore, in order to account for the heightened fiscal deficits, the governments were forced to further increase their debt positions, or a least

[11] The data, provided by Eurostat (2012), supports this development and shows a negative annual growth rate of GDP of 4.25 percent for the whole Euro area at the peak of the subprime crisis in 2009.
[12] Between 2008 and 2010, public debt of the whole Euro zone increased from 70.1 to more than 85 percent of GDP (Eurostat 2012).

dependent on the ability to roll over maturing bonds (Higgins/ Klitgaard 2011, p.8). As a result, public debt and fiscal deficits increased over the years, massively jeopardizing the stability and trustworthiness of the EMU (See Figure 1).[13] The reaction of the financial markets came abrupt, expressing the distrust into these countries with soaring interest rates (see Figure 3). Considering these developments, it becomes clear that the fiscal problems are for the most part the consequence, rather than the cause of the problems currently prevailing in the Euro zone (Dadush/ Stancil 2010, p.9). Not without good reason, Schuknecht et al. (2011, p.10) characterize the first decade of the Euro as “wasted good times”, laying the foundation for the present crisis in the EMU.

2.2 Central Bank Policy Interventions

Due to the happenings at the beginning of 2010, the EMU suddenly found itself confronted with several member countries in severe economic distress. The recent developments brought up problems which could not be handled by themselves and therefore forced the European Central Bank to intervene. The challenge was to determine measures, which induce a reduction of government spending, but without eliminating economic growth. This way, market distrusts can be calmed and additional liquidity enables the governments to reduce the debt burden and improve competitiveness (Pessoa 2011, p.3).

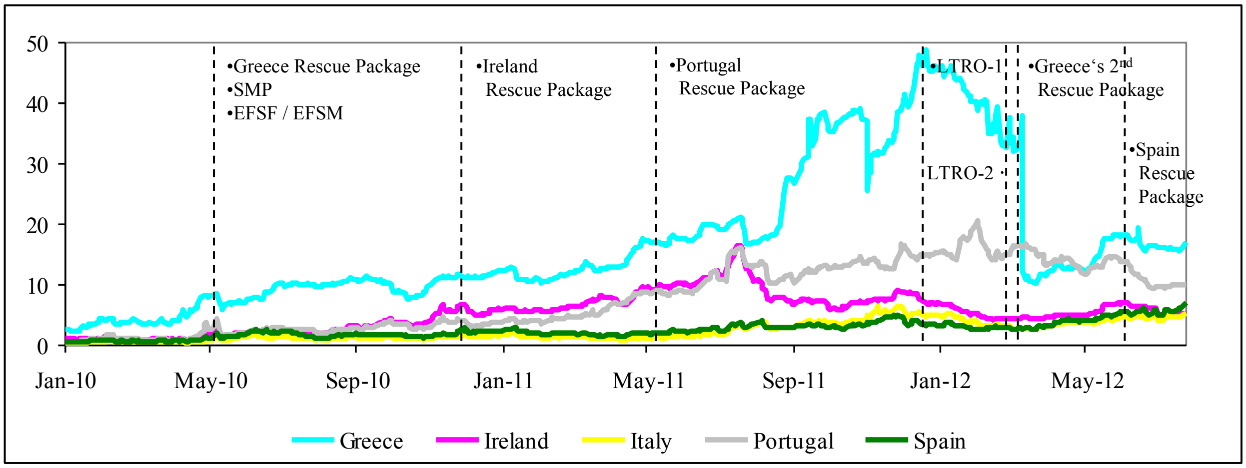
The trigger for these interventions was the tension observable on the financial markets on the 6th and 7th of May 2010. Concerns about the fiscal situation of some countries in the Euro area caused disturbances and increased the investors’ fear. The sudden drop of the Dow Jones on the 7th of May substantiated this anxiety, as it served as an evidence for spillover effects (contagion) from the troubled countries to the financial markets more generally.[14] The volatility on the markets rose, while liquidity decreased, induced through a flight to safety behaviour on behalf of the investors (ECB 2010b, p.36). The risk for an impairment of the financial markets became possible and the European Central Bank realised that a downturn would have consequences on the transmission of monetary policy impulses. The primary objective of price stability was endangered, which made an intervention inevitable (ECB 2011, p.52).

The purpose of this section is to deal with the applied policy measures of the European Central Bank, the European Commission and the International Monetary Fund from the year 2010 up until now. At this point it is important to notice, that only non-standard measures are being concerned, as their exceptional set up can have an impact on the countries’ decisions concerning future debt policy. Sections 2.2.1 and 2.2.2 consider the policy measures applied on a national basis, i.e. the Securities Market Programme (SMP) and the country specific rescue packages provided through the European Financial Stability Facility (EFSF), the European Financial Stabilisation Mechanism (EFSM) and the International Monetary Fund (IMF). In section 2.2.3, the focus will then be laid on the Longer Term Refinancing Operations (LTRO’s) introduced in December 2011, a transnational policy measure mainly designed to support the troubled domestic banks of the EMU. The reason for choosing this order is that it allows us to go through the specific measures chronologically, which enables a detailed analysis of their respective effects. Section 2.3 then sums up and shows the outcome of this turbulent time, laying the foundation for the model

[13] Debt to GDP levels of the GIIPS countries in 2010: Greece 143%; Ireland 92%; Italy 118%; Portugal 93%, Spain 61% (IMF 2012b).
[14] The exact cause for the sudden plunge of the Dow Jones on the 6th of May is still under review (ECB 2010b, p.36).

theoretic analysis in section 3. Figure 4 provides a chronologic overview, depicting what impact those measures had on the spreads of the individual countries' government bonds.

Figure 4: Third party policy interventions (2010-2012)



Source Data: Thompson Reuters (2012)

2.2.1 Securities Market Programme

The first action was taken on the 10th of May 2010, through the ECB's introduction of the Securities Market Programme (SMP). The purpose of this measure is to conduct interventions in the Euro area debt market, by buying bonds of troubled countries whose bond prices fall too low (high yields).[15] With the use of this tool, the ECB aims to ensure liquidity and depth in troubled market segments through a decrease in the interest rates, which in turn leads to an appropriate functioning of the monetary policy transmission mechanism (Arellano/ Conesa/ Kehoe 2012, p.12) (ECB 2010a, p.8). While making this decision, the ECB has taken into consideration the troubled governments' statement saying that they "*will take all measures needed to meet their fiscal targets this year and the years ahead in line with excessive deficit procedures"* (ECB 2010d). The introduction of this programme raised questions, whether these interventions violate the ECB's "no-bailout clause" (Art. 123) and endanger the price stability of the monetary union.[16] Regarding these concerns, the mechanics of this instrument need to be lighted in more detail. It is crucial to understand that the European Central Bank is not providing liquidity to the governments' directly, but to the holders of these government bonds, i.e. the monetary financial institutions (MFI's). As these holders are typically domestic and foreign banks, legally no direct financing of government budget deficits has happened, circumventing the "no-bailout clause" (De Grauwe 2011b, p.11). Concerning the second question, the money base remained unaffected, as several bond purchases were settled on the secondary markets and financed through a sale of assets on behalf of the European Central Bank. This way, the consequences of buying bonds from distressed countries were sterilized and

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[15] A detailed description of the eligibility criteria for debt instruments and counterparties is provided by the ECB in the "Decision of the European Central Bank of 14 May 2010 establishing a securities markets programme" (ECB 2010c).

[16] Art. 123 of the Treaty states that it is prohibited for the for the European Central Bank to issue direct credits to member states or to purchase government bonds on the primary market (Schäfer 2012, p.11).

no inflationary effects were observable in the end (Schäfer 2012, p.11). The difference between the SMP and classic asset purchase programmes, designed to supply the economy with additional monetary stimulus (increase in money supply), is therefore given (ECB 2010b, p.24 f.). However, the side effect of this specific set up is in turn that the ECB's capability to help distressed countries is limited, as no increase in the money base is realizable (Schäfer 2012, p.12). So was the total amount of purchased bonds until the 27th of July 2012 worth around 211 billion Euro (ECB 2012b). Unfortunately, a detailed deployment, showing the respective counterparties of the SMP is not publicly accessible, which makes an assessment of the individual transactions' impact impossible. Nevertheless, taking a look at the deployment of the bond spreads in the months after the introduction, the upward trend foreshadows that the need for further policy actions was inevitable.

2.2.2 EFSF and EFSM

In contrast to the Securities Market Programme, the intend of the European Financial Stability Facility is not to buy bonds on the secondary market, but to build up a fund in order to provide rescue packages to governments of troubled countries.[17] However, these rescue packages must no be understood as a simple provision of liquidity. Its purpose is to provide piecewise interest bearing loans in order to replace the countries' maturing debt contracts (Chamley/ Pinto 2011, p.1). Considering the debt maturity structure of the respective countries, it becomes clear that their borrowing needs for the forthcoming years were almost entirely covered through these bailouts (See Table 1). Important to notice in this context is that these credits were provided at interest rates lower than the ones obtainable on the international bond markets (Arellano/ Conesa/ Kehoe 2012, p.12). As this set up immediately lowers the costs of borrowing, additional resources become free to be used for an improvement of the countries' public finances (Arellano/ Conesa/ Kehoe 2012, p.1).

Actuator for the implementation of this facility was the financial assistance demanded by Greece on the 9th of May 2010. As a reaction on this request, a consortium, composed of the International Monetary Fund, the ECB and the European Commission (thenceforth called "Troika"), jointly agreed to provide a rescue package with a total amount of 110 billion € to the Greek government. 30 billion € of this package were supplied by the IMF, while the 80 billion represent a collective amount, committed by several EU member countries (Seitz/ Jost 2012, p.3 f.). Through the application of this measure, the IMF and the EU promised themselves to achieve a reduction of the fiscal deficit (15.4% in 2009) and the debt burden (115% in 2009) to the levels defined in the Maastricht Treaty by 2014. In order to prevent against irrational behaviour, the package involved several requirements aimed to restructure Greece's public finances and bring back economic growth (fiscal austerity measures). These requirements included budget cuts and tax increases to account for the fiscal and debt problems, as well as freezes in salaries and deep reforms in order to strengthen the country's competitiveness (IMF 2010a).[18]

Soon it became clear that due to the interconnectedness of the financial markets, such financial aid would be needed as well by other countries in the Euro zone (Chamley/ Pinto 2011, p.1). Two funds with a total

[17] This institution is comparable to the idea of a European Monetary Fund, serving as a substitute for the European Central Bank, as first proposed by Gros and Mayer (2010).
[18] A detailed overview over the terms and conditions demanded from Greek's government and citizens can be found in (IMF 2010a).

capacity of 500 billion € were established. While the European Financial Stability Facility (EFSF) should lend up to 440 billion €, the European Financial Stabilisation Mechanism (EFSM) would provide the remaining 60 billion € in terms of a balance-of-payments support. The available amounts of this fund are comprised of guarantees from the Euro countries themselves.[19] With an additional contribution of 250 billion € on behalf of the IMF, a total amount of 750 billion € was available to support troubled countries within the monetary union.[20]

Since the inception of this fund, three rescue packages (excluding the one granted to Greece in 2010) have been provided, all conditional on fiscal austerity requirements in order to prevent against moral hazard.[21] An overview over these packages is provided in Table 1 and brings the granted amounts in relation to the maturing debt of the receiving countries.

Table 1: Granted rescue packages

Date	Country	Total Amount in Bio €	Maturing Debt until 2020 in Bio €
May 09, 2010	Greece	110	233.4
Dec 16, 2010	Ireland	85	83.4
May 20, 2011	Portgual	78	105.3
Mar 15, 2012	Greece	130	233.4

Source Data: (Seitz/ Jost 2012, p.4) (Statista 2012)

The first package was issued on the 16th of December 2010, when Ireland's real estate market collapsed and funds were needed to finance the state guarantees given to Ireland's banking system. The package involved a total amount of 85 billion €, whereas 62.5 billion € were provided by the EFSF, EFSM and the Irish nations cash reserves and the remaining 22.5 billion € by the IMF (IMF 2010b) (Seitz/ Jost 2012, p.4 ff.). Then, the second country who demanded the help of the "Troika" was Portugal in May 2011. The recession, the weak competitiveness and the high fiscal deficits directed the country into a situation, where government funding costs attained a level not bearable anymore. A total package of 78 billion €, equally apportioned between the EFSF, the EFSM and the IMF, was put together on the 20th of May 2011 to calm the situation on the capital markets (IMF 2011). Last but not least, Greece again asked for financial aid in July 2011 (Seitz/ Jost 2012, p.4 ff.). However, the conditions of this last package differed from the ones provided before. Already prior to the request, the European Union planned to force banks to accept a "voluntary" haircut of 50% on Greek bonds. Should enough bondholders agree to this policy, a fundamental write-down of Greek's debt would be realizable. On the 15th March 2012, this policy was agreed and in combination with lengthened maturities and lowered interest rates on bonds, a total haircut

[19] A list of guarantor Euro-area members with their respective guarantee commitments can be found in the "EFSF framework agreement" (EFSF 2012).
[20] Later in time (2011), it was decided that this constellation should only be temporary and that a new permanent rescue fund, called the European Stability Mechanism (ESM), will replace it in 2013. With this introduction, the EU wanted to account for the fact that troubled countries would need much more time to get back to the capital markets in order to be able to finance themselves (Seitz/ Jost 2012, p.2 ff.).
[21] A fourth rescue package to Spain has been agreed in June 2012. However, up until this point in time, it is not yet clarified whether these funds will be provided through the EFSF or the ESM. Therefore, consideration of this fourth package is neglected (Financial Times 2012).

of 75% was accomplished (Arellano/ Conesa/ Kehoe 2012, p.3) (IMF 2012a).[22] As a result, a total package of 130 billion € was put together and provided to the country (Seitz/ Jost 2012, p.5 f.). At this stage it needs to be noticed that Greece cancelled the requirements of the last package, as a adaptation of the public finances in accordance to the Maastricht Treaty was not achievable within the agreed time (IMF 2012a).

However, the country specific rescue packages and the associated fiscal austerity measures indeed showed their impact and the countries Greece, Ireland and Portugal managed to slightly run down their fiscal deficit to 9.2, 13.1 and 4.2 percent of GDP in 2011. Nevertheless, taking a look at the longterm interest rates on those countries' debt, one can see that the bond spreads continued to increase, even though the "Troika" was set up to calm the market's distrust by guaranteeing the roll over of the countries' maturing debt positions. There are two reasonable explanations, considering the set up of the rescue packages on one hand, and on the other the influence of fiscal austerity measures.

First, through the provision of interest bearing loans, countries are provided with additional liquidity without the explicit guarantee of an increase in the net present value of fiscal resources. Therefore, in case of a malfunctioning, the only ones gaining from these rescue packages are the bond holders whose contracts mature at the time, when the official loans through the Troika flow in. This discourages new investors, as the newly provided loans are perceived to be senior, devaluating their own claims becoming due in the future. The increase in the risk premium is therefore attributable to a lack of confidence into the success of the rescue packages (Chamley/ Pinto 2011, p.1 ff.) In context of the Russian crisis in 2001, this argumentation has already been provided by Kharas, Pinto and Ulatov (2001, p.43) stating that "*A debt-based liquidity injection that aims to boost confidence could worsen public debt dynamics while offering heavily exposed investors a convenient selling opportunity. In this sense the financing portion of the package could actually trigger a crisis if the market is sufficiently sceptical about the implementation of fiscal and structural reforms. This argument is even stronger if the liquidity injection involves debt that is perceived to be senior to the existing claims of private creditors.*" The development of the credit default swap premiums during these years supports this argumentation, as shown in Appendix C.

The second explanation refers to the fiscal austerity measures and their risk of being counterproductive in the end. Cutting down government spending automatically reduces growth and consequentially increases unemployment, while an increased tax base additionally worsens the investment behaviour of a country's citizens (Pessoa 2011, p.10). The intent of achieving cost deflation (relative to the trading partners e.g. Germany), through decreased domestic demand and an internal devaluation of unit labour costs, can lead to lower economic growth, exacerbating the already prevailing debt problem (Favaro et al. 2011, p.230 f.). Is this case occurring, the mentioned counterproductive effects result and financial markets react by getting even more nervous (Pessoa 2011, p.10). Important to notice at this point in time is that especially short term funding became more expensive. Interest rates on e.g. two year government bonds massively exceeded the ones' of longer term instruments and domestic banks entered a situation, where they became

[22] Through the fact, that more than 50% of bondholders agreed to the haircut, the Greek government has allowed to invoke collective action clauses (CAC's), forcing the remaining bond holders to participate in the haircut (Arellano/ Conesa/ Kehoe 2012, p.3)

unable to raise unsecured funds. A spillover of the sovereign debt crisis on the banking sector was the outcome (BIS 2012a, p.1) (Chamley/ Pinto 2011, p.1 f.).[23]

Since mid of 2010, the domestic banks of the countries Greece, Ireland Portugal were suffering from constant deposit outflows. The necessity for a rescue package in these countries has set a negative sign to the markets and investors decided to withdraw their money and deposit it in the few remaining save havens of the core euro countries. However, a dramatic situation was observed in the countries Italy and Spain. Although those countries recorded continuous capital inflows in 2010, suddenly investors got afraid and the domestic banks suffered a loss of literally 120 billion Euros within the fourth quarter of 2011 (Dittli 2012a). These developments made evident that the crisis in Europe more and more takes on a systemic character, affecting the banking system as well as other member countries of the EMU (Pessoa 2011, p.10 f.). The prevailing situation increased the investors' fear and made them back out of peripheral countries' bonds, denotable as self-fulfilling movements. Through this behaviour, countries became trapped in a liquidity crisis resulting in a collapse of bond prices through a substantial increase in interest rates between June and November 2011 (See Figure 4). As financial markets typically become highly integrated with the entry into a monetary union, most of these government bonds were held by banks in the Euro zone, which turned the sovereign debt crisis into a banking crisis, leading to a literal closure of Europe's interbank market (De Grauwe 2012, p.1).[24] The initial assumption of the EU authorities, considering the problems of the governments to be national, demanding for national policy measures, turned out to be wrong (Pessoa 2011, p.10 f.). Therefore, in order to avoid a credit crunch and assure further funding of the troubled economies, a transnational infusion of liquidity became unavoidable.

2.2.3 Longer Term Refinancing Operations

On the 8th of December 2011, Mario Draghi, the president of the European Central Bank, announced to supply the European banking sector with two money injections involving a maturity of 36 months and an interest rate, fixed at one percent.[25] The instrument used to provide these two tranches is denoted as "Longer Term Refinancing Operations" (LTRO's). This measure belongs, besides other instruments, to the repo auctions, which are the predominant instrument of the ECB to implement monetary policy. Through the provision of these credits, the ECB was able to assure the liquidity of the domestic banks, which was important for the continuous maintenance of price stability (Linzert/ Nautz/ Bindseil 2004, p.7). In order to receive these loans, banks of the respective countries had to hand in bids, containing the amount of money desired to receive. The first loan was provided on the 21st of December 2011 with a total amount of 489 billion €, while the second package of 529.5 billion € was injected on the 1st of March 2012 (ECB 2012b). This additional liquidity was urgently needed by the Euro-area's domestic banks, as it enabled them to cover their funding needs of the next few years in large parts.

Main recipients of the first LTRO tranche were banks in Italy and Spain with an amount of 300 billion €, and to a smaller extent Belgium and French Banks (150 billion €). Banks in Germany, Finland and

[23] A more detailed analysis of these spillover effects, particularly in the context of sovereign rating changes, is provided by Arezki, Candelon and Sy (2011).

[24] Appendix D shows the foreign bank claims on assets of the GIIPS countries.

[25] In the meantime, on 5th of July 2012, this interest rate has been decreased again by 25 basis points to an amount of 0.75 percent (ECB 2012f, p.199).

Luxembourg didn't bid for considerable amounts.[26] However, their funding conditions improved indirectly, as large parts of the additional liquidity flowed as deposits into their books, increasing the liquidity on their balance sheets.[27] Little change was also observable in the LTRO balance of Greek, Portuguese and Irish central banks. The reason for that was, that those banks already bid for a considerable amount of 165 billion € in the previously announced 12-month LTRO and may have been short on collateral to participate in the bidding (BIS 2012a, p.3). The situation didn't change significantly in the second tranche (1st of March 2012). The main recipients were again domestic banks of Italy and Spain, asking together for an additional amount of 300 billion €. Contrary to the first LTRO, monetary financial institutions (MFI's) of Greece applied for additional money, increasing their liabilities to the central bank over about 70 billion € (BIS 2012a, p.4).[28]

Immediately after the introduction of this non-standard measure, bank funding conditions improved. Investors returned to the long-term bank debt markets and sales of uncollaterized bonds increased (BIS 2012a, p.4). But the strengthening of the domestic banking sector was not the only objective the ECB promised itself, even though this was not officially stated. By implementing these measures, the European Central Bank circumvented the "no-bailout clause" and delegated the power to buy bonds of the troubled countries to the domestic banks (De Grauwe 2012, p.1). This was indeed done, particularly in the countries Spain, Italy and Ireland. The reaction of the interest rates on government debt came promptly and funding conditions improved, as visible in Figure 4 (BIS 2012a, p.4).

Even though the implementation of the Longer Term Refinancing Operations helped to avoid a collapse of the European banking system and lead to a significant reassurance of the markets, it contained as well a few major disadvantages. The first and probably most significant problem is that the introduction of this measure didn't address the main problems of the peripheral countries. Instead, it provided the countries and, above all, their respective banks with time and short-term market quietness to take care of the fundamental inefficiencies. This was as well supported by Mario Draghi himself, stating that the most significant achievement of the LTRO's was "*that it gave banks room for deleveraging in an orderly fashion*" (ECB 2012d) (Meier 2012). This brings us to the second disadvantage of this measure. Banks were thought to use this additional liquidity to buy bonds of the troubled governments, denotable as "carry trade". As government bonds provide a much higher return, this was an invitation to an almost riskless return, with the disadvantage of fostering moral hazard trough the availability of short-term gains. As a result, banks again exposed themselves and included assets of troubled countries into their already weak balance sheets. Once the concerns over the solvency of these countries return, the effect turns around and the domestic banks may find itself in the same situation as prior to the introduction of the LTRO's (Meier 2012). However, only a fraction of this liquidity was actually channelled into the government bond markets. The problem is yet that the remaining parts were mainly kept in the books of the domestic banks or parked at the ECB over night, instead of being invested into the real economy, helping to achieve an improvement of their respective fundamentals (De Grauwe 2012, p.1).

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[26] Belgium and France needed to apply for additional funds, as due to the developments at the end of 2011, U.S. funds stopped providing their domestic banks with considerable amounts of U.S. Dollars (BIS 2012a).

[27] A more detailed analysis of this circumstance can be found in BIS (2012a).

[28] The total use of these 36-month repo auctions is depicted in Appendix E, showing the LTRO liabilities on the balance sheets of the domestic monetary and financial institutions (MFI's).

2.3 Current Situation

The last three sections have chronologically dealt with the different policy interventions, their mechanics and their respective effects on the economies, as well as on the financial markets. As we went along, it became clear that steadily new policy measures on behalf of the European Central Bank were needed in order to counteract against self-fulfilling crises to occur. With the implementation of the Longer Term Refinancing Operations in the end, a short-term relieve of the markets was achieved. Funding conditions for domestic banks and governments have improved and the investors appetite for risk returned, due to decreased concerns about the downside risks of market participants (BIS 2012b, p.1). In Greece, this relieve was additionally triggered trough the assurance of the second rescue package at the 1st of March 2012. However, as explained above, the LTRO's contained some substantial disadvantages, leading to doubts about the long term success of this operation. Recent developments of the markets have shown that these doubts were not unfounded. At the end of March 2012, the concerns about the Longer Term Refinancing Operations began to vindicate. The GIIPS countries still found itself in a recession and the actors on the financial markets realized that monetary policy measures (e.g. LTRO's) alone wouldn't be sufficient to address the fundamental problems of the Euro-area countries. As a result, interest rates on government bonds returned to move upwards, as investors were afraid of distorting growth effects through the introduction of fiscal austerity measures and the slow pace on the domestic labour markets. This trend was particularly observable in the countries Italy and Spain, the new victims of the systemic crisis. The downgrading of Spain to a Standard and Poor's rating of BBB+ on the 26th of April 2012 clearly supported this development. It wasn't long in coming until the domestic banking sector reacted, especially as it increased it's exposure to the governments earlier in the year. During the months April and May, euro bank equity prices constantly underperformed and the markets tend to move into a comparable situation observed in the months of October and November 2011. However, the situation is still considered to be less worrisome by market participants, as with the additional liquidity of the Longer Term Refinancing Operations, banks have built up a certain liquidity buffer (BIS 2012b, p.5 ff.).

Being aware of the different policy measures, established by the European Central Bank, the International Monetary Fund and the European Commission, the focus turns now to the specific situation of the Euro-area countries and their progress during these turbulent times. By the end of April 2012, the condition looks rather depressing. With a total of 17 member countries, no more than four (Finland, Germany, Luxembourg and the Netherlands) enjoy the highest Standard and Poor's rating of AAA. Contrary to that, the rating for government bonds of the five GIIPS was downgraded to junk bond status with BBB+ or lower. A clear exception thereby builds Greece, which was rated CCC in July 2011, then shortly became listed as selectively defaulted (SD) and returned, after the introduction of the second rescue package, again to CCC by May 2012 (Arellano/Conesa/Kehoe 2012, p.1) (Standard & Poor's 2012).

However, cutting it down to the essentials, the situation of the GIIPS countries is over all comparable. At the end of 2008, the subprime crisis erupted and drove the countries of the European Union into the deepest recession since the 1930's. While core countries like Germany managed to achieve positive growth through a recovery of its private sector, the recession in the GIIPS countries is still ongoing. As a result of this recession, government revenue dropped, forcing the governments to bridge this missing liquidity through the issuance of public debt, as there is no foreign exchange market, nor a flexible exchange rate to counteract against these losses. Through this necessity, countries became highly dependent on the willingness of the actors on the financial markets to roll over their debt contracts, as otherwise a sovereign default would be inevitable. However, instead of running down public borrowing to avoid the high costs caused through the steadily increasing interest rates, the debt positions continued to soar in all GIIPS countries (See Figure 1) (Conesa/ Kehoe 2012a, p.2 ff.).

One argumentation for this behaviour might simply be a lack of rationality. Thereby, governments, as well as lenders, may have fooled themselves by thinking that "this time is different" and therefore a default of the respective countries cannot occur with positive probability (Reinhart/ Rogoff 2009). However, the developments and reactions in the markets during the last years provide reasons to believe that the actors in the markets perfectly understood the risks. Therefore another argumentation for this development can be that the policy measures introduced by the EU authorities may have been ill designed and fostered moral hazard. The idea is that through the expectation of a future bailout, driving down the interest rates on government bonds, countries may have decided to refrain from sharp cuts in expenditures and run up the their debt positions, gambling on a recovery of the private sector. The allegedly irrational behaviour is thereby the optimal response to the developments in the markets. The purpose of the theoretical framework in the next section is to investigate exactly this theory (Conesa/ Kehoe 2012a, p.1 ff.).

3. The Model

The sections above have shown the prevailing economic situation of the European Union and the consequential policy decisions taken by the European Central Bank and the governments of the respective countries. As we move on, we want to analyse those countries' behaviour, and more specifically their reactions on the third party policy measures, within the dynamic stochastic general equilibrium model provided by Juan Carlos Conesa and Timothy J. Kehoe in 2011.

Before starting with the illustration of the environment, a short instruction into the main deliberations of this framework needs to be given, to make the understanding of the following theoretical sections more comprehensible. As already denoted above, the government of a respective country has a need to roll over its debt into the next period, in order to stay capable of acting. Is this possibility not given, default is the only resulting option. However, the inability to meet its liabilities does not necessarily need to be a result of missing liquidity or insolvency, it can be as well induced through the fear of international investors, expecting a country to become unable to pay. Is this the case, we talk about a self-fulfilling prophecy, as the lenders' or more specifically the banks' fear of a forthcoming crisis, actually initiates suchlike. One option to proceed against this situation is to start running down the debt positions with the hope that the banks' fear, and in the same move, the interest rates decline. The other option is a martingale gambling strategy, referred to as gambling for redemption. If a country finds itself in a recession, it wants to avoid the costs of running down its debt positions, as it assumes that the economy will recover at a certain point in time. This expectation gives a government the incentive to continue running up its borrowing level, speculating for the end of the recession and the subsequent recovery in tax revenues. Is this the case, the costs of repaying the debt lower, as the interest rates decrease. However, is the recession still prevailing, the country has built up such a high borrowing level that default is the only reasonable option. The basic assumption allowing for such strategies is the lack of coordination among the international bondholders (Conesa/ Kehoe 2012a, p.1 ff.).

In a first step, in section 3.1, the fundamental basics will be presented. Using this foundation enables us then to determine the optimal government policies for two special cases in section 3.2 and 3.3. Based on the knowledge gained from these two cases, in section 4, a calibration of the model is applied in order to calculate the equilibrium solution for a government, finding itself an environment comparable to the economy in Europe.

3.1 Fundamentals

Section 3.1.1 focuses on the environment of the theoretical framework, pointing out the similarities, as well as the distinctions of the basic and the extended framework mentioned in the introduction. With the use of this knowledge, the section 3.1.2 – 3.1.3 will then deal with the definition of the equilibrium and the derivation of certain debt thresholds and bond pricing functions. As a last step, in section 3.1.4, the general constraints of this model are being explained, guiding is to the two special cases referred to above.

3.1.1 Environment

There exists a continuum of infinitely lived agents, time is discrete and in each period $t=0,1,\dots,T$, a single good is consumed (Cole/ Kehoe 2000, p.93 f.) (Conesa/ Kehoe 2011, p.6 ff.). Individuals are subdivided into two types of people: consumers and international bankers. Additional to that, there is a benevolent government, taking the role of the strategic agent (Cole/ Kehoe 1996, p.310 ff.). In the following, the economy and each type of agents are described separately, to give an insight into the actors' incentives and consequentially their behaviour in the framework.

First, the influencing factors to a country's current economic situation need to be defined, as this information is crucial to understand the triggers for an upcoming debt crisis. This aggregate state is denoted as $s=\{B,a,z_{-1},\zeta\}$, containing information about the current level of government debt B, the condition of the private sector ($a=1$ normal, $a=0$ recession), whether the government has already defaulted in the past z_{-1} ʽ $\{0,1\} = \{default, repayment\}$, and the value of an exogenous sunspot variable ζ, acting as an indicator for a future crisis (default) to occur. While B, a and z_{-1} are given through the outcomes of the last period, the value of the sunspot variable is randomly drawn at every period from the independent and uniform distribution on the interval $[0,1]$. Using part of the information, given through the aggregate state, enables then the theoretical derivation of a country's GDP:

$$y(a,z) = A^{1-a} Z^{1-z} \bar{y},$$

where $0<A$ and $Z<1$. The setup of this equation points out the fact that a recession, as well as default, has a suppressive impact on the gross domestic product of a country. Considering first the variable z, one needs to know that the default of a country has an immediate and infinitely long lasting penalty-effect on its GDP, as by definition, once $z=0$, it stays equal to zero for all future periods (Conesa/ Kehoe 2011, p.6).[29] There are two intuitive justifications for this supposition. On one hand, the default of a country involves a reputation damage. In this context, it is assumed that the government immediately loses access

[29] Please notice that the default penalty occurs in the same period as the crisis. Anyhow, substituting z against z_{-1} in the GDP equation wouldn't have any qualitative effect on the results (Conesa/ Kehoe 2011, p.6).

to all international borrowing and lending for an infinitely long time, hurting its ability to operate, visible in a decreased output (Cole/ Kehoe 1998, p.7 f.). In the paper of Harold L. Cole and Patrick J. Kehoe (1998), these reputation spillover effects are analyzed in a more detailed way. On the other hand, the fall in productivity after a default can be induced through the reduction of the value of output, evoked by the government's inability to engage in international trade (Cole/ Kehoe 1998, p.7). Reasons for such trade sanctions are provided by Obstfeld and Rogoff (1996), including loss of access to short term credits and the confiscation and seizure of goods and trade related assets. Being aware of these restrictions, the costs induced through the default penalty become clearer, as they deduct important resources available for government spending, private consumption and repayment of debt (Arellano/ Conesa/ Kehoe 2012, p.9).
Going back to the equation explaining a country's GDP, one can see that the second relevant variable is the condition of the economy's private sector, *a*. If a country finds itself in a recession, its labour market experiences a downturn, which in turn lowers the government's revenue generated through taxation. This again increases a country's vulnerability to a self-fulfilling crisis, as the decreased tax revenue needs to be bridged by additional foreign borrowing. The mathematical coherence to this circumstance will become clearer in this and further sections of this thesis.

Before moving on to the individual agents in the model, the country's initial aggregate state $s=\{B,a,z_{-1},\zeta\}$ in $t=0$ has to be defined, as this represents a major difference to the basic framework provided by Cole and Kehoe (2000). Prior to period $t=0$, the private sector is in normal conditions $(a=1)$ and no default has happened so far $(z=1)$. Then, in $t=0$, suddenly, the economy falls into a recession $(a=0)$, which results in a decrease of GDP from $y(a,z)= \bar{y}$ to $y(a,z)=A\bar{y} < \bar{y}$. This is contrary to Cole and Kehoe (2000), as in their model a stays equal to *one* all the time. Inserting this slight difference allows for the adaptation of the basic model on the European Sovereign Debt Crisis. The probability for an economic recovery is represented through the variable p, where with probability $0<p<1$, a can become 1 in every period $t=1,2,\ldots,T$, staying equal to one forever. Being aware of the fact that the economy might recover at a certain point time triggers the incentive of a government to gamble for redemption, given the assumption that it is able to continue rolling over its debt positions (Conesa/ Kehoe 2011, p.6).

Having taken a look on the initial setup of the economy, we can now move on to the individual type of agents mentioned above. First, let's focus on the consumers, representing the citizens of the domestic economy by paying taxes and consuming the private and public good. The consumer's problem is

$$V^c(s;p,\pi) = \max u(c,g) + \beta EV(s';p,\pi),$$

subject to its budget constraint $c \leq (1-\theta)y(a,z)$. c is private consumption, g reflects government expenditures (public good), $0<\theta<1$ is the constant proportional tax rate on domestic income and $0<\pi<1$ denotes the probability of a self fulfilling crisis (Cole/ Kehoe 2000, p.93) (Conesa/ Kehoe 2011, p.7). Further, the discount factor β is assumed to be $0<\beta<1$ and u is continuously differentiable, strictly concave and monotonically increasing (Cole/ Kehoe 1996, p.311). Variables evaluated in the following period, e.g. tomorrows aggregate state of the economy s', are denoted with a "prime". Comparing the consumers in this model to the ones in the original framework, provided by Cole and Kehoe in 1996, on can see that the citizens of a country have no capital stock to optimize and therefore do not face private investment decisions. Abstaining from this detail leads to the fact that consumers do not save and therefore consume all of the GDP remaining after taxation (Arellano/ Conesa/ Kehoe 2012, p.9).

Now, let's move on to the international bankers, acting as a lender, by purchasing government bonds period by period, dependent on the probability they assign to a crisis. Whether to buy future bonds or not is triggered through the random realization of the previously mentioned sunspot variable ζ, acting as a synonym for bad news about a country's economic condition (Conesa/ Kehoe 2011, p.7) (Arellano/ Conesa/ Kehoe 2012, p.9 f.). If $\zeta_t > 1-\pi$, bankers expect there a crisis to occur and therefore won't buy government bonds, in case this crisis turns out to be self-fulfilling. However, is the contrary case occurring, $\zeta_t \leq 1-\pi$, the risk they assign to a crisis, and a consequential default, is not stopping them from buying additional bonds. Being aware of this causality, the probability of a crisis π, $0 \leq \pi \leq 1$, becomes arbitrary, given the assumption that the level of debt is high enough, making such a crisis even possible (Conesa/ Kehoe 2011, p.7). This again shows, due to the fact that the sunspot variable is randomly drawn and varies over time, how countries are at the mercy of financial markets (Arellano/ Conesa/ Kehoe 2012, p.10). Another supposition needed to be mentioned at this point in time is the risk neutrality of the international bankers with a discount rate β (Conesa/ Kehoe 2011, p.8). The reason for this lies in the assumption that the bonds of this particular country only make a small fraction of the investor's total portfolio (Arellano/ Conesa/ Kehoe 2012, p.7).[30] The resulting problem function is the following:

$$W(b, B', s; p, \pi) = \max x + \beta E W(b', B'', s'; p, \pi)$$

subject to its budget constraint, $x \leq \bar{x} + zb + qb'$, and the constraint that the amount of purchased bonds does not exceed its current asset holdings, $qb' \leq \bar{x}$ (Conesa/ Kehoe 2011, p.8) (Cole/ Kehoe 1998, p.5). x is the banker's private consumption, $\bar{x}$ is it's endowment in each period and q is the price of a one-period government bond paying back b' in $t+1$, if the government decides to repay ($z=1$) and otherwise *zero* (if $z=0$) (Cole/ Kehoe 1996, p.311).[31] The height of this bond price q changes, depending on the probability the international bankers assign to the occurrence of a self-fulfilling debt crisis and therefore depending on the future debt level (Conesa/ Kehoe 2011, p.8). Anyways, a detailed derivation of these prices follows in section 3.1.3. In period $t=0$, each banker holds b_0 units of government bonds (Cole/ Kehoe 1996, p.311). If the banker decides, not to spend the whole endowment, along with the earnings received through the bonds of the previous period, he consumes the remains later on in the period (Cole/ Kehoe 2000, p.93). Additional to the budget constraint, the following inequality $b' \geq -A$ needs to hold in order to rule out Ponzi schemes (Conesa/ Kehoe 2011, p.8).[32]

Last but not least, we need to take a look at the benevolent government, supplying a valued public good g to the citizens of the country. Its responsibility lies in the maximization of the consumers' present values, without violating its budget constraint. The choices made in each period comprehend the future borrowing level B', the default decision z and the level of government spending g, subject to its budget constraint:

$$g + zB \leq \theta y(a, z) + q(B', s; p, \pi) B'.$$

So in every period, the government has to meet the requirement that the sum of resources spent in form of government expenditures (supplying the public good) and payments on bonds becoming due has to be less

[30] If we would assume that the bonds of this specific country represent a major part of the banker's portfolio, then he or she would be risk averse and therefore pay less for a bond with the de facto same risk (Arellano/ Conesa/ Kehoe 2012, p.7). The foreign bank claims in Appendix D support this assumption.

[31] The endowment $\bar{x}$ is chosen to be large enough, in order to ignore corner solutions to the banker's utility maximization problem (Cole/ Kehoe 1996, p.311).

[32] With $A > 0$, this constraint does not bind otherwise in the equilibrium (Cole/ Kehoe 2000, p.93).

than or equal to the collected tax revenues and receipts from the newly issued bonds (Arellano/ Conesa/ Kehoe 2012, p.6).[33] Being aware of this inequality, the impact of a default, as well as of a recession, is now more visible than at the beginning of this section. As already indicated, both lead to an immediate drop in government revenue. This is due to the fact that the tax rate is assumed to be fixed, building a constant fraction of a country's GDP (Conesa/ Kehoe 2011, p.6) (Arellano/ Conesa/ Kehoe 2012, p.9). The intuition behind this was explained by Cole and Kehoe (1998), arguing that the period over which crises can occur is assumed to be short, compared to the time it takes a government to adjust its tax policy.[34] As the level of taxation is fixed, two major financing instruments remain to be investigated. One way to generate revenue is the usage of inflation, through an increase in money supply. However, against the background that the optimal government policy within an economic and monetary union is being derived, this possibility needed to be ruled out. Therefore, in accordance with the facts, public debt remains the only financing instrument, which can be varied in order to account for fluctuations in revenue and expenditures. Is the government therefore running a fiscal deficit, meaning that the tax revenues cannot cover the current expenditures, it has a need to frequently sell large quantities of bonds in order to avoid a default to occur (Arellano/ Conesa/ Kehoe 2012, p.6).[35]

As mentioned at the beginning of this section, the government acts as the only strategic agent in this framework. Several decisions are made considering those decisions' effects on the future debt price, upcoming government revenue and the level of private consumption (Cole/ Kehoe 2000, p.94). Being able to do this highly depends on the timing of actions in each period, denoted below (Cole/ Kehoe 2000, p.94) (Conesa/ Kehoe 2011, p.7):

1. The sunspot variable ζ is realized, determining the aggregate state of the economy $s=\{B,a,z_{-1},\zeta\}$.
2. The government decides how much public debt B' to issue, given the price schedule q.
3. International bankers choose how much debt b' to purchase.
4. The government makes its default decision z and chooses the level of government expenditures g.
5. Consumers, taking $y(a,z)$ as given, choose how much to consume c.

This timing of actions demands now for the introduction of one of the model's key facts: the limited commitment assumption. The government is not able to commit for future policy decisions and therefore announces its policy, consisting of the choice whether to repay the debt or not, towards the end of a period. This is an important element of this framework, as it allows the government to issue new bonds B', before deciding whether to pay back the old ones. Noticing this order, policy decisions can be made, dependent on the amount of money currently available and the expected expenditures in the future. Still, as bonds are assumed to have a maturity of one period, the government has a need to roll over its debt period by period, as otherwise it would be vulnerable to a self-fulfilling crisis (Cole/ Kehoe 2000, p.94). The fact, that the trigger for such a crisis is the amount of debt needed to be repaid B, underlines this idea (Conesa/ Kehoe 2011, p.8).

[33] The benevolent government in the model is therefore not required to balance its budget in a traditional manner by equalizing tax revenues with expenditures (Arellano/ Conesa/ Kehoe 2012, p.6).

[34] However, changing this assumption would have no significant effect on the qualitative results (Cole/ Kehoe 1996, p.95).

[35] This allows for a modification of the budget constraint to the following term: *Fiscal deficit = expenditures – tax revenues = receipts from new bonds – payments on maturing bonds* (Arellano/ Conesa/ Kehoe 2012, p.6)

3.1.2 Recursive Equilibrium

Given the fact that the government has a limited ability to commit for future policy choices, a recursive-Markov-equilibrium is applied. This way, all agents choose their actions sequentially in reverse order to the timing of actions defined above. Utilizing this backward induction is in line with the environment's set up, allowing the government, and any other agent in the framework, to choose its policy with awareness of the optimal choices done by the other respective agents in the economy (Cole/ Kehoe 2000, p.95). This way, an optimal time-consistent policy can be found to account for the possibility of confidence crises, encouraged through the limited commitment assumption (Cole/ Kehoe 2000, p.92).

At the beginning of each period, the aggregate state, $s=\{B,a,z_{-1},\zeta\}$, is defined. Taking this information and considering other variables, affecting an agent's maximization problem and its state in a future period, leads us to the current individual state of an agent. With respect to this individual state, the solution to an agent's maximization can be found, expressed through a value function, maximizing the agent's utility, and a policy function, leading to this maximized utility through the optimal selection of its choice variables (Cole/ Kehoe 2000, p.95). In summary, the following functions are being derived within the recursive equilibrium: the government's value function $V(s)$ and its policy functions $B'(s)$, $z(B',s,q;p,\pi)$ and $g(B',s,q;p,\pi)$; the bankers' value function $W(b,B',s;p,\pi)$ and its policy function $b'(b,B',s;p,\pi)$; and the bond price function $q(B',s;p,\pi)$ (Conesa/ Kehoe 2011, p.12). As we can see at this point in time, the choices of the individual consumer's are omitted. This is due to the fact that, contrary to the basic framework developed by Cole and Kehoe (2000), the model of Conesa and Kehoe (2011) abstains from the fact that individual consumers have a capital stock to optimize. Therefore, with the assumption that the government is benevolent, consideration of their decisions is already assured and can therefore be neglected.

Now we can start with the definition of the recursive equilibrium, analyzing first the problems of the government. The government has to make its decisions at two points in time: at the beginning and at the end of a period. As we work backwards, we start with its default choice z, having a direct impact on GDP $y(a,z)$ and the level of government spending g. The policy functions for those two variables can be derived, using the consumers' problem and the government's budget constraint. The resulting problem at the end of the period takes the following form:

$$V^g(B,a,z_{-1},\zeta;p,\pi) = \max_{g,z} u(c,g) + \beta E V'^g(B',a',z,\zeta';p,\pi)$$

subject to:

$$c = (1-\theta)y(a,z)$$

$$g + zB = \theta y(a,z) + qB'$$

$z=0$ or $z=1$.

Solving this problem then leads to the policy functions for $z(B',s,q;p,\pi)$ and $g(B',s,q;p,\pi)$, building the foundation for a country's optimal spending path, given the fundamentals explaining its current situation (Cole/ Kehoe 2000, p.96 f.) (Conesa/ Kehoe 2011, p.12). Being aware of this coherence, we can now move on to the problem of the international bankers.

At the point in time, the banker faces its decisions, he is aware of its current holdings of government debt b, the aggregate state of the economy s and the amount of new debt B', the government is offering at the beginning of the period (Cole/Kehoe 2000, p.96). Given this information, the individual state of a banker is defined as *(b,B',s;p,π)*. Using now the bankers' dynamic programming problem enables us to perform the calculation of the bond price *q*, dependent on the aggregate state of the economy *s*, and the amount of bonds *B'*, the government is offering for sale (Conesa/ Kehoe 2011, p.9). The problem takes the following form:

$$W(b,B',s;p,\pi) = \max_{b'} \bar{x} + z(B',s,q(B',s;p,\pi);p,\pi)b - q(B',s;p,\pi)b' + \beta EW'(b',B'',s';p,\pi)$$

subject to

$$q(B',s;p,\pi)b' \leq \bar{x}$$

where *b'(b,B',s;p,π)* represents the bankers policy function (Conesa/Kehoe 2011, p.12). Using the first order condition solved after *q*, leads to the bond price function (perfect foresight condition):

$$q(B',s;p,\pi) = \beta Ez(B',s,q(B',s;p,\pi);p,\pi),$$

allowing for the calculation of the implicit interest rate *i* through *i = (1/q)-1* (Conesa/ Kehoe 2011, p.9). As long as the price for a bond satisfies this function, an international banker is willing to purchase the amount of debt offered by the government. Simply said, the expected gross return on a bond needs to be at least *1/β*, underlining the bankers' risk neutrality mentioned before (Cole/ Kehoe 2000, p.96). In order to define the exact bond prices, conditional on the fundamentals of the environment, and especially the newly offered amount of foreign debt, we have to define certain debt thresholds first. Section 3.1.3 will cover that.

As a last step, we can now move on to the government's decision at the beginning of the period: the choice of the new debt amount *B'*. At this point in time, the government is aware of the bond price *q(B',s;p,π)*'s dependency on the aggregate state of the economy and the new borrowing level. Additional to that, it knows what policy decisions, concerning expenditures *g(B',s,q;p,π)* and the default *z(B',s,q;p,π)*, it will face later on in the period and what effect these decisions have. Baring in mind its objective to maximize the utility of the consumers, the decision over *B'* is being made (Cole/ Kehoe 2000, p.96). Anyhow, due to the limited commitment assumption, the government is not able to promise a repayment of debt later on in the period. Therefore, if the sunspot variable indicates a self-fulfilling crisis to occur, bankers are not willing to roll over the debt, guiding the government into immediate default without the ability to react (Conesa/ Kehoe 2011, p.12 f.). Considering these reflections, the government's problem at the beginning of the period takes the following form:

$$V^g(B,a,z_{-1},\zeta;p,\pi) = \max u(c,g) + \beta EV'^g(B',a',z,\zeta';p,\pi)$$

subject to

$$c = (1-\theta)y(a,z(B',s,q(B',s;p,\pi);p,\pi))$$

and

$$g(B',s,q(B',s;p,\pi);p,\pi) + z(B',s,q(B',s;p,\pi);p,\pi)B$$
$$= \theta y(a,z(B',s,q(B',s;p,\pi);p,\pi)) + q(B',s;p,\pi)B'$$

with *B'(s)* as the resulting policy function (Conesa/ Kehoe 2011, p.12).

This set up of the individual value functions and the respective agents' corresponding maximization choices defines the recursive equilibrium of this environment. Strategic decisions being made by the government are underlined through the selection of this certain pattern, expressed by the increasing amount of information and dependencies from step to step. Again, the specific reason for the choice of this set up is the government's lack of ability to commit to a repayment of its debt at the beginning of the period. The uncertainty prevailing in the market allows for crises of confidence which need to be taken care of through an optimal time-consistent policy (Cole/ Kehoe 2000, p.92). Literature examining these theories has been provided by Lucas and Stokey (1983) and Chari and Kehoe (1990).

However, evaluating the described equilibrium from a sustainability and credibility perspective, as defined by Chari and Kehoe (1990, 1993) for the former and by Stokey (1991) for the latter, two crucial assumptions differ. The first is the major attribute of a recursive-Markov-equilibrium, visible in the ability of the agents to react on the policy choices made by other individuals. The second one is that prices and quantities are not set by the government and the agents, as the individuals' decisions can be changed, depending on the government's decision on future borrowing (Cole/ Kehoe 2000, p.98). Being aware of this defined equilibrium, the exact bond prices are now being derived, laying the foundation for the determination of government policy in section 3.2 and 3.3.

3.1.3 Bond Pricing

As already mentioned above, the crucial factor, driving the results of the model, is the government's need to roll over its debt to the next period (Cole/ Kehoe 1996, p.312). Whether this is possible, solely depends on the current borrowing level and the international bankers' willingness to buy additional bonds, while this lending behaviour is triggered by the probability the banker assigns to the default of a country π. Being aware of the fact that π is an expression of the sunspot variable's realization ζ, we can now focus on the determination of current debt levels B, making such a crisis (and consequential default) even possible. In order to this, we work with so called thresholds (Cole/ Kehoe 2000, p. 98 f.).

There are five state space regions described by four different threshold (cut-off) functions $\bar{b}(a;p,\pi)$ and $\bar{B}(a;p,\pi)$ with $a=0,1$. Function $\bar{b}(a;p,\pi)$ defines the country's maximum borrowing level for which no default can occur, called "the upper safe debt limit". $\bar{B}(a;p,\pi)$ in turn is denoted as "the upper sustainable debt limit", describing the maximum level of debt where an equilibrium, with positive lending on the part of international bankers, is still realizable (Cole/ Kehoe 2000, p.98 f.) (Conesa/ Kehoe 2011, p.8 f.) (Arellano/ Conesa/ Kehoe 2012, p.9).

Taking a look at the general case without consideration of the private sector's condition, three zones can be identified. The first one is the "no-crisis-zone" with a current debt level of $B \leq \bar{b}(a;p,\pi)$. As in this situation a country is not vulnerable to a potential default, the risk premium on bonds is particularly low. Bankers do not run the risk of a government's default and are therefore willing to lend, independent of the sunspot variable's realization. The second zone is called the "crisis-zone". With debt levels between the two thresholds, $\bar{b}(a;p,\pi) < B \leq \bar{B}(a;p,\pi)$, a crisis can occur with positive probability. This has an immediate effect on the behaviour of the international bankers, as their decision becomes dependent on the realization of the sunspot variable. If $\zeta_t > 1-\pi$, international bankers' forecast the government's default and are therefore not willing to lend additional money in the equilibrium. This inability to roll over the current debt drives the government into a default. However, is the contrary case, $\zeta_t \leq 1-\pi$, observable, then no inability to pay is predicted by the international bankers, giving the government the chance to roll over its debt and meet its commitments. Last but not least, let's consider the situation, where current debt is higher than the upper threshold, $B \geq \bar{B}(a;p,\pi)$, denoted as the "default-zone". Here, immediate default is the only possible outcome, as bankers' correctly predict the government's default in the current period, no matter the realization of the sunspot variable (Cole/ Kehoe 2000, p.98 f.) (Conesa/ Kehoe 2011, p.7 ff.).

Being aware of these assumptions, conclusions over a government's default decision can be made, accounting for the current debt level, the condition of the private sector and the international bankers' lending behaviour. The strategies for the four thresholds are summarized as follows, keeping in mind that each rule has to be considered for $a=0$ and $a=1$ (Conesa/ Kehoe 2011, p.8 f.):

1. If $B \leq \bar{b}(a;p,\pi)$, the government is able to repay its debt (no default) even if international bankers do not lend, while with $B > \bar{b}(a;p,\pi)$, the government is not able to meet its obligations when further borrowing is not possible.
2. If $B \leq \bar{B}(a;p,\pi)$, the government is able to repay its debt only if international bankers lend, while with $B > \bar{B}(a;p,\pi)$, it defaults, even though the bankers would be willing to roll over the debt.

Looking at the different thresholds underlying the model, one can see that for a government, finding itself in the crisis-zone, $\bar{b}(a;p,\pi) < B \leq \bar{B}(a;p,\pi)$, it is absolutely necessary to have bankers agreeing to roll over the public debt, as otherwise immediate default is the only resulting option (Cole/ Kehoe 2000, p.98).

Being aware of the different thresholds underlying the model, and particularly the role of the sunspot variable within the "crisis-zone", we can now move on to the exact calculation of the bond prices. As already betoken above, the prices of foreign debt are depending on the amount of new debt being offered by the government B', and the condition of the private sector a. However, whether a crisis in a country takes place is triggered by the amount of debt the government has to repay B (Conesa/ Kehoe 2011, p.8). In the previous section, the price function for bonds q was deduced through the solution of the international bankers' dynamic programming problem. The resulting equation defines the price as the risk neutral discount rate of an international banker β times the expected value of a country's default z, depending on the fundamentals of the given period.

$$q(B',s;p,\pi) = \beta Ez(B',s,q(B',s;p,\pi);p,\pi)$$

Using the four different debt thresholds defined above, the objective of this section is now to assign a bond price to each of the five state space regions. In this context, two major conditions, related to the assumption of an infinitely long lasting default penalty z, need to hold. First, if a country has already defaulted once in the past ($z_{-1}=0$), it is excluded from international borrowing from this period onwards, resulting in a zero bond price, due to the unwillingness of bankers to lend. The second assumption refers to the crisis-zone, $\bar{b}(a;p,\pi) < B \leq \bar{B}(a;p,\pi)$. If the realization of the sunspot variable indicates a forthcoming crisis, $\zeta > 1-\pi$, bankers as well desist from rolling over the government's debt into a future period, setting $q=0$:

$$q(B',(B,a,0,\zeta);p,\pi) = q(B',(B,a,1,\zeta);p,\pi) = 0.$$

Therefore, in the crisis zone, the sunspot variable needs to predict an advantageous outlook $\zeta \leq 1-\pi$, in order to make further lending even possible. Is this given, the bond price q can be assigned to the different zones (Conesa/ Kehoe 2011, p.8 f.). The way to do this is intuitive, without the use of complex mathematics. The only thing needed in advance is a basic equation, denoting the expected default decision of a country. As all variables in the environment are conditioned on the probability of an economic recovery p and the probability of default π, those indicators will be used for the formulation, representing the risk premium on public debt. The basic expression resulting is the following:[36]

$$q(B',(B,a,z_{-1},\zeta);p,\pi)=\beta(p(1-\pi)+(1-p)(1-\pi))$$

Now considering the information given from the debt thresholds above, the probability for a crisis can be assumed, given the potential probability for a recovery. As the economy is in a recession ($a=0$) from period $t=0$ onwards, this probability is again dependent on the last threshold that has been exceed by the new level of debt B'. So, in order to be able to orientate ourselves, these thresholds have to be sequenced first. From the characterisation of the cut off levels, four structural conditions can be elicited:

$\bar{b}(0;p,\pi) < \bar{b}(1;p,\pi)$; $\bar{b}(0;p,\pi) < \bar{B}(0;p,\pi)$; $\bar{b}(1;p,\pi) < \bar{B}(1;p,\pi)$ and $\bar{B}(0;p,\pi) < \bar{B}(1;p,\pi)$.

However, still not entirely clarified is the relationship between $\bar{b}(1;p,\pi)$ and $\bar{B}(0;p,\pi)$. The further investigation will be restricted to the following case:

$$\bar{b}(0;p,\pi) < \bar{b}(1;p,\pi) < \bar{B}(0;p,\pi) < \bar{B}(1;p,\pi)$$

as the other two possibilities, $\bar{b}(1;p,\pi) > \bar{B}(0;p,\pi)$ and $\bar{b}(1;p,\pi) = \bar{B}(0;p,\pi)$, are only realizable for very low values of A. Nevertheless, for the purpose of completeness, bond prices for those cases are derived in Appendix F. Applying now the basic equation defined above, the following bond prices result (Conesa/ Kehoe 2011, p.9 ff.):

$$q(B',(B,0,1,\zeta);p,\pi)=\begin{cases}\beta & \text{if } B' \leq \bar{b}(0;p,\pi)\\ \beta(p+(1-p)(1-\pi)) & \text{if } \bar{b}(0;p,\pi) < B' \leq \bar{b}(1;p,\pi)\\ \beta(1-\pi) & \text{if } \bar{b}(1;p,\pi) < B' \leq \bar{B}(0;p,\pi)\\ \beta p(1-\pi) & \text{if } \bar{B}(0;p,\pi) < B' \leq \bar{B}(1;p,\pi)\\ 0 & \text{if } B' > \bar{B}(1;p,\pi)\end{cases}$$

Taking a look at the equations, it is visible that prices decrease with an increase in future debt. The model therefore assumes a positive correlation between the current debt level and the risk premium on government debt.

The number of thresholds varies to the framework of Cole and Kehoe (2000), as the economy unexpectedly runs into a recession in period $t=0$. Through the introduction of the variable, denoting the probability for a recovery, cut-off levels increase from three to five. Applying the basic equation defined above, it is straight forward to derive the bonds prices fitting to this environment, where $s=\{B,1,1,\zeta\}$:

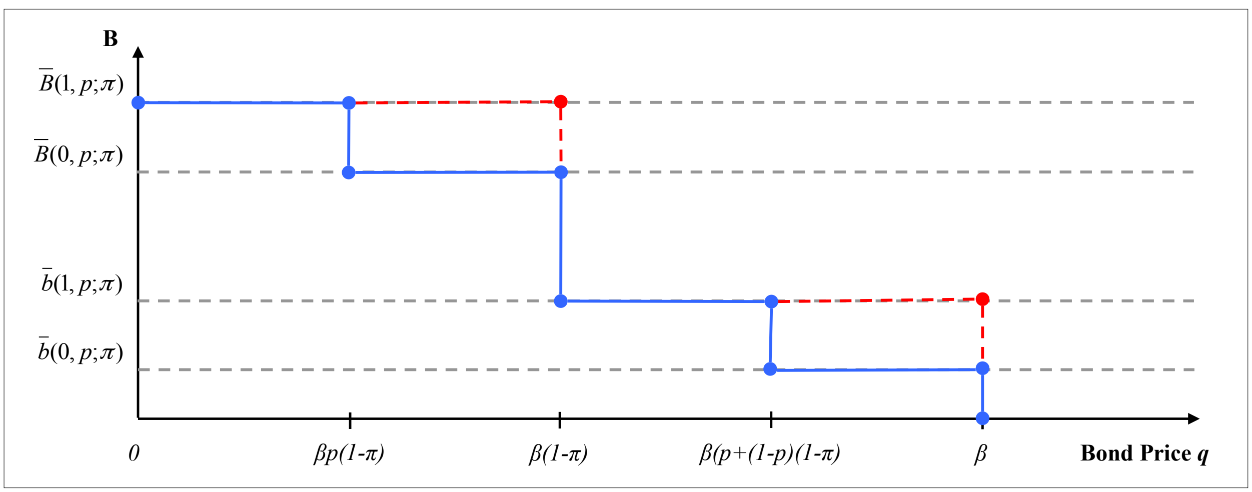
[36] Please notice that the approach of using a basic equation has not been confirmed by the authors Cole, Conesa and Kehoe. However, given their definition of the respective bond prices dependent on the level of future debt, the application of this function makes sense in my opinion.

$$q(B',(B,1,1,\xi);p,\pi)=\begin{cases}\beta & \text{if } B' \leq \bar{b}(1;p,\pi)\\ \beta(1-\pi) & \text{if } \bar{b}(1;p,\pi) < B' \leq \bar{B}(1;p,\pi)\\ 0 & \text{if } B' > \bar{B}(1;p,\pi).\end{cases}$$

The gained knowledge over the sequence of the thresholds and the resulting bond prices allows now for a clarification of the martingale gambling strategies applied in this framework. As stated above, default is the only outcome as soon as the current debt level exceeds the upper threshold, $B \geq \bar{B}(a;p,\pi)$. However, through the consideration of the private sector's conditions and the prevalent recession since period $t=0$, this assumption changes. The idea behind this is the following: Is the private sector of the economy staying in a recession, immediate default is the only possible outcome, due to the effects on the implicit interest rates. However, does the aggregate state predicate that the private sector has turned into normal conditions, default does not necessarily need to happen, as the upper threshold increases to $\bar{B}(1;p,\pi) > \bar{B}(0;p,\pi)$, enabling the government to return back into the new "crisis zone", where $B \leq \bar{B}(1;p,\pi)$ (Cole/ Kehoe 2000, p.98 f.) (Conesa/ Kehoe 2011, p.8 ff.). The challenge thereby is to find the optimal point in time to exceed this upper sustainable debt limit, as otherwise default is the only reasonable option.

The knowledge gained from this section is summarized in Figure 5, containing the defined thresholds and the implications extracted from the bond pricing functions. Please notice that the red dotted lines refer to the limited case, where the economy finds itself in normal conditions, resulting in three instead of five bond prices.

Figure 5: Debt thresholds and bond pricing functions



Idea for this figure provided by Conesa and Kehoe (2011, p.11)

3.1.4 General Constraints

Having defined the fundamentals of the model, last but not least two major constraints need to be introduced, ensuring rational behaviour on behalf of the government in an equilibrium with positive lending.

The functions $V_n^g(B,a,q;p,\pi)$ and $V_d^g(B,a,q;p,\pi)$ define a government's payoff in the case of not defaulting and defaulting, labelled through the subscripts. The realization of these functions happens after the issuance of the new debt *B'*. Is the payoff of not defaulting bigger or equal than the value of defaulting, the government is willing to meet its repayment liabilities with the bankers, as long as it is able to perform further borrowing. The satisfaction of this inequality is denoted as the "*participation constraint*":

$$V_n^g(B,a,q;p,\pi) \geq V_d^g(B,a,q;p,\pi).$$

Equalizing those two value functions in the equilibrium, enables then the definition of the upper sustainable debt limit, $\bar{B}(a;p,\pi)$, for which positive lending in an equilibrium is still possible.

The second constraint is referred to as the "*no-lending continuation condition*", building the fundament for making a crisis even possible. The basic idea is that a government in the crisis zone, which is not able to issue further debt, indicated through a bond price $q=0$, strictly prefers defaulting instead of repayment. The inequality related to this condition is the following:

$$V_n^g(0,a,0;p,\pi) < V_d^g(0,a,0;p,\pi).$$

Contrary to the participation constraint, the no lending constraint is used to derive the upper safe debt limit $\bar{b}(a;p,\pi)$, the country's maximum borrowing level for which no default can occur (Cole/ Kehoe 1996, p. 319 f.) (Cole/ Kehoe 2000, p. 100 f.).

Having derived the fundamentals of the theoretical framework, we can now move on to the determination of government policy, contingent on the conditions prevailing in the investigated economy. Main objective is to account for the situation of the countries currently involved in the European Sovereign Debt Crisis, in order to get an understanding of their behaviour in terms of external financing. This issue is denoted as the general model, characterized through a prevailing recession $(a=0)$, an existing probability of a country's default $(0<\pi<1)$ and a positive possibility for a recovery of the private sector within the next years $(0<p<1)$. However, the derivation of a unique equilibrium, given these characteristics, involves technical complexities due to two essential factors. The first problem arises through the discontinuity of the bond prices $q(B',s;p,\pi)$, caused through the debt thresholds. As no constant price for foreign debt can be applied during the solution of the model, the government's value function $V(s;p,\pi)$ has kinks which directly affects its policy function $B'(s;p,\pi)$. The second problem is connected to the limited commitment assumption introduced at the beginning of this chapter. As the government is not able to commit for future policy choices, the manual derivation of a continuous value function $V(s;p,\pi)$ becomes impossible (Conesa/ Kehoe 2012b, p.33).

In order to account for these facts, the deduction of the equilibriums has to be done by focussing on two limited cases, accounting for self-fulfilling crises and for martingale gambling strategies separately. The first case (section 3.2) considers an economy where the private sector has recovered $(a=1)$, asking for a neglection of the government's incentive to gamble for redemption. This is a modified version of the basic model provided by Cole and Kehoe (2000). The second case (section 3.3) then refers to an economy, where there is indeed a recession prevailing ($a=0$ and $0<p<1$), but without the possibility for self-fulfilling crises to occur. This set up allows for the analysis of the governments' incentives to gamble for redemption (Conesa/ Kehoe 2011, p. 13 ff.). Based on the results derived within these two cases, assumptions about the behaviour of the countries' governments in the general model can then be made. However, an assurance of these assumptions can only be guaranteed with the subsidy of numerical experiments. These calibrations will then be part of section 4.

3.2 Self fulfilling Debt Crises

To sum up again, a financial crisis is considered to be self-fulfilling if the fear of a country's default, on the part of the international bankers, actually leads to this default, even though the government would have been able to pay for the bonds becoming due. For a such an event to be possible, the current debt level has to be within the "crisis zone", as for borrowing rates below the upper safe debt limit or above the upper sustainable debt limit, the expression of the sunspot variable has no impact on the decisions of the international bankers (Arellano/ Conesa/ Kehoe 2012, p.9). The purpose of this section is to deduce a country's optimal strategies and analyse what main thoughts, on the part of the government, are behind them. The way to do this is the following: We derive the value maximizing government policy and its resulting payoffs, considering this sections fundamental assumption, the recent recovery of the private sector from the prevailing recession (*a=1*). The fact that the economy finds itself in normal conditions disposes the government's incentive to gamble for redemption, making the probability for a recovery *p* irrelevant. As a result, two policy options remain to be considered, dependent on the prevalent level of foreign debt and the realization of the sunspot variable. These two options include either keeping the current borrowing level constant, or running it down and exiting the danger of a default (Conesa/ Kehoe 2011, p.13).

In the following, the derivation of the equilibrium is subdivided into three major steps. In a first step, the general problem of the government is solved. In order to do this, we assume for the time being that the possibility for a self-fulfilling crisis to occur is set to zero ($\pi=0$). This enables on one hand the determination of the equilibriums in the "no-crisis zone" and the "default zone", but on the other as well the definition of the thresholds applying in this particular setting. As we proceed then to the "crisis zone", this assumption well be neglected ($\pi>0$), in order to assure an appropriate derivation of this equilibrium.

3.2.1 The Problem of the Government

The problem of the government takes the following form (Conesa/ Kehoe 2011, p.13 f.):[37]

$$V^g(B,1,z,\zeta;p,\pi) = \max_g u(c,g) + \beta E V'^g(B',a',z,\zeta;p,\pi) = \max_{z,g} \sum_{t=0}^{\infty} \beta^t u(c_t,g_t)$$

subject to:

$$c_t = (1-\theta)\bar{y}$$
$$g_t + B_t = \theta\bar{y} + qB_{t+1}$$
$$B' = B$$
$$B_t \leq \bar{B}(1;p,\pi).$$

Plugging the first budget constraint, solved after c_t, into the problem function of the government and using a Lagrange multiplier for the second constraint, the new problem results, denoted as:

$$\max \sum_{t=0}^{\infty} \beta^t u((1-\theta)\bar{y}, g_t) - \lambda_t(g_t + z_t B_t - \theta\bar{y} - qB_{t+1}).$$

[37] Please note that the specific set up discussed in this section has an immediate effect on the budget constraint in the government's problem set.

After the differentiation with respect to g, we receive the following first order conditions:

$$\beta^t u_g((1-\theta)\bar{y}, g_t) = \lambda_t \quad \text{and} \quad \lambda_{t+1} = \beta\lambda_t .$$

Taking a look at these conditions, one can see that a constant government spending rate of $g_{t+1}=g_t=\hat{g}$ is the immediate consequence, as $u_g((1-\theta)\bar{y},g_t)=u_g((1-\theta)\bar{y},g_{t-1})$. Applying this knowledge to the government's second budget constraint, solved after g, and considering the assumption of a constant level of borrowing $B_t=B$ over the future periods, the following expression results:

$$\hat{g} = \theta\bar{y} - (1-\beta)B .$$

This implies that the optimal strategy for a government, finding itself in the situation where no vulnerability to a self-fulfilling crisis exists, is a stationary debt and expenditures policy. In order to support this, attention needs to be given to the transversality condition (Conesa/ Kehoe 2011, p.13 ff.):

$$\lim_{t\to\infty} \lambda_t B_{t+1} \geq 0 .$$

Its purpose is, in combination with the first order conditions, to define a unique optimum in this maximization problem, by assuring that the deviation of the variables from this solution won't lead to an increased utility (Becker 2012). This in turn shows, that the equilibrium solution does not allow for negative debt levels over all periods in time. Relating to this condition, one can see that a too low government spending path $\hat{g}$ would violate this condition, while a massive increase in spending would demand for a debt level, running the danger of hitting the upper sustainable debt limit $\bar{B}(1;p,\pi)$. It is understandable that both of these spending paths cannot lead to an optimal solution (Conesa/ Kehoe 2011, p.14).

Having solved the government's problem function in a general way, we are now able to apply this on the "no-crisis zone" and on the "default zone", as for those two, the assumption of $\pi=0$ is still justified. Let's start with the "no-crisis zone", where the underlying assumption is that the current debt level has not passed the lower threshold ($B\leq\bar{b}(a;p,\pi)$). Under the premise that future borrowing will as well stay below this upper safe debt limit, no crisis is possible, leading to a bond price of $q=\beta$. In this case, the future level of foreign borrowing B' satisfies the "participation constraint". Keeping in mind the general results derived above, it becomes clear that the optimal behaviour is a stationary debt policy by maintaining a borrowing level constant below the lower threshold (Cole/ Kehoe 2000, 101 ff.). As a result, the government's expected payoff in the state $s=\{B,a,z_{-1},\zeta\}=\{B,1,1,\zeta\}$ results (Conesa/ Kehoe 2011, p.14):

$$V^g(s;p,\pi) = V^g(B,1,1,\zeta;p,0) = \frac{u((1-\theta)\bar{y}, \theta\bar{y} - (1-\beta)B)}{1-\beta} .$$

In a next step, the government's optimal behaviour within the "default zone" needs to be investigated. Here we consider a hypothetical case in which the government has chosen to default in the previous period, even though it would have had the possibility to perform further borrowing ($s=\{B,1,0,\zeta\}$). Although this behaviour will not be observed within the equilibrium, it is still useful to be examined, as it helps to define the upper safe debt limit with the help of the "no lending continuation condition" (Cole/ Kehoe 2000, p.101). As by definition, after a default the government is excluded from further international borrowing and lending, the default probability π and the sunspot variable ζ become irrelevant (supporting our restriction). Applying this knowledge, including the implications of the GDP defined in the environment, the derivation of *c*, *g* and the government's payoff is straight forward (Conesa/ Kehoe 2011, p. 14 f.).

$$c = (1-\theta)Z\bar{y} \qquad \text{and} \qquad g = \theta Z\bar{y}$$

$$V^g(B,1,0,\zeta;p,0) = \frac{u((1-\theta)Z\bar{y},\theta Z\bar{y})}{1-\beta}$$

As visible in this value function, a government which is not able to perform further borrowing due to a bond price *q=0* will choose immediate default instead of repayment, as this option leads to a higher expected payoff. This is as well expressed through the variables *g* and *c*, showing that a government, having defaulted in an earlier period, only consumes its revenue generated through taxation, as it is excluded from the international credit market by definition (Cole/ Kehoe 2000, p.101).

3.2.2 Defining the Thresholds

Having defined the expected payoff of a government finding itself in the "no-crisis zone" and in the "default zone", we now want to move on to the analysis of the optimal behaviour within the "crisis zone". The fundament for doing this investigation requires first the mathematical definition of the upper safe- and the upper sustainable debt limit, as with the help of those it can be proven that this particular zone, where the participation constraint as well as the no-lending continuation constraint is satisfied, actually exists. As a basis for this definition, the information derived above is used, still applying the limiting case where *a=1* and *π=0*. We start with the no-lending continuation condition, where under a stationary debt policy *B=B'* the government prefers to default instead of repaying, if the price of new debt *B'* is zero:

$$V_n^g(\bar{b}(1;p,0),1,0;p,0) < V_d^g(\bar{b}(1;p,0),1,0;p,0)$$

$$u((1-\theta)\bar{y},\theta\bar{y}-\bar{b}(1;p,0)) + \frac{\beta u((1-\theta)\bar{y},\theta\bar{y})}{1-\beta} < \frac{u((1-\theta)Z\bar{y},\theta Z\bar{y}}{1-\beta}$$

Equalizing these expressions and solving them after the current debt level, defines the upper safe debt limit $\bar{b}(1;p,0)$. The resulting value is the largest level of debt, where the no-lending continuation constraint is not satisfied, meaning that the government prefers to repay its debt liabilities, even though it is not able to perform further borrowing (*q=0*) (Cole/ Kehoe 2000, p.103 f.) (Conesa/ Kehoe 2011, p.15):

$$u((1-\theta)\bar{y},\theta\bar{y}-\bar{b}(1;p,0)) = \frac{u((1-\theta)Z\bar{y},\theta Z\bar{y}}{1-\beta} - \frac{\beta u((1-\theta)\bar{y},\theta\bar{y})}{1-\beta}$$

Having defined the upper safe debt limit, we can now move on to determine the upper sustainable debt limit $\bar{B}(1;p,0)$ with the help of the participation constraint. However, the upper threshold cannot be defined with the use of a constant $\hat{g}$, as within the crisis zone, the optimal government policy does not necessarily involve a constant debt level. In order to account for this circumstance, an upper *stationary* debt limit $\bar{B}^s(1;p,0)$ is introduced, describing the highest level of government debt for which repayment can still be achieved through a stationary debt and expenditures policy, on condition that future debt can be sold for a positive price. Later on it can be seen that $\bar{B}^s(1;p,0) = \bar{B}(1;p,0)$ (Conesa/ Kehoe 2011, p.15).

Using the same methodology than above, this approach results in the following equation (Cole/ Kehoe 2000, p.103 f.) (Conesa/ Kehoe 2011, p.15 f.):[38]

$$V_n^g(\bar{B}^s(1;p,0),1,\beta;p,0) = V_d^g(\bar{B}^s(1;p,0),1,\beta;p,0)$$

$$\frac{u((1-\theta)\bar{y},\theta\bar{y}-(1-\beta)\bar{B}^s(1;P,0))}{1-\beta} = u((1-\theta)Z\bar{y},\theta Z\bar{y}+\beta\bar{B}^s(1;p,0)) + \frac{\beta u((1-\theta)Z\bar{y},\theta Z\bar{y})}{1-\beta}$$

Assuming now that the government finds itself with a debt level $b>\bar{B}^s(1;p,0)$, its optimal strategy, considering the participation constraint, would be the immediate default. Being aware of this, one can see that there is a nonempty interval of stationary debt levels, $\bar{b}(1;p,0)<b\leq\bar{B}^s(1;p,0)$, where crises with zero probability are possible. The upper stationary debt limit defines the maximum debt level for this interval, as at this point the participation constraint holds as equality (Cole/ Kehoe 1998, p.20).

3.2.3 Equilibrium

Having derived the thresholds, we can now move on to the investigation of the government's optimal behaviour in the "crisis-zone". As the amount of current debt is now between the two debt limits, $\bar{b}(1;p,\pi)<B\leq\bar{B}(1;p,\pi)$, a country's vulnerability to a crisis has increased, expressed through the probability variable $\pi>0$. The fact that default is only a matter of time finds expression in the risk premium on government bonds, depressing the prices to $\beta(1-\pi)<\beta$. Being aware of this situation, a government has the following policy choices in period $t=0$: immediate default, run down the future debt to $B_T\leq\bar{b}(1;p,\pi)$ in $t=1,2,\ldots,T$ periods, or last but not least, never run down its debt positions. The objective of this paragraph is to find this policy option, which leads to the maximum expected payoff, on condition that such a one even exists (Cole/ Kehoe 2000, p.101). Please notice at this point in time that the lower threshold defined before doesn't change, given the fact that the government now finds itself in the crisis zone, $\bar{b}(1;p,0)=\bar{b}(1;p,\pi)$. Therefore, besides the objective to find the optimal policy decision of a government, only the upper threshold $\bar{B}(1;p,\pi)$ needs to be taken care of, as a stationary debt policy not necessarily needs to be the best policy option in this respective situation (Conesa/ Kehoe 2011, p.16 ff.).

First, let's suppose that the government decides to run down its debt positions to the lower cut-off level $\bar{b}(1;p,\pi)$ in T periods (Conesa/ Kehoe 2011, p.17). Under the assumption that the current debt level is below the upper stationary debt limit ($B_0\leq\bar{B}^s(1;p,0)$), the government's problem can be solved through the application of a stationary expenditures policy (Cole/ Kehoe 1998, p.25). In order to determine this particular spending level g, we use the first order conditions derived above (section 3.2.1) and the government's budget constraint, resulting in the following expression:[39]

$$g^T(B_0;\pi) = \theta\bar{y} - \frac{1-\beta(1-\pi)}{1-(\beta(1-\pi))^T}(B_0 - (\beta(1-\pi))^{T-1}\beta\bar{b}(1;p,\pi)).$$

[38] Please notice that the bond price applied is $q=\beta$, as the assumption that no self-fulfilling crisis can occur is still prevalent. The detailed steps of this derivation have been displaced to Appendix G.
[39] Please notice that the detailed steps of this derivation are displaced to Appendix H.

So, if the government manages to hold this lowered level of expenditures constant over the defined periods, the decreasing debt level will reach the lower threshold exactly in period T, $B_T=\bar{b}(1;p,\pi)$. As the vulnerability to a default is still existent for all the periods except $t=T$, we can rule out that there is an optimal solution, where B_T falls below the upper safe debt limit. Nevertheless, such a solution would anyway require to lower g^T, which cannot be utility maximizing (Cole/ Kehoe 2000, p.102).

Now, let's consider the second policy option of the government: the choice to never run down the debt. Following the same procedure, the government expenditures g are defined as follows:

$$g^{\infty}(B_0;\pi) = \theta\bar{y} - (1-\beta(1-\pi))B_0$$

As $lim_{T\to\infty}\, g^T(B_0;\pi)$ equals $g^{\infty}(B_0;\pi)$, we are now ready to compute the expected payoff of running debt down, as the periods can simply be extended to infinity, $T=1,2,\ldots,\infty$. In order do this, a backward induction is applied, using the value functions $V_t^T(B_0;\pi)$, where t denotes the remaining number of periods (Conesa/ Kehoe 2011, p.17 ff.). Here again, the results are presented below, while the detailed derivation has been displaced to Appendix I, in order to save space. The resulting value function of running debt down in T periods is:

$$V^T(B_0;\pi) = \frac{1-(\beta(1-\pi))^T}{1+\beta(1-\pi)} u((1-\theta)\bar{y}, g^T(B_0;\pi)) + \frac{1-(\beta(1-\pi))^{T-1}}{1+\beta(1-\pi)} \frac{\beta\pi u((1-\theta)Z\bar{y}, \theta Z\bar{y})}{1-\beta}$$
$$+ (\beta(1-\pi))^{T-2} \frac{\beta u((1-\theta)\bar{y}, \theta\bar{y})}{1-\beta}.$$

Analyzing this expression, the following implications can be read out. In the first part of the equation, dealing with the periods $t=1,2,\ldots,T\text{-}2$, one can see that the government expenditures remain constant at the level $g^T(B_0;\pi)$ while the vulnerability to a default is still omnipresent ($\pi>0$). Entering period $T\text{-}1$, there is still a positive probability for a crisis, but as soon as the sunspot variable indicates an advantageous outcome $\zeta_t\leq 1-\pi$, the government will announce its future borrowing level of $B_T=\bar{b}(1;p,\pi)$ automatically setting π equal to zero (Cole/ Kehoe 2000, 102). As an immediate effect, in period T, the bond prices increase to $q=\beta$, providing the government additional liquidity which results in a rise of consumption and government spending to $c=(1-\bar{y})\theta$ and $g=\bar{y}\theta$ (Conesa/ Kehoe 2011, p.18). At this point in time, the government enters the equilibrium of the "no-crisis zone" (Cole/ Kehoe 2000, 102).

Last but not least, the upper threshold $\bar{B}(1;p,\pi)$ needs to de derived. As a method to find this limit, and in the same move the optimal number of periods T to run the debt down, we maximize over all value functions from $t=0$ until $t=\infty$ (Cole/ Kehoe 2000, p.103) (Conesa/ Kehoe 2011, p.18 f.). As $lim_{T\to\infty}\, V_g^T(B_0;\pi) = V_g^{\infty}(B_0;\pi)$, with the value function for an infinite time period:

$$V^{\infty}(B_0;\pi) = \frac{u(1-\theta)\bar{y}, \theta\bar{y} - (1-\beta(1-\pi))B_0}{1+\beta(1-\pi)} + \frac{\beta\pi u((1-\theta)Z\bar{y}, \theta Z\bar{y})}{(1-\beta)(1+\beta(1-\pi))}$$

the existence of such a maximum is guaranteed (Cole/ Kehoe 2000, p.103):

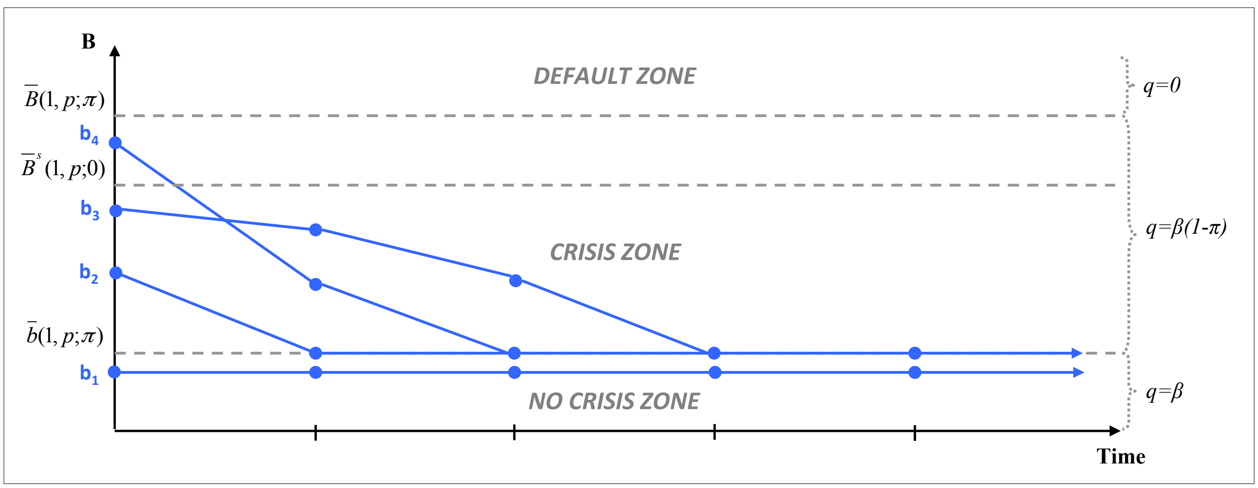
$$\max\left\lfloor V_g^1(\bar{B}(1;p,\pi)), V_g^2(\bar{B}(1;p,\pi)), \ldots, V_g^{\infty}(\bar{B}(1;p,\pi)\right\rfloor$$
$$= u((1-\theta)Z\bar{y}, \theta Z\bar{y} + \beta(1-\pi)\bar{B}(1;p,\pi)) + \frac{\beta u((1-\theta)Z\bar{y}, \theta Z\bar{y})}{1-\beta}.$$

3.2.4 Results

During this section, we have addressed several policy possibilities and resulting expected payoffs for an economy, whose private sector has recovered from the prevailing recession. Depending on the initial debt level in period $t=0$, different strategies needed to be applied in order to maximize the expected payoff. The resulting value functions are summarized below. The corresponding strategies, under the supposition that a crisis is being avoided during the periods, are then again visualized in Figure 6 and consequentially described underneath (Conesa/ Kehoe 2011, p.19).

$$V(B,1,1,\zeta;p,\pi)=\begin{cases}\dfrac{u((1-\theta)\bar{y},\theta\bar{y})}{1-\beta} & \text{if } B\leq\bar{b}(1;p,\pi)\\ \max\left[V^{1}(V;\pi),V^{2}(B;\pi),\ldots,V^{\infty}(B;\pi)\right] & \text{if } \bar{b}(1;p,\pi)<B\leq\bar{B}(1;p,\pi) \text{ and } \zeta\leq 1-\pi\\ \dfrac{u((1-\theta)Z\bar{y},\theta Z\bar{y})}{1-\beta} & \text{if } \bar{b}(1;p,\pi)<B\leq\bar{B}(1;p,\pi) \text{ and } \zeta>1-\pi\\ \dfrac{u((1-\theta)Z\bar{y},\theta Z\bar{y}}{1-\beta} & \text{if } \bar{B}(1;p,\pi)<B\end{cases}$$

Figure 6: Value maximizing debt paths when a=1 and π>0



Idea for this illustration provided by Conesa and Kehoe (2011, p.19)

First, let's start with the case where the government finds itself in the no-crisis zone in $t=0$, expressed through the path b_1. If $B_0\leq\bar{b}(1;p,\pi)$, all variables in the equilibrium are stationary for the periods $t>0$: $c=(1-\theta)\bar{y}$, $\hat{g}=\theta\bar{y}-(1-\beta)B$, $B=B_0$ and $q=\beta$. In this case, there is no probability for a default, making the occurrence of a crisis impossible (Cole/ Kehoe 2000, p.105 f.). The reason for this behaviour is that the government wants to avoid the costs of a default by all means and is therefore willing the repay the bonds becoming due, even though this requires a drop in government expenditures. As this behaviour can be anticipated, international bankers are willing to pay a comparably high price for the government bonds, due to the low default risk. Additional to that, bad news, indicated through the sunspot variable, have no influence on the behaviour of the other bankers, offsetting the danger of an uncoordinated investment stop (Arellano/ Conesa/ Kehoe 2012, p.9).

Moving on to the case, where the government finds itself in the crisis zone ($\bar{b}(1;p,\pi)<B_0\leq\bar{B}(1;p,\pi)$), the incentives of the investors change, as there is a positive probability for a future crisis to occur $\pi>0$. An indicator for this vulnerability is given through the decreased bond prices, encouraging the government to run down its debt level in order to escape the exposure of a self-fulfilling crisis. Depending on the height of the initial debt B_0, in relation to the defined thresholds, two different policy options result:

The first option considers the case, where the initial debt level is lower or equal to the stationary debt limit, $B_0\leq\bar{B}^s(1;p,\pi)$, indicated through the paths b_2 and b_3. In this situation, the participation constraint does not bind and therefore the government should reduce its foreign borrowing to the upper safe debt limit within T periods, by smoothing its expenditures. The reason behind this smoothing strategy is that sharp cuts in expenditures are relatively painful. Therefore, accounting for the government's objective to maximize the consumers' utility, the repayment of debt is portioned over multiple periods, increasing in B_0. As soon as the debt level has reached this particular lower threshold and the sunspot has not indicated a crisis so far ($\zeta_t>1-\pi$), the equilibrium converges to the one, where $B\leq\bar{b}(1;p,\pi)$ and $q=\beta$, as the probability for a default disappears (Cole/ Kehoe 2000, p.106 f.) (Arellano/ Conesa/ Kehoe 2012, p.10).

The other case within the "crisis zone" can be, that the initial level of debt is above the stationary debt limit, $B_0>\bar{B}^s(1;p,\pi)$, expressed through the path b_4. Has this level been reached, the ability to run down the debt with the help of a standard consumption smoothing policy disappears, demanding for a sharp drop of debt in period $t=0$. The reason for that is that in period $t=0$ the participation constraint binds, encouraging the government to repay its debt, as new debt still can be sold for a positive price. However, in order to be able to repay constantly over periods, B_1 has to be set below B_0 immediately requiring the government expenditures to be more restricted than in the case of a stationary debt policy ($g_0<g^T(B_0;\pi)$). Has this action been taken in the first period, the government can follow the path of a standard consumption smoothing policy ($B\leq\bar{B}^s(1;p,\pi)$) from period $t=1$ onwards (Cole/Kehoe 2000, 105 ff.).

Having defined the two strategies within the "crisis zone", we can now move on the last case: $B_0>\bar{b}(1;p,\pi)$. If the government finds itself above the upper sustainable debt limit, the only possible outcome is default, as the "no lending continuation condition" binds. In this equilibrium, all variables are stationary from period $t\geq1$ onwards: $c=(1-\theta)Z\bar{y}$, $\hat{g}=\theta Z\bar{y}$, $B=0$ and $q=0$ (Cole/ Kehoe 2000, p. 106).

Please notice, that the studied case, where the economy has recovered from the recession can as well be modified to the special case, where the recession is still prevailing ($a=0$), but without the possibility for a recovery of the private sector ($p=0$). In order to do this, GDP simply has to be changed from $\bar{y}$ to $A\bar{y}$. The government of the respective country is now confronted with a vulnerability to self-fulfilling crises, but has no incentive to gamble for redemption, as the probability for a recovery is set to zero. The resulting optimal strategies in this case are similar to the ones discussed above: Either leaving the debt positions constant or running them down in order to avoid an upcoming default (Conesa/ Kehoe 2011, p.13).

3.3 Gambling for Redemption

Having derived the equilibrium outcomes for an economy in normal conditions ($a=1$), we now want to proceed to the analysis of the contrary case, a prevailing recession, expressed through the aggregate state $s\{B,a,z_{-1},\zeta\}=s\{B,0,1,\zeta\}$ (Conesa/ Kehoe 2011, p.20). Initiated through this bad economic condition, it is assumed that the incentives of a country's government change, as there is now the opportunity to speculate on an economic boom. Its strategy may therefore include only a slight cut in government spending, but a steady increase in public debt, gambling for a recovery of the private sector and the concomitant increase in tax revenues, enabling a reduction of the debt positions afterwards. This strategy is as well valued by the citizens of the country, as due to the fact that government spending is smoothed over time, sharp drops in utility are being avoided. However, is the recovery not arriving, the country has exposed itself to a much riskier position, leading to immediate default due to the comparably high debt positions. Please notice at this point in time that a default following this strategy cannot be considered as self-fulfilling, as foreign investors and the government correctly predict the economic situation of the respective country (Arellano/ Conesa/ Kehoe 2012, p.10 f.). Being aware of this approach, one can imagine that the variable p, describing the probability for a recovery, becomes very important and sets the fundament for these strategies. Anyhow, the crucial factor, allowing for a theoretical derivation of this game theoretic behaviour, is the fact that the probability for a self-fulfilling crisis is set to zero $\pi=0$. This way, a country has no restrictions on foreign borrowing, allowing for an unrestricted increase of indebtedness. Against the background of the European sovereign debt crisis and the resulting central bank policy actions, this set up can also be interpreted as an environment where a country's default is quite possible, but where all the actors, namely the government and the international bankers, assign zero probability to it (Conesa/ Kehoe 2011, p.20).[40]

3.3.1 Defining the Thresholds

In order to perform an analysis of the government's incentives to gamble for redemption and run the risk of a potential default, one can imagine that the definition of the thresholds, particularly the upper ones $\bar{B}(1;p,0)$ and $\bar{B}(0;p,0)$, is absolutely key. In the previous section, the upper sustainable debt limit in good times, $\bar{B}(1;p,0)$, has already been defined. For the determination of the upper threshold in a recession $\bar{B}(0;p,0)$, a similar approach is being applied. The strategic assumption is that the government increases its foreign borrowing up to a point, where it is less than or equal to the mentioned upper threshold. Then, it again heightens its debt burden to $\bar{B}(0;p,0)<B\leq\bar{B}(1;p,0)$ at a bond price $q=\beta p(1-\pi)$, speculating on a recovery of the economy.[41] If the recession ends, the governments is able to perform the repayment of debt, while otherwise, in an ongoing struggling economy, it defaults. Being aware of the methodology, the resulting equation, defining the upper sustainable debt limit $\bar{B}(0;p,0)$ is as follows, while the preceding steps are displaced to Appendix J (Conesa/ Kehoe 2011, p.20 f.):

[40] In the paper written by Cristina Arellano (2008), default can also be a result of a too low GDP. The idea behind it is that a country, managing an absolutely save debt level, suddenly rushes into a recession. This recession leads to such a huge drop in GDP that the upper sustainable debt limit lowers to the extent, that the default of the managed debt becomes unavoidable.

[41] Please notice that the bond price reduces to βp, as the probability for a crisis has been set to zero.

$$V_n^g(\overline{B}(0;p,0),0,1;p,0) = V_d^g(\overline{B}(0;p,0),0,1;p,0)$$

$$u((1-\theta)A\bar{y},\theta A\bar{y} + \beta p\overline{B}(1;p,0) - \overline{B}(0;p,0) + \frac{\beta p}{1-\beta}u((1-\theta)\bar{y},\theta\bar{y} - (1-\beta)\overline{B}(1;p,0))$$

$$= u((1-\theta)AZ\bar{y},\theta AZ\bar{y} + \beta p\overline{B}(1;p,0)) + \frac{\beta p}{1-\beta}u((1-\theta)Z\bar{y},\theta Z\bar{y})$$

Considering the restrictions of this limited case, one could assume that due to the zero probability of a crisis, the government might have an incentive to borrow even more than the upper sustainable debt limit in good times, violating the constraint $B' \leq \overline{B}(1;p,0)$. However, borrowing at the upper sustainable debt limit in a recession ($\overline{B}(0;p,0)$) is the highest value of debt B', at which the bond price is still $q=\beta(1-\pi)$, which means in fact $q=\beta$, as $\pi=0$. Nevertheless, if the constraint $B' \leq \overline{B}(1;p,0)$ is not binding, the optimal future debt B' can simply be determined by maximizing the government's value function V_n^g after the future debt level:

$$\max_{B'} u((1-\theta)A\bar{y},\theta A\bar{y} + \beta pB' - B) + \beta(1-p)\left(\frac{u((1-\theta)AZ\bar{y},\theta AZ\bar{y})}{1-\beta(1-p)} + \frac{\beta pu((1-\theta)Z\bar{y},\theta Z\bar{y})}{(1-\beta)(1-\beta(1-p))}\right)$$
$$+\frac{\beta p}{1-\beta}u((1-\theta)\bar{y},\theta\bar{y} - (1-\beta)B'),$$

resulting, after a few cancellations, in the following first order condition:

$$u_g((1-\theta)A\bar{y},\theta A\bar{y} + \beta pB' - B) = u_g((1-\theta)\bar{y},\theta\bar{y} - (1-\beta)B'),$$

where a constant debt level $\hat{B}'(B)$ would be the solution to the problem: $B'(B)=min[\hat{B}'(B), \overline{B}(1;p,0)]$. However, as the government finds itself in a recession and a debt exceeding the threshold $\overline{B}(0;p,0)$ would lead to an immediate default, unless the private sector recovers, this possibility needs to be ruled out. Noticing this restriction, two policy options remain to be considered (Conesa/ Kehoe 2011, p.21 f.):

First, the government decides to never exceed the upper threshold $\overline{B}(0;p,0)$, avoiding the vulnerability to an immediate default, if no recovery occurs (section 3.3.2). The result is a convergence of the debt level to $\overline{B}(0;p,0)$ for an infinitely long time, as π was set to zero. The second option is the gambling for redemption strategy. Here, the government runs up its debt, violating the constraint $B \leq \overline{B}(0;p,0)$, an defaults, unless the private sector recovers (section 3.3.3).

Whether the first or the second option is chosen, depends on the height of the default penalty Z. If a default has a severe downside effect on the country's GDP, saying that Z is only slightly above zero, the government will choose not to gamble for redemption, as the costs associated to a default are too high. However, is Z particularly near to *1*, the costs of a default are moderate and trigger the incentive to run up the future debt level (Conesa/ Kehoe 2011, p.23). Each of these two policy options will now be investigated in a more detailed way, defining their respective equilibrium's expected payoffs.

3.3.2 Equilibrium without Default

Generally, in order to allow the strategy $B \le \bar{B}(0;p,0)$ to be an equilibrium, two conditions need to hold. First, the government's expected payoff of holding a debt level at the upper sustainable debt limit in a recession needs to be higher than the payoff of defaulting, after the international bankers have rolled over the bonds $B' = \bar{B}(0;p,0)$ at the price $q=\beta$. Second, the decision to violate the constraint $B \le \bar{B}(0;p,0)$, by buying additional debt at the price $q=\beta p$ and repaying it in case of a recovery, or defaulting otherwise, does not lead to a higher expected payoff than holding the debt level constant at $B' = \bar{B}(0;p,0)$. If those two conditions are satisfied, the dynamic programming problem takes the following form (Conesa/ Kehoe 2011, p.22 f.):[42]

$$V^g(B,a) = \max_{B,B'}((1-\theta)A^{1-a}\bar{y}, \theta A^{1-a}\bar{y} + \beta B' - B) + \beta EV(B',a')$$

$$\text{subject to: } B \le \bar{B}(0;p,0).$$

The equation above is commonly referred to as a Bellman's equation, stating that the future decisions must always depict an optimal policy with regard to the initial state occurring through the first decision (Sargent 1987, p.11 ff.). Considering the first order condition $V_{B'}^g(B,a)$ and the envelope condition $V_B^g(B,a)$:

$$V_{B'}^g(B,a): \beta u_g((1-\theta)A^{1-a}\bar{y}, \theta A^{1-a}\bar{y} + \beta B' - B) - \beta EV_{B'}(B',a') = 0$$

$$V_B^g(B,a): -u_g((1-\theta)A^{1-a}\bar{y}, \theta A^{1-a}\bar{y} + \beta B' - B) = 0$$

the envelope condition shows, that the government's expected payoff is decreasing in B, which implies that $V(B,a)$ is concave in B. Turning the attention to the first order condition, it can be seen that the policy function $B'(B,a)$ is increasing in B, while the policy function for government spending $g(B,a)$ decreases with an increase in current debt (Conesa/ Kehoe 2011, p.22 ff.).

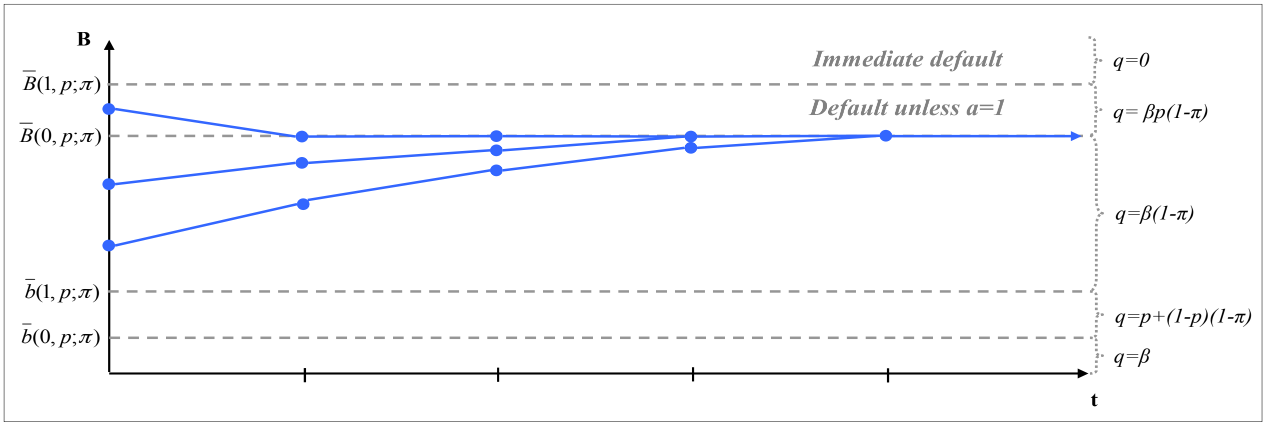
At this point a major assumption of this model needs to be introduced, describing the incentive for martingale gambling. Basically, it is assumed, that for any level of current debt in a recession, such that $A\bar{y} - b$ is the realizable amount of government expenditures g, the following condition holds:

$$u_g((1-\theta)A\bar{y}, \theta A\bar{y} - b) > u_g((1-\theta)\bar{y}, \theta\bar{y} - b)$$

This condition states that during a recession, a government always has the incentive to transfer resources from future recovered periods into the current period, in which the recession is still prevailing (Conesa/ Kehoe 2011, p.7). Noticing this incentive makes clear, that it is impossible to achieve a stationary debt policy during a recession, unless the constraint $B \le \bar{B}(0;p,0)$ binds. Is this constraint not binding, the government of a troubled country has an incentive to run up its public debt, implying $B'(B,0) > B$. As shown earlier, this incentive is not given in an economy with a recovered private sector, as a stationary debt policy $B'(B,1) = B$ is considered to be the optimal policy decision (Conesa/ Kehoe 2011, p.24). Figure 7 summarizes these thoughts again and depicts a few exemplary government strategies as a function of the initial debt level.

[42] The inequalities behind those two assumptions have been displaced to Appendix K.

Figure 7: Value maximizing debt paths when $B \leq \bar{B}(0;p,0)$



Idea for this illustration provided by Conesa and Kehoe (2011, p.25)

3.3.3 Equilibrium with Default

Having defined the equilibrium, where the government decides not to exceed the upper threshold $\bar{B}(0;p,0)$, we now consider the contrary case. Purpose of this section is to find out, whether there is an equilibrium solution, where a government sells additional debt to the international bankers in period t, speculating on a recovery of the private sector in $t+1$, and defaults otherwise. The implications in the section above have already shown that the government has an incentive to transfer potential resources from the future into the current period. However, the challenge in applying this strategy is to find the optimal point in time, as otherwise default is the only resulting option. This set up demands for a finite horizon dynamic programming problem, as it is able to account for the strategy proposed above (Conesa/ Kehoe 2011, p.25). This is comparable to an uncertainty tree with branches of $a=0$ and $a=1$. The optimal policy we're looking for is the one, where the government moves along the branch with $a=0$ until $a_t=0$, betting one a recovery of the private sector in period $t+1$ (Conesa/ Kehoe 2012a, p.26).[43]

$$V_t(B_t,0) = \max_{B_{T+1}} u((1-\theta)A\bar{y}, \theta A\bar{y} + \beta B_{t+1} - B_t) + \beta(1-p)V_{t+1}(B_{t+1},0) + \beta p \frac{u((1-\theta)\bar{y}, \theta\bar{y} + (1-\beta)B_{t+1})}{1-\beta}$$

subject to:

$$\bar{B}(0;p,0) \leq B_{T+1} \leq \bar{B}(1;p,0)$$
$$B_T \leq \bar{B}(0;p,0)$$

In order to solve this problem, a backward induction needs to be applied, using the terminal value function $V_T(B_T,0)$, which builds the basis for choosing the optimal number of periods T before crossing the threshold $\bar{B}(0;p,0)$, maximizing $V_0(B_0,0)$:

$$V_T(B_T,0) = \max_{B_{T+1}} u((1-\theta)A\bar{y}, \theta A\bar{y} + \beta p B_{T+1} - B_T) + \beta(1-p)\frac{u((1-\theta)Z\bar{y}, \theta Z\bar{y})}{1-\beta} + \beta p \frac{u((1-\theta)\bar{y}, \theta\bar{y} + (1-\beta)B_{T+1})}{1-\beta}$$

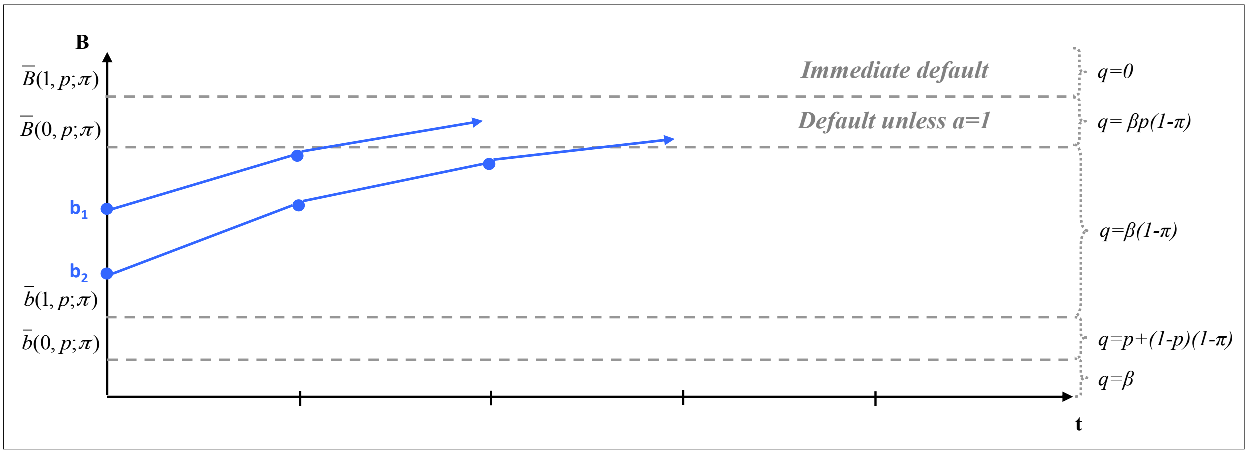
subject to: $B_{T+1} \leq \bar{B}(1;p,0)$.

[43] Please notice that for the solution of this dynamic programming problem, the notation has changed using t's instead of primes in order to be able to denote the steps of the following backward induction properly.

The bottom line delivered through this terminal value function is, that as long as the constraint $B_{T+1} \geq \bar{B}(0;p,0)$ binds, the expected payoff in period $t=0$, $V_0(B_0,0)$, can be increased by increasing the number of periods T staying below the threshold $\bar{B}(0;p,0)$ (Conesa/ Kehoe 2011, p.25 f.). Figure 8 depicts two exemplary optimal policy paths, depending on B_0. In b_1 T is set to one, while in b_2 T is set to two.

However, in order to calculate the optimal policy path, a computed policy function iteration is inevitable. The detailed algorithm to perform this application can be found in the Appendix L.

Figure 8: Value maximizing debt paths when $B > \bar{B}(0;p,0)$



Idea for this illustration provided by Conesa and Kehoe (2011, p.26)

3.3.4 Results

Within the last sections, we have attained the special case, referring to an economy in a prevailing recession ($a=0$), but without being threatened of bankers, suddenly ceasing to roll over the debt contracts ($\pi=0$). Within this general scheme of things, the preferences of a country's government change. The hope that the ongoing recession might have an end within a short period of time ($0<p<1$) triggers the incentive to cut government spending very slowly but to remain in the same time at a high level of indebtedness, or even to further increase the liabilities against international bankers. Whether to run this strategy or not is influenced by two factors, summarized as the costs of default: the default penalty and the interest rates on government bonds. If the costs of a default are high, the government has an incentive to maintain a level of indebtedness, at which it is not running the danger of being forced to default. As the probability of a self-fulfilling crisis is set to zero in this limiting case, this level is exactly at the upper sustainable debt limit $\bar{B}(0;p,0)$. However, are the costs following a country's default moderate, gambling for redemption dominates (Arellano/ Conesa/ Kehoe 2012, p.10 f.) (Conesa/ Kehoe 2011, p. 1 ff.). Exactly these costs of default are going to be the key element while evaluating the behaviour of the European GIIPS countries against the policy decisions of the ECB, the IMF and the European Commission, as encouraging the wrong incentive can prolong or even worsen the crisis in the European Monetary and Economic Union.

4. Analysis

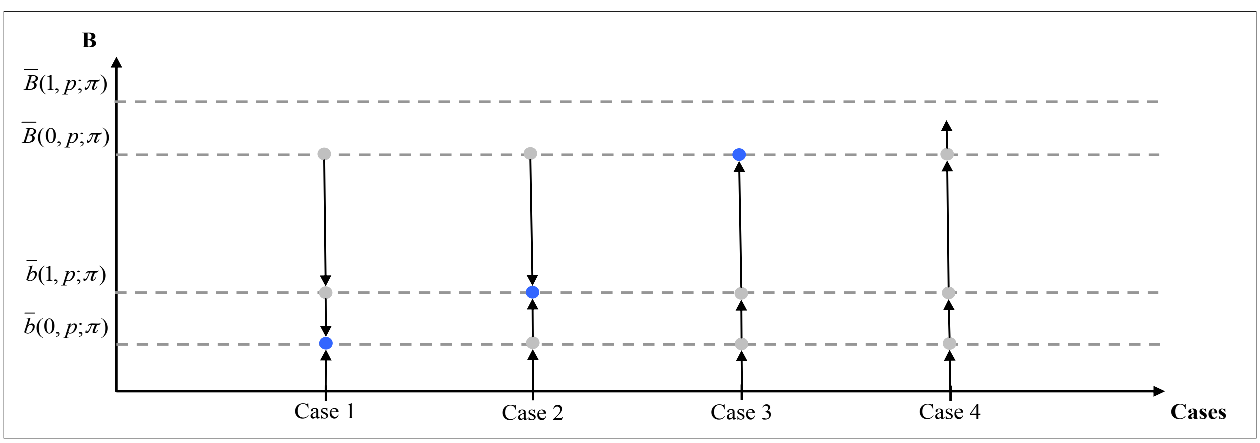
The two limited cases discussed above have shown the optimal policy paths for a government, dependent on the fundamentals of the environment. While explaining these fundamentals, it became evident that the solution of the general model for an economy, suffering from a recession, but being threatened by self-fulfilling crises in the same time ($a=0$; $0<\pi<1$ and $0<p<1$), is not deducible in a theoretic manner, which makes an underpinning through numerical experiments unavoidable.

The purpose of this chapter is to realize these numerical experiments through a calibration of the general model (section 4.1).[44] The results thereby serve as a benchmark scenario, providing the optimal policy decisions for a country, finding itself in either good (no recession) or bad times (recession). By inserting this direct comparison, the change in incentives, caused through an alteration of the country's private sector, can be analyzed. The variables used for these calibrations represent a benchmark economy with the characteristics of the troubled countries in the European Economic and Monetary Union. This allows for a first assessment of the GIIPS countries' behaviour during the European Sovereign Debt Crisis.

Being aware of the results, provided through the general model, attention will then be turned to the policy interventions applied by the European Central Bank, the International Monetary Fund and the European Commission (section 4.2). The incorporation of these policy measures' characteristics in to the theoretical framework allows for an assessment of their individual influence on the benchmark economy's optimal debt policy. Noticing the outcomes, last but not least a country specific calibration is applied (section 4.3), followed by a discussion to qualify the results (section 4.4).

Before starting with the calibration of the general model, Figure 9 depicts the optimal policy paths, achievable in the theoretical framework. These four cases include stationary debt policies on the three thresholds $\bar{b}(0;p,\pi)$, $\bar{b}(1;p,\pi)$ and $\bar{B}(0;p,\pi)$, as well as gambling for redemption, visualized in case 4. While analyzing the forthcoming results, this figure serves as a point of orientation and will be referred to in the prospective argumentations.

Figure 9: Phase diagrams in the general model



Idea for this illustration provided by Conesa and Kehoe (2011, p.28)

[44] At this point in time, I would like to thank Juan Carlos Conesa and Timothy J. Kehoe for providing me with the Fortran-codes, allowing me to perform the calibration of the general model on my own. The used codes are denoted in Appendix M.

4.1 Calibrated Results of the General Model

In order to proceed, a functional form of the utility function *u(c,g)* has to be determined. The specification applied by Conesa and Kehoe (2012a) is denoted below, while the used parameter values are pooled in Table 2.

$$u(c,g) = \frac{c^{\rho}}{\rho} + \gamma \frac{(g - \bar{g})^{\rho}}{\rho}$$

Table 2: Parameter values used for the benchmark scenario

Parameter	Value	Comments
$\bar{y}$	*100*	Normalization of the Gross Domestic Product
A	*0.90*	Average government revenue loss between 2007 and 2009
Z	*0.95*	Default Penalty (Proposed by Cole and Kehoe in 1996)
p	*0.20*	Assumed average recovery of the private sector within 5 years
β	*0.97*	Riskless return on safe bonds is 3% (i.e. Germany in 2009)
π	*0.03*	Average risk premium on bonds at the beginning of 2010
γ	*0.50*	Share of consumption in the utility function
θ	*0.4041*	Tax revenues as a share of the GDP
ρ	*-1*	Standard value from the literature
$\bar{g}$	*28*	Assumption[45]

Conesa/ Kehoe (2012a, p.30) (IMF 2012b) (Thompson Reuters 2012)

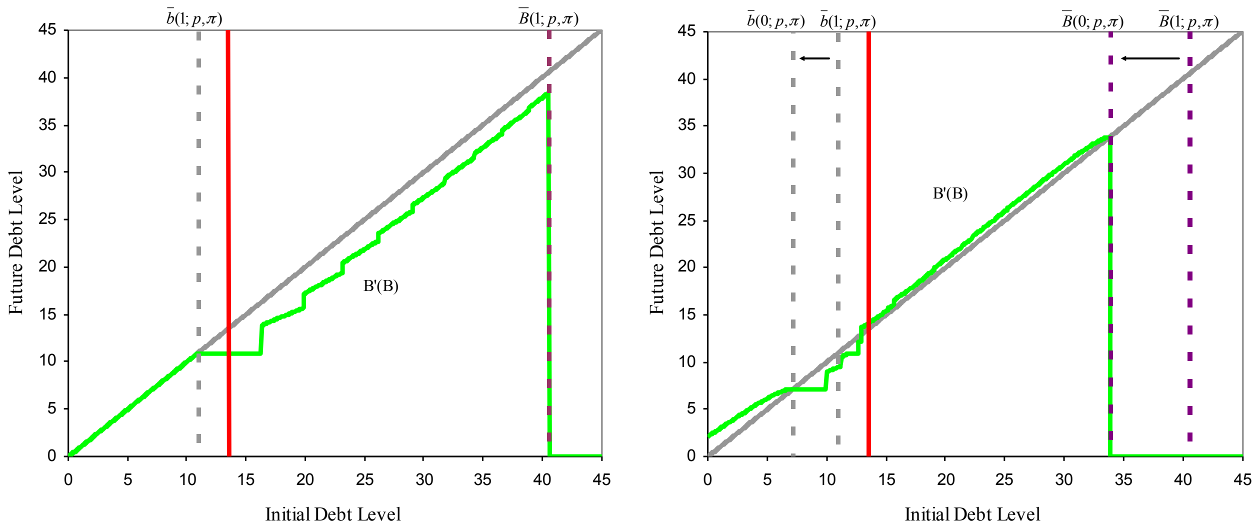
While taking a look at these variables, it becomes clear that those represent a benchmark economy, which reflects an average of all GIIPS countries at the end of 2009. The reason for choosing this base year is the following. After the eruption of the global financial crisis, countries in Europe were forced to immediately run up their indebtedness. However, to what extent this increase in borrowing was an expression of martingale gambling is albeit questionable, as initially those funds were needed in large parts to avoid a collapse of the domestic banking sectors, biasing the results. Nonetheless, when the European Sovereign Debt Crisis began to deploy, with the announcement of Greece's fiscal deficit for 2009, a continuation of this trend was observable, which persisted up until now. Whether this applied strategy is considered to be optimal by the theoretical framework shall be identified with the help of the forthcoming calibrations. Further, it allows for a direct assessment of the introduced policy measures, as the provision of those started at the beginning of 2010. Even though the countries' individual indicators are not yet separately incorporated, the results reveal the outcomes of the theoretical framework's general model, laying the foundation for a country specific analysis later on (section 4.3).

In order to enable a correct interpretation of the results, one important implication needs to be introduced. During the derivation of the model in the sections above, it was assumed that the government faces its policy decisions period by period, while one period was typically referring to one year. However, as this would apply for the analysis of developing countries, typically borrowing short-term, it does not reflect

[45] The value of this parameter has been chosen arbitrarily. It can interpreted as the level of expenditures, which has been committed by the respective county's government and cannot be changed within a short period of time. However, it is understandable that the higher this value is, the harder it is for a government to decrease spending, increasing the incentive to gamble for redemption (Conesa/ Kehoe 2012a, p.30).

the average maturity of the GIIPS countries' debt (See Appendix B). In order to account for this fact, the model has been rescaled to a period length of 6 years. The resulting effect of this modification is that theoretically only 1/6 of the total debt burden is due to be financed at the beginning of each period.[46] Clearly, a direct mapping between the average maturity and the fraction of debt becoming due is not given, as it depends on the composition of the total portfolio. However, inserting simplification does not significantly change the qualitative results, but allows for a proper analysis of the calibrated outputs (Conesa/ Kehoe 2011, p.31). Being aware of this implication, we can now proceed to the assessment of the results depicted in Figure 10.

Figure 10: Benchmark economy in good (left panel) and bad times (right panel)



The provided graphs show the upper safe (grey) and the upper sustainable debt limits (violet), as well as the government's optimal debt policy function *B'(B)* (green). The current level of debt is thereby provided on the abscissa, while the ordinate denotes the optimal future borrowing amount, given the initial level of indebtedness. In order to facilitate the assessment of the results, a 45 degree line is inserted. Is the policy function below this line, the optimal strategy is to run down the debt positions, while in the contrary case, gambling for redemption dominates. To enable a connection to the European Economic and Monetary Union, the red vertical line depicts the average debt to GDP level of the GIIPS countries at the end of 2009 (~81%) (OECD 2012).

First, let's focus on the left graph of the figure, representing the optimal policy response of an economy in good times (*a=1*). The thresholds show, that a country exceeding a roll over amount of 11% (which refers to a debt to GDP level of 66% in total) enters the crisis zone and exposes itself to the decisions of the international bankers.[47] However, is the country exceeding the upper sustainable debt limit, fixed at 41% (246% debt to GDP), immediate default is the only reasonable option (Conesa/ Kehoe 2012a, p.31). Now, what is the optimal strategy for a country, given these fundamentals? For a current borrowing level below the upper safe debt limit ("no-crisis zone"), a stationary debt policy maximizes the utility of the consumers

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[46] In general, an increase in the maturity reduces the vulnerability to self-fulfilling crises, as the roll over amounts decrease, which in turn heightens the upper safe debt limit (Cole/ Kehoe 2000, p.108).
[47] The actual amount of debt is calculated as the calibrated amount times 6, as only 1/6 of the total debt burden is due to be refinanced every period.

in the environment. Has the country entered the crisis zone, however, the optimal policy changes. The vulnerability to an uncoordinated investment stop on behalf of the international bankers and the increased interest rates on bonds provide an incentive to run down the debt positions to the upper safe debt limit. This decrease is done step by step, as sharp cuts in government spending are rather painful for the government itself, as well as for the consumers in the environment. Once having reached the level of indebtedness, where this vulnerability to a crisis disappeared, the country proceeds to a stationary expenditures and debt policy at the upper safe debt limit. In the same turn, bankers perceive the country's increased safety and are therefore willing to pay a high price for government bonds, no matter the realization of the sunspot variable (Arellano/ Conesa/ Kehoe 2012, p.9 f.). This is in line with the basic model provided by Harold L. Cole and Timothy J. Kehoe derived in section 3.2 and is reflected though case 2 in the phase diagram above (Conesa/ Kehoe 2011, p. 30).

Now, let's turn the attention to the contrary situation: a country in a recession ($a=0$) (right graph). As soon as this economic downturn erupts, the upper and the lower thresholds decrease immediately (highlighted with arrows). A country, which was initially save at 11% is suddenly vulnerable to a self-fulfilling crisis and has to pay a risk premium on its sovereign debt. The same logic applies to a country with an initially high indebtedness, but with the danger, that the occurrence of a recession may force it to immediate default.[48] The new lower and upper thresholds are now 7% and 33% respectively, which refers to an actual debt to GDP level of 42% and 198%. However, the most important fact observable in the results for this particular situation is the change in the optimal debt policy. A country, finding itself in the "no-crisis zone" after the eruption of the recession, suddenly has an incentive to run up its debt positions to the upper safe debt limit $\bar{b}(0;p,\pi)$. This result is intuitively understandable, as due to the economic downturn and the consequential decrease in tax revenues, liquidity gaps have to be bridged through additional borrowing. However, has the debt level exceeded this threshold, the optimal policy changes. Running down future debt to the upper safe debt limit becomes utility maximizing, as the payment of increased interest rates wants to be avoided (Case 1 in Figure 9 above). Nevertheless, this strategy only applies for intermediate roll over amounts (up to 13%). For amounts higher than 13%, the incentives change, and the government indeed starts to gamble for redemption, running up the debt positions until reaching the upper sustainable debt limit $\bar{B}(0;p,\pi)$ (Case 3 in Figure 9). The trigger for this result is the large drop in expenditures, which would be necessary in order to compensate for the decreased liquidity, released trough the restrictive debt policy. If, however, the government manages to outstand the recession, the thresholds rise again and tax revenues prosper due the recovery of the private sector. In the same turn, the costs of rolling over the debt positions decline and the additional liquidity allows the government to run down its debt positions, leading to the equilibrium in case 2. Interesting to observe within this context is that this complete change in strategy is only depending on the level of debt and the corresponding interest rate at the point in time, when the recession erupts. So can two countries, suffering from the exact same loss in revenue and the same probability for a panic, decide to run completely contrary strategies from this point onwards. Please notice, that this does not apply when the recession has different downturn effects, as then the situation is not directly comparable anymore (Conesa/ Kehoe 2012a, p.32 f.). Exactly this circumstance will be part of the discussions in the country specific analysis in section 4.3.

The results discussed above have clearly shown how a prevailing recession changes the optimal policy path of a country. The main basis of decision-making is the current debt level and the corresponding risk

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[48] This circumstance is analyzed in a more detailed way in the paper provided by Christina Arellano in 2008.

premium at this point in time. Is the risk premium still low, a country has an incentive to run down its debt positions in order to avoid being vulnerable to self-fulfilling movements on the part of the international bankers. However, is the country facing a high indebtedness, gambling for redemption turns out to be the optimal strategy. Applying these results on a surrogate of the troubled GIIPS countries in Europe, it becomes visible that at the end of 2009, the optimal policy involved gambling for redemption, speculating on a recovery of the private sector in the near future. The history shows that this strategy has indeed been followed in the last years. Whether the policy measures introduced by the EU authorities reinforced this behaviour shall be identified in the next sections' analysis.

4.2 Central Bank Policy Implications

In section 2.2, we turned our attention to the third party policy interventions in the European sovereign debt crisis. The development of the GIIPS during the years 1999-2007 has revealed that the deep seated problem lies in those countries' weakened competitiveness and in the inability to counteract with a devaluation of the own currency. This was as well perceived by the actors on the financial markets, answering with constantly rising risk premiums on government bonds during the years 2010 – 2012. However, even though the EU authorities intervened in the markets, the steady increase in indebtedness persisted. This raises the question, whether these policy measures may have been ill designed and provided the wrong incentives.

Altogether, they share two common characteristics. Those involve lowered yields on government bonds and a decrease of the costs in case of a default. With the use of the calibration, we will separately insert both of these specifications, in order to enable an individual assessment of the effects those policy measures have on a country's optimal debt policy. This will as well be done as a calibration of the defined benchmark economy, representing the GIIPS countries as a whole (Figure 11).[49]

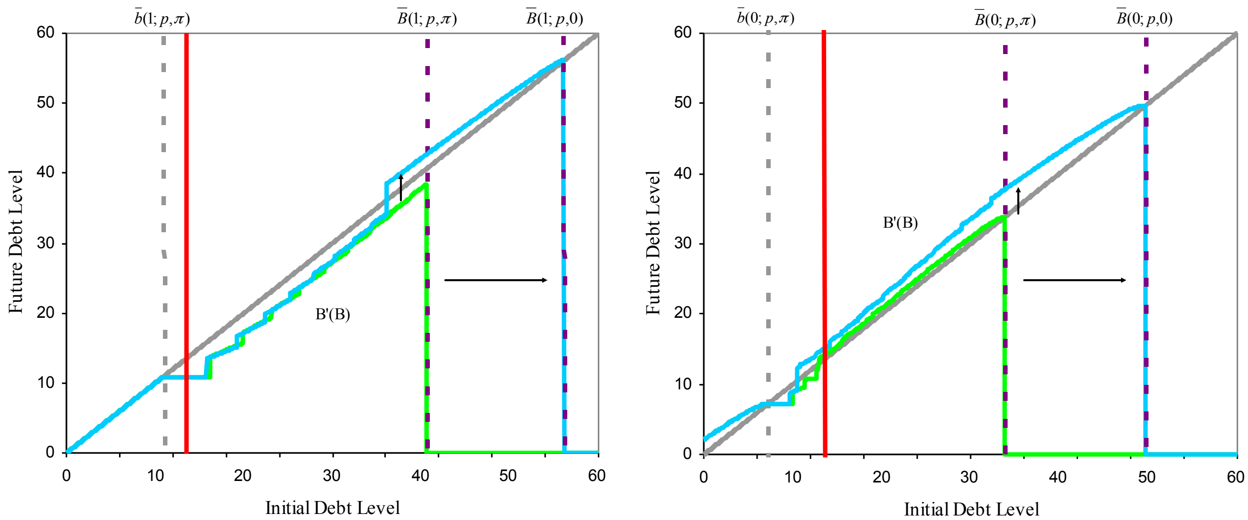
First, let's focus on the implications a reduced yield has on the policy decision of country. A theoretic way to implement this in the model is to set the risk premium within the crisis zone to zero. Keeping all other variables equal to the general model, the following calibration outcomes result, depicted in Figure 11.

The green policy function represents the results received from the general model, while the blue function refers to the new case provided above. Considering the thresholds first, one can see that an omission of the risk premium increases the upper sustainable debt limit and therefore makes the default of a country less likely. This is intuitively understandable, as the ability roll over the debt contracts under better conditions increases liquidity and allows the government to accumulate more public debt before being forced to default. This fact has a direct implication on the optimal policy of a country. In a recession, the country's incentive to gamble for redemption reinforces and becomes beneficial at considerably lower debt levels. Therefore, a country, which was initially in the act of leaving the crisis zone, may suddenly have an incentive to apply martingale gambling strategies, as further lending does not necessarily lead to increased roll over costs. Next to this, an interesting observation is that even in good times, above a certain level of debt, the optimal policy response changes and the government maximizes the value of the economy by

[49] Please notice that only the Fortran-code for the calibration of the general model has been provided in Appendix M. However, as all variables are equipped with comments, the calibrated results for these and the forthcoming analyses can be rebuilt and will therefore not be included in the Appendix.

increasing public borrowing, until reaching the upper threshold (Conesa/ Kehoe 2012a, p. 34 f.). However, as this policy is only optimal for a debt level above 216% of GDP, it is likely that a self-fulfilling crisis forces a government to default earlier, before being able to follow this despair-strategy.

Figure 11: Calibrated results for a bailout below market rates in good (left) and bad times (right)



Being aware of these results, the underlying theory can now be generalized with the bond pricing functions of the theoretical framework. Suppose the benchmark economy is hit by a recession and finds itself in the low range of the crisis zone, following the strategy of running down its current debt positions to the upper safe debt limit. However, during this process, the sunspot variable changes and indicates a self-fulfilling crisis to occur within the upcoming period. The provision of a loan above the interest rate:

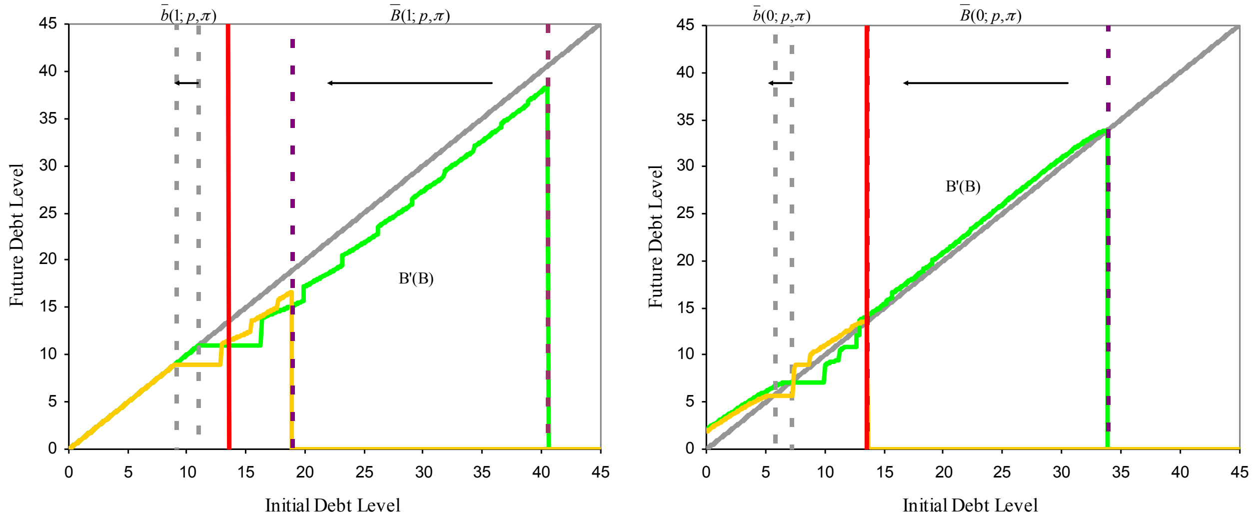
$$\frac{1}{\beta(p+(1-p)(1-\pi))}-1$$

would be able to prevent this crisis to become self-fulfilling and keep the government on its current strategy of running debt down.[50] However, is the interest rate on the loan provided by the third party below this certain value (which means below market standard), the bailout may admittedly avert the panic to become self-fulfilling, but it would in the same turn give the government the incentive to change its strategy and start gambling for redemption (Conesa/ Kehoe 2012a, p.33). Whether such a complete change in the optimal policy installs itself in some countries of the European Economic and Monetary Union will be seen in the country specific calibration.

Being aware of the interest rates' influence on the optimal debt policy within a recession, the attention is now turned to a decrease of the default penalty (*Z=0.98*), as visualized trough the orange functions in Figure 12 below.

[50] This was done by the government of the United States to avoid a self-fulfilling crisis in Mexico in 1995, as examined by Cole and Kehoe (1996).

Figure 12: Calibrated results for Z=0.98 in good (left) and bad times (right)



Considering first the thresholds, one can see that the upper safe and the upper sustainable debt limit decline in good times, as well as in a recession. This is reasonable, as a decrease in the default penalty lowers the incentive of a government to undertake painful cuts in expenditures. So is a government, which has to face decreased costs in the case of a default, running the same strategy as in the general model, but with the important difference, that the decision to file bankruptcy already dominates at debt to GDP levels of 114% and 78% in good and bad times respectively. However, this result does not automatically allow for the inference that the optimal strategy for Greece would have involved immediate default after the introduction of the haircut (with a debt level of 142% of GDP), as the fundamentals for the calibration of these graphs reflect an average of the GIIPS countries.

In summary, the analyses above have shown that policy measures, which involve increased default penalties and risk premiums, provide governments of troubled countries with the incentive to run down their debt positions and exit the crisis zone. In contrast, is the policy decreasing the interest rates and lowering the costs following a default, gambling for redemption dominates (Arellano/ Conesa/ Kehoe 2012, p.12). Important to notice is that a country within a recession already has an incentive to follow this martingale gambling strategy, but with the introduction of policy measures, like the ones implemented by the EU authorities and the IMF, the application of such a strategy becomes beneficial at considerably lower debt levels. So can a country be given the incentive to run up its debt positions in situations, where running down the borrowing level would have been optimal in the first place.

Being aware of these results, the policy measures introduced during the years 2010 and 2012 are now being assessed separately. Considering first the Securities Market Programme, it is much likely that this policy encouraged gambling for redemption. As explained above, the intent of the SMP is to buy bonds of countries, whose yields reach too high. By intervening in the market this way, the ECB aims to ensure liquidity in troubled market segments through a decrease in the interest rates. However, in the same turn, this reduces the incentive of the countries to run down their debt positions and escape the crisis zone, as there is prevention against high risk premiums (Arellano/ Conesa/ Kehoe 2012, p.12).

The same logic applies to the rescue packages provided by the EFSF and IMF, with the slight difference that these institutions intervene directly in the countries' debt roll over. This fact even advances the countries' alleged safety, as through the provision of these packages, a full replacement of the countries' maturing debt of the next years is guaranteed. However, in this context it needs to be remembered that the eligibility to receive a rescue package was coupled on the requirement to enforce fiscal austerity measures. Nonetheless, these requirements were violated later on, as pointed out when Greece applied for a second bailout in July 2011. As discussed, this second package included a 75 percent haircut. By implementing this measure, the Greek government indicated to the other GIIPS countries that such a decrease in the default penalty can be achievable. The calibration above has shown what impact this might entail (Arellano/ Conesa/ Kehoe 2012, p.12).

Considering the Longer Term Refinancing Operations, the methodology is slightly different, as this measure was introduced on a transnational basis. With the cut down of the repo interest rate, the European Central Bank provided nearly complimentary credits to the domestic banks of the troubled countries, reinforcing their incentive to buy government bonds. This was indeed done and showed its impact through a short term decrease in the risk premiums. Even though the actual intervention was not coupled on lower interest rates on bonds, its outcome enforced the incentive to gamble for redemption (Arellano/ Conesa/ Kehoe 2012, p.12). In the same turn, banks had an incentive to increase their exposure, as there was an invitation to a nearly riskless trade (Meier 2012).

However, the calibrations above focussed on a benchmark economy, representing an average of the GIIPS countries at the end of 2009. Therefore, country specific statements were not possible up until this point in time. In order to account for this circumstance, individual calibrations have been applied and will be examined in the next section.

4.3 Country Specific Calibration

In order to enable an individualized assessment of the respective countries, the variables A, π and θ are customized, as they determine the variety of the starting points at the end of 2009 (Table 3).[51] Equal to the previous sections the green policy function reflects the general model, the blue policy function refers to a decrease in the interest rates and the red vertical line depicts the countries' current debt level. To account for the haircut policy, an orange policy function has been inserted in the graph for Greece (Figure 13).

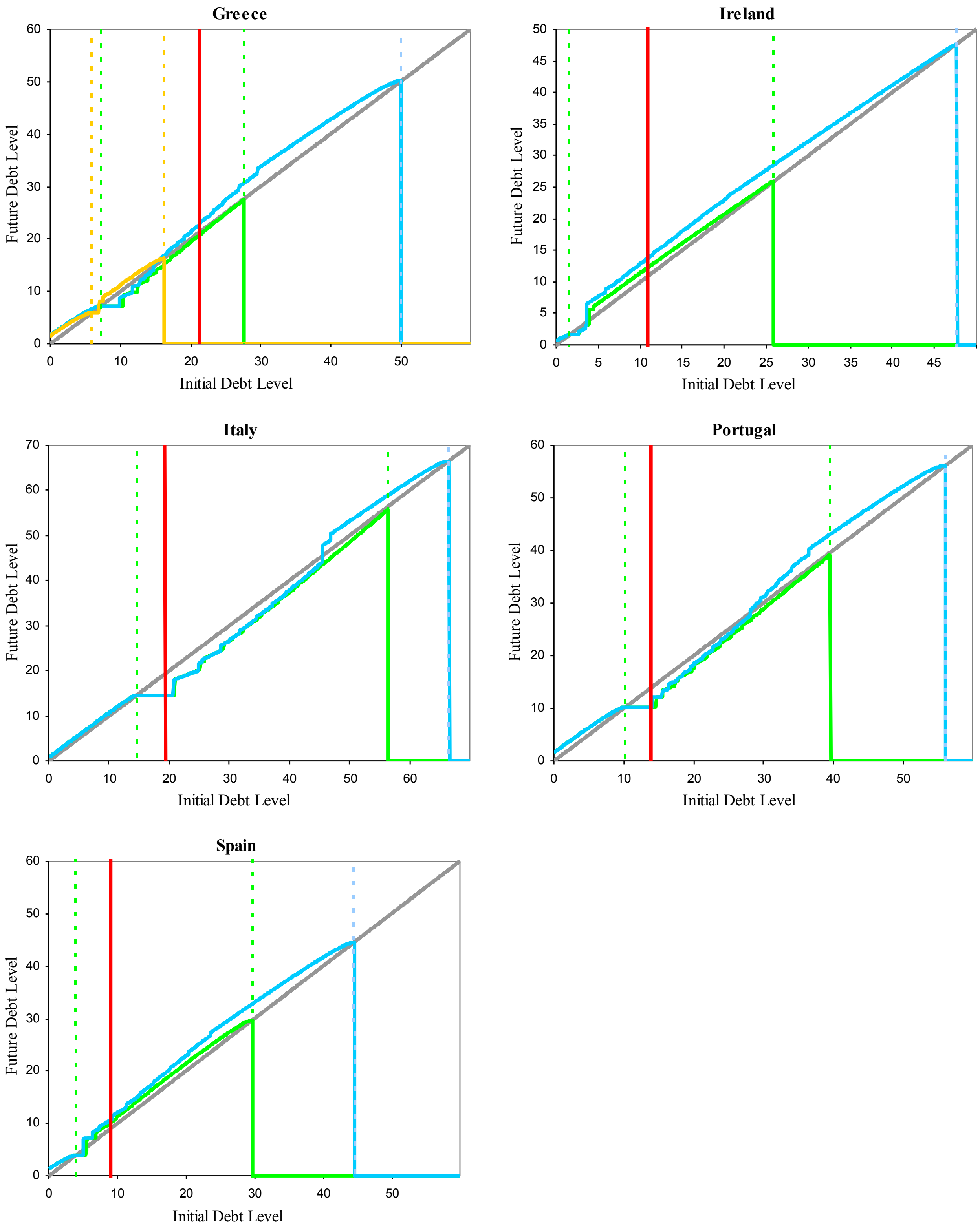
Table 3: Calibration inputs for the country specific analysis

Variables	Greece	Ireland	Italy	Portugal	Spain
A	*0.93*	*0.85*	*0.98*	*0.95*	*0.89*
π	*0.06*	*0.03*	*0.01*	*0.03*	*0.02*
θ	*0.39*	*0.34*	*0.46*	*0.42*	*0.36*

Source Data: (IMF 2012b) (ECB 2012e)

[51] The remaining inputs align with the ones used to calibrate the general model.

Figure 13: Country-specific calibrated results



Overall it can be seen that according to the theoretical framework, literally all GIIPS countries found themselves in the crisis zone at the beginning of 2010, running the danger of suffering from speculative attacks on behalf of the international bankers. This observation is supported by the history, as during the years 2010 and 2011, the governments' problems more and more began to spill over to the domestic banking sectors. As a result, actors on the financial markets started to get nervous and threatened to back out of the GIIPS countries' bonds, justifying the intervention of the EU authorities (BIS 2012a, p.1) (De Grauwe 2012, p.1). Nevertheless, according to the model only Ireland and Spain would have had an incentive to gamble for redemption in the first place. The optimal debt policy for the remaining GIIPS countries Greece, Italy and Portugal in turn involved running down the future debt positions to avoid an upcoming default. These results seem counterintuitive, considering the observations in section 2. The crucial factor driving these contrary strategies is the output loss experienced trough the recession (A). The incentive to gamble for redemption is only given, when the revenue loss takes on a substantial amount, as otherwise a slight drop in revenue can be counteracted through a bearable decrease in expenditures. Additional to that, the tax rate θ needs to be considered, as it serves as an indicator for the dependence on sovereign debt. Is the share of tax revenues from the total GDP rather high, the need for further borrowing is lower, which alleviates the incentive to transfer future resources into the current period. Comparing these implications to Table 3 above, this mentioned tendency can be recognized. The sensitivity analysis in Appendix N supports this argumentation.

Being aware of this initial position, we now turn our attention to the impact of the introduced policy interventions. The results of section 4.2 have shown that these interventions fostered the incentive to apply martingale gambling strategies. However, this generalization is not observable in the country-specific results. While the effects were simply reinforced for countries already running up the debt positions (Ireland and Spain), the optimal policy for Italy and Portugal did not change. Running down the future borrowing level still remains the dominant strategy, as the economic downturn through the recession was particularly low. However, turning the attention to Greece, a significant outcome resulted. According to the model, prior to the introduction of the third party policy measures, the optimal strategy for Greece would have involved running down its liabilities in order to leave the crisis zone. With announcement of financial aid, however, martingale gambling strategies became dominant. Incorporating as well the 75 percent haircut, an immediate default would have been the equilibrium solution. This serves as a proper illustration of how the wrong set up of third party policy measures can induce a country to follow a strategy, which was not considered to be optimal in the first place. The bond pricing theories in the previous section support this statement (Conesa/ Kehoe 2012a, p.33).

Overall, the calibrated results show that the expansive debt policy in the last years was not the optimal strategy for all the GIIPS countries. So have governments decided to start gambling for redemption, even though efforts to leave the crisis zone would have lead to an increased expected utility. Clearly, the calibrations above serve as a snapshot at the end of 2009. But against the background that increasing interest rates and an increased probability for a panic provide incentives to escape the crisis zone (see the sensitivity analysis in Appendix N), it can be assumed that no sudden change in strategy would occur at a later point in time.

The behaviour of the countries can have many reasons. In forthcoming section, a reasonable argument-tation is being elaborated, incorporating the thoughts of the theoretical framework, but as well the observations on the markets from section 2. The main attribute thereby is the distinction between liquidity and solvency problems and its influence on the incentive to display opportunistic behaviour (i.e. moral hazard).

4.4 Discussion

The history has shown that all of the five GIIPS countries have increased their debt positions since the inception of the crisis. While the immediate necessity to bridge the missing liquidity in 2008 would deliver an explanation for a sudden short-term increase in debt, it does not outline why this enhancement persisted over the years. Certainly, an essential factor for this development was the continuous increase of the risk premiums and the countries' necessity to replace maturing debt with new bonds at a higher price. Low productivity and weakened international competitiveness prevented a recovery of the private sector and made the countries dependent on the ability to roll over their debt contracts. However, while this serves as an explanation for the bad condition of those countries' domestic economy, it does not entirely explain why all of the GIIPS followed this risky strategy instead of enforcing measures to decrease the exposure to the actors in the financial markets. Therefore, it is likely that the conveyed safety though the introduced policy measures may have played a part in contributing to the increased indebtedness.

With the elaboration of the European Financial Stability Facility (EFSF), the European Central Bank, the European Commission and the International Monetary Fund set a sign to the members of the union that financial aid would be available in times of distress. Before that establishment, this kind of assurance was not available, keeping in mind the no-bailout clause of the ECB, defined in the Maastricht Treaty (De Grauwe 2007, p.162 ff.). Even though the financial markets' data does not directly reflect this circumstance, it is assumable that governments of the troubled countries perceived this increased security (Conesa/ Kehoe 2011, p.4). Against this background, it needs to be remembered that the available amounts of this fund are comprised of guarantees from the Euro-area countries themselves (Seitz/ Jost 2012, p.2 ff.). This provides reasons to believe that the strategies followed by the countries during these years might be an expression of moral hazard. However, in order to evaluate whether this motivation was triggering the continuous increase in debt, the respective development the GIIPS countries has to be assessed in a more detailed way.

From 2010 onwards, the countries most suffering from increasing interest rates were Greece, Ireland and Portugal. This heightening of debt servicing costs forced the countries to substantially increase their indebtedness, as due to the low productivity and the weakened competitiveness, maturing bonds were needed to be financed through the issuance of further debt (Higgins/ Klitgaard 2011, p.8). As a result, the lack of confidence extended and forced one after another country to apply for a bailout at the Troika (Seitz/ Jost 2012, p.4 ff.). However, even though the roll over of the maturing debt contracts was guaranteed through the provision of these rescue packages, risk premiums continued to soar. As explained in section 2.2.2, this serves as an indicator for fundamental solvency problems, as it expresses the investors' distrust in the sustainable success of the coherent fiscal austerity measures. Additional to that, it might as well signalize that long term investors got afraid of being perceived as junior by the markets, as the official loans would be the ones first served in case of a default. As in this case the only ones profiting from the bailout packages are the ones currently holding bonds, this might induce a sell-off in order to avoid losses in the future. The consequential drop in liquidity after such a sudden stop forces the countries to accumulate new debt at considerably higher interest rates, especially when the requirements of the austerity measures have not been implemented until this point in time (Chamley/ Pinto 2011, p.4). Following this theory, it is assumable that the behaviour of those three countries is not attributable to moral hazard in the first place, but rather to efforts preventing an upcoming default. However, Greeks cancellation of the fiscal austerity measures provides reasons to believe that opportunistic behaviour may have influenced the decision. In this context it needs to be remembered that contrary to the monetary policy, fiscal policy remained decentralized, triggering during this political turmoil the incentive to use debt instead (Collignon 2012, p.5). The fact that this action leads to a doom loop, as a steadily increasing indebtedness even worsens the evaluation on the financial markets, was obviously not considered.

Moving on to Italy and Spain, the circumstances change. From 2010 onwards, the funding costs for those two countries remained more or less stable, even though their public finances as well recorded worrisome levels at the beginning of the crisis (Eurostat 2012). Through the provision of these low risk premiums, the market may have signalled to the lenders, as well as to the governments, that the debt problems are considered to be an expression of missing liquidity, without having serious concerns about an upcoming default. Additional to that, it might as well express that the actors in the markets perfectly understand the risks a default of such a country would entail, as their health is of high economic importance for the core countries of the European Union.[52] The height of the foreign bank claims (Appendix D) and the export volumes are just a few factors to be mentioned.[53] This delivers reasons to believe that the strategy followed by these countries may be an expression of moral hazard, as the awareness of a potential bailout in times of financial distress triggers the incentive to gamble for redemption (Conesa/ Kehoe 2011, p.4). The recent commitment of the rescue package for Spain supports this theory. Another way to think about this circumstance is that Italy and Spain may have been less optimistic concerning their future recovery and therefore considered their risk premiums as under-priced. In this case, the costs of public borrowing lower relatively, which makes martingale gambling strategies a reasonable policy decision. The contrary case has been analyzed by Conesa and Kehoe (2012b), introducing panglossian borrowers into the theoretical framework. The result was a decrease in future debt.

However, in order to assess the argumentations above, it needs to be analyzed what interest may have been followed through this increased indebtedness. In order to do this, the fundamentals of the model have to be reconsidered. In the theoretical framework, the triggering factor leading to an end of the recession is the variable p. Given this set up, the model does not explain why at a particular point in time the private sector recovers, but with which probability. The incentive of the countries in the model to run up their debt positions is therefore only to avoid sharp cuts in expenditures, maximizing the utility of the whole economy (Conesa/ Kehoe 2011, p.6 ff.). However, the motivation of the GIIPS countries might not have been entirely the same during the years 2010-2012. So can increases in debt be an expression of Keynesian stimulus policies, trying to increase aggregate demand and lower unemployment (Seidman 2011, p.4). This is reasonable in view of the fact that unemployment steadily increased in the last years (Eurostat 2012).[54] Is this the case, the probability of a recovery would then be dependent on government expenditures $p'(g)>0$, making the increase in public debt economically legitimate. This is as well backed by the theoretical framework, as incorporating this feature would intensify the incentive to gamble for redemption (Conesa/ Kehoe 2012a, p.36 f.).[55] The history has revealed that Spain applied such deficit spending in 2008. Considering the other GIIPS countries, it is visible that investments increased in Italy about 10% from 2009 to 2010, while in the other countries this measure steadily declined, even though the

[52] In this context it needs to be noticed that the Italian and the Spanish economy have about 9 and 5 times the size of the Greek economy, as measured by GDP (IMF 2012b).
[53] Germany's exports to Italy and Spain formed nearly 10% of its total Export volume in 2011 (Statistisches Bundesamt 2012).
[54] In Spain, the unemployment rate reached a level of 24.8 percent in June 2012 (Eurostat 2012).
[55] The underlying utility function, used for this special case, can be found in Conesa and Kehoe (2012b, p.63 f.).

continuous reduction of the ECB base rate would have encouraged such proceedings (IMF 2012b) (ECB 2012f, p.196 ff.).[56]

Overall, the country individual calibrations have revealed that martingale gambling strategies are not considered to be the optimal policy path for each of the GIIPS countries at the beginning of 2010. Therefore, the behaviour of those countries during the European sovereign debt crisis is not entirely explainable through the applied theoretical framework. While part of the steadily increasing debt positions can be explained through the continuously increasing interest rates on bonds, it is not clarified why the GIIPS followed this risky strategy instead of adopting measures to run down the exposure to the actors in the financial markets. In the paragraphs above, an argumentation, affiliating the increased indebtedness to moral hazard, triggered through the indicators provided by the market participants, has been elaborated. All in all, it is assumable that countries, whose problems are considered to be an expression of missing liquidity, had an incentive to display opportunistic behaviour due to their relevance in the markets. In contrast, the increased indebtedness in countries with fundamental solvency problems can be addressed to efforts avoiding an upcoming default. However, the theories above only point out a few assumptions, based on the observations in the markets. This does not mean that there may not have been other actuators for this behaviour. In this context, e.g. political aspects cannot be neglected and certainly played a key role in the developments of the last years. However, to completely clarify the exact cause of the expansive debt policy in Europe during the prevailing crisis, further research is indispensable.

5. Conclusion

During the European sovereign debt crisis, troubled countries, lacking the ability to overcome the prevailing recession, continuously increased their indebtedness, which exposed them to the financial institutions' willingness to roll over these contracts. Increasing risk premiums and the threat of a country's default through a self-fulfilling crisis made an intervention of the European central bank inevitable. However, even though the intention of the ECB's policy measures was to calm the markets, the countries' debt positions continued to rise over the years. The purpose of this paper was to deliver an explanation for these developments through the application of a dynamic stochastic general equilibrium model, providing the optimal debt policy for a country being vulnerable to self-fulfilling movements on the financial markets.

Overall, the results have shown that the exceptional set up of the third party interventions provided countries in the EMU with the incentive to apply martingale gambling strategies. The provision of lowered risk premiums and decreased penalties diminished the costs of an upcoming default, inducing a government to take additional risk, expressed through an increase in indebtedness. The application on a benchmark economy, representing an average of the troubled GIIPS countries, confirmed these results. However, with an individualized calibration for each country, this unambiguousness disappeared. Reasonable argumentations for this deviation incorporate the countries' persistent budget deficits, as well as the incentive to display opportunistic behaviour, initiated through the awareness of a potential bailout in turbulent times. The influence of political aspects should thereby not be overlooked. Nonetheless, against the background that the crisis is still prevailing, further research is indispensable to confirm these theories.

[56] Lately, on the 5th of July 2012, the ECB base rate has been lowered to 0.75% (ECB 2012f, p.199).

While a general assessment of the third party policy interventions' influence on the individual countries' debt policy was enabled with the use of the theoretical framework, it raised in turn a few open questions, which demand for future work. The developments in Europe have revealed the increasing inter-connectedness of the financial markets, induced through the entrance into the monetary union. While the theoretical framework enables the determination of the optimal policy path for one member country, it does not account for the effects these decisions have on other members or the union as a whole. A related paper, dealing with contagion of self-fulfilling crises due to diversification of investment portfolios, has been established by Goldstein and Pauzner (2004), but without taking into consideration the limitations of a monetary union. Further, Vitek and Bayoumi (2011) examined spillovers effects from the European sovereign debt crisis on other member countries, pointing out that a spread of financial distress can entail serious consequences for the union as a whole. Therefore it would be interesting to observe how the introduction of a certain responsibility towards the union would change the optimal debt policy in the theoretical framework. In this context, the lack of fiscal integration can be taken into account.

Another observation in the financial markets was that risk premiums on government bonds increased steadily, which substantially contributed to the increased indebtedness on the affected countries. This circumstance is not fully incorporated in the theoretical framework. Through the introduction of such time varying risk premiums, the optimal policy may change, as a country, continuously being confronted with increasing borrowing costs, may be tempted to reconsider its chosen strategy. This idea was as well raised by Conesa and Kehoe (2012b), but has not been incorporated into the model until now. An introduction of this circumstance would in the same turn allow decoupling the interest rate from the future debt level, as positive correlation between those variables is not observed in every country.

References

Abel, A. B./ Bernanke, B. S./ Croushore, D. (2008): “Macroeconomics”, 6. Edition, Boston, Pearson Education

Ameco (2012): “The Annual Macro-economic Database”, http://ec.europa.eu/economy_finance/db_indicators/ameco/index_en.htm, 02.08.2012

Araujo, A./ Santos, R./ Leon, M. (2012): “Welfare analysis of currency regimes with defaultable debts”, in: Journal of International Economics, Accepted Manuscript

Arellano, C. (2008): “Default Risk and Income Fluctuations in Emerging Economies”, in: American Economic Review, Vol. 98, pp. 690-712

Arellano, C./ Conesa, J. C./ Kehoe, T. J. (2012): “Chronic Sovereign Debt Crises in the Eurozone, 2010-2012”, in: Economic Policy Paper 12-4, Federal Reserve Bank of Minneapolis

Arezki, R./ Candelon, B./ Sy, A. N. R. (2011): “Sovereign Rating News and Financial Markets Spillovers: Evidence from the European Debt Crisis”, in: IMF Working Paper, Vol. 11/68

Baldwin, R. E./ Wyplosz, C. (2009): “The Economics of European Integration”, 3. Edition, Maidenhead, Mc Graw Hill Education

Banca d’Italia (2012): “Statistical Database”, http://bip.bancaditalia.it/4972unix/homebipentry.htm?dadove=corr&lang=eng, 16.08.2012

Banco de Espana (2012): “Biest – Time Series Search Engine”, http://app.bde.es/tsi_www/vercarrito.do, 16.08.2012

Banco de Portugal (2012): “BP Stat – Statistics online”, http://www.bportugal.pt/EstatisticasWeb/%28S%28ozb4koifzgz1yoi24oo40045%29%29/Default.aspx?Lang=en-GB, 16.08.2012

Bank of Greece (2012): “Statistics”, http://www.bankofgreece.gr/Pages/en/Statistics/default.aspx, 16.08.2012

Banque de France (2012): “Statistics”, http://www.banque-france.fr/en/economics-statistics/database.html, 16.08.2012

Becker, R. A. (2008): “Transversality Condition”, in: Durlauf, S. N./ Blume, L. E. (Eds.): “The New Palgrave Dictionary of Economics Online”, http://www.dictionaryofeconomics.com/article?id=pde2008_T000218, 09.07.2012

BIS (2012a): “European bank funding and deleveraging”, in: BIS Quarterly Review – March 2012, http://www.bis.org/publ/qtrpdf/r_qt1203a.pdf, 16.07.2012

BIS (2012b): “Optimism evaporates”, in: BIS Quarterly Review – June 2012, http://www.bis.org/publ/qtrpdf /r_qt1206a_de.pdf, 18.07.2012

BIS (2012c): “Locational Banking Statistics”, http://www.bis.org/statistics/bankstats.htm, 20.08.2012

Blundell-Wignall, A. (2011): “Solving the Financial and Sovereign Debt Crisis in Europe”, in: OECD Journal: Financial Market Trends, Vol. 2011, No. 2

Calvo, G. A. (1988): “Servicing the Public Debt: The Role of Expectations”, in: The American Economic Review, Vol. 78, No. 4, pp. 647-661

Central Bank of Ireland (2012): “Statistics”, http://www.centralbank.ie/polstats/stats/Pages/default.aspx, 16.08.2012

Chamley, C./ Pinto, B. (2011): “Why Official Bailouts Tend Not to Work: An Example Motivated by Greece 2010”, in: The Economists’ Voice, Vol. 8, No. 1

Chari, V. V./ Kehoe, P. J. (1990): “Sustainable Plans”, in: Journal of Political Economy, Vol. 98, No. 2, pp. 783-802

Chari, V. V./ Kehoe, P. J. (1993): “Sustainable Plans and Mutual Default”, in: Review of Economic Studies, Vol. 60, No. 1, pp. 175-195

Cohen, D./ Villemot, S. (2011): “Endogenous Debt Crises”, in: CEPR Discussion Paper, No. 8720

Cole, H. L./ Kehoe, P. J. (1998): “Models of Sovereign Debt: Partial versus General Reputations”, in: International Economic Review, Vol. 39, No. 1, pp. 55-70

Cole, H. L./ Kehoe, T. J. (1996): “A self-fulfilling model of Mexico’s 1994-1995 debt crisis”, in: Journal of International Economics, Vol. 41, No. 3-4, pp. 309-330

Cole, H. L./ Kehoe, T. J. (1998): “Self-Fulfilling Debt Crises”, in: Research Department Staff Report 211, Federal Reserve Bank of Minneapolis

Cole, H. L./ Kehoe, T. J. (2000): “Self-Fulfilling Debt Crises”, in: Review of Economic Studies, Vol. 67, No. 1, pp. 91-116

Collignon, S. (2012): “Europe’s Debt Crisis, Coordination Failure, and International Effects, in: Asian Development Bank Institute Working Paper Series, No. 370

Conesa, J. C./ Kehoe, T. J. (2011): “Gambling for Redemption and Self-Fulfilling Debt Crises”, Mimeo

Conesa, J. C./ Kehoe, T. J. (2012a): “Gambling for Redemption and Self-Fulfilling Debt Crises”, in: Research Department Staff Report 465, Federal Reserve Bank of Minneapolis

Conesa, J. C./ Kehoe, T. J. (2012b): “Gambling for Redemption and Self-Fulfilling Debt Crises”, in: Lecture Presentation, http://www.econ.umn.edu/~tkehoe/classes/GamblingOverheadsFeb12.pdf, 12.08.2012

Dadush, U./ Eidelman, V. (2010): “Germany: Europe’s Pride or Europe’s Problem”, in: Dadush U. (Ed.): “Paradigm Lost – The Euro in Crisis”, Washington D.C., Carnegie Endowment for International Peace, pp. 17-21

Dadush, U./ Stancil, B. (2010): “Europe’s Debt Crisis: More than a Fiscal Problem”, in: Dadush U. (Ed.): “Paradigm Lost – The Euro in Crisis”, Washington D.C., Carnegie Endowment for International Peace, pp. 8-15

De Grauwe, P. (2007): “Economics of Monetary Union”, 7th Edition, Oxford, Oxford University Press

De Grauwe, P. (2009): “The Politics of the Maastricht Convergence Criteria”, http://www.voxeu.org/article/politics-maastricht-convergence-criteria, 16.08.2012

De Grauwe, P. (2011a): “The Governance of a Fragile Eurozone”, in: Centre for European Policy Studies Working Document, No. 346

De Grauwe, P. (2011b): “The European Central Bank: Lender of Last Resort in the Government Bond Markets?”, in: CESifo Working Papers, No. 3569

De Grauwe, P. (2012): “How not to be a lender of last resort”, in: Centre for European Policy Studies Commentary, http://www.ceps.eu/book/how-not-be-lender-last-resort, 16.08.2012

Dittli, M. (2012a): “Europas Rendezvous mit dem Kollaps”, in: Never Mind the Markets – Here comes the Economy Blog, http://blog.bazonline.ch/nevermindthemarkets/index.php/7216/europas-rendezvous-mit-dem-kollaps/, 16.07.2012

Dittli, M. (2012b): “Die Liebesaffäre der Börsen mit Mario Draghi”, in: Never Mind the Markets – Here comes the Economy Blog, http://blog.bazonline.ch/nevermindthemarkets/index.php/6644/die-liebesaffare-der-borsen-mit-mario-draghi/, 16.07.2012

ECB (2010a): “Monthly Bulletin - May 2010”, http://www.ecb.int/pub/pdf/mobu/mb201005en.pdf, 29.06.2012

ECB (2010b): “Monthly Bulletin - June 2010”, http://www.ecb.int/pub/pdf/mobu/mb201006en.pdf, 28.06.2012

ECB (2010c): “Decision of the European Central Bank of 14 May 2010 establishing a securities markets programme”, http://www.ecb.int/ecb/legal/pdf/en_dec_2010_5__f_sign.pdf, 14.07.2012

ECB (2010d): “ECB decides on measures to address severe tensions in financial markets”, in: Press Release 10 May 2010, http://www.ecb.int/press/pr/date/2010/html/pr100510.en.html, 14.07.2012

ECB (2011): “Monthly Bulletin - September 2011”, http://www.ecb.int/pub/pdf/mobu/mb201109en.pdf, 28.06.2012

ECB (2012a): “Economic and Monetary Union (EMU)”, http://www.ecb.int/ecb/history/emu/html/index.en.html, 11.06.2012

ECB (2012b): “Open Market Operations”, http://www.ecb.int/mopo/implement/omo/html/index.en.html, 02.07.2012

ECB (2012c): “ECB announces measures to support bank lending and money market activity”, in: Press Release - 8 December 2011, http://www.ecb.int/press/pr/date/2011/html/pr111208_1.en.html, 02.07.2012

ECB (2012d): “Introductory statement to the press conference (with Q&A)”, http://www.ecb.int/press/pressconf/2012/html/is120404.en.html, 16.07.2012

ECB (2012e): “Statistical Data Warehouse”, http://sdw.ecb.europa.eu/browse.do?node=9484572, 02.08.2012

ECB (2012f): “Monthly Bulletin – August 2012”, http://www.ecb.int/pub/pdf/mobu/mb201208en.pdf, 14.08.2012

EFSF (2012): “EFSF Framework Agreement”, http://www.efsf.europa.eu/attachments/20111019_efsf_framework_agreement_en.pdf, 16.07.2012

European Commission (2009): “Economic Crisis in Europe – Causes, Consequences and Responses”, in: European Economy 7/2009, http://ec.europa.eu/economy_finance/publications/publication15887_en.pdf, 16.08.2012

Eurostat (2012): “Statistics Database”, http://epp.eurostat.ec.europa.eu/portal/page/portal/statistics/search_database, 02.08.2012

Favaro, E./ Li, Y./ Pradelli, J./ Van Doorn, R. (2011): “Europe’s Crisis: Origins and Policy Challenges”, in: Primo Braga, C. A./ Vincelette, G. A. (Eds.): “Sovereign Debt and the Financial Crisis – Will This Time Be Different?”, Washington D.C., The World Bank, pp. 221-239

Financial Times (2012): “Spanien – EFSF oder ESM?”, http://www.ftd.de/politik/europa/:schuldenkrise-spanien-efsf-oder-esm/70048526.html, 14.08.2012

Gibson, H. D./ Hall, S. G. / Tavlas, G. S. (2011): “The Greek financial crisis: growing imbalances and sovereign spreads”, in: Working Paper No. 124, Bank of Greece

Goldstein, I./ Pauzner, A. (2004): “Contagion of Self-Fulfilling Financial Crises Due to Diversification of Investment Portfolios”, in: Journal of Economic Theory, Vol. 119, No.1, pp. 151-183

Gros, D./ Mayer, T. (2010): "How to deal with sovereign default in Europe: Towards a European Monetary Fund", in: Centre for European Policy Studies Policy Brief, No. 202

Higgins, M./ Klitgaard, T. (2011): "Saving Imbalances and the Euro Area Sovereign Debt Crisis", in: Federal Reserve Bank of New York - Current Issues In Economics and Finance, Vol. 17, No. 5

IMF (2010a): "Europe and IMF Agree €110 Billion Financing Plan with Greece", in: IMF Survey Magazine: Countries & Regions, May 02, 2010, http://www.imf.org /external/pubs/ft/survey/so /2010/CAR050210A.htm, 25.06.2012

IMF (2010b): "IMF Executive Board Approves €22.5 Billion Extended Arrangement for Ireland", in: Press Release No. 10/496, December 16, 2010,http://www.imf.org/external/np/sec/pr/2010 /pr10496.htm, 06.07.2012

IMF (2011): "IMF Executive Board Approves €26 Billion Extended Arrangement for Portugal", in: Press Release No. 11/190, May 20, 2011,http://www.imf.org /external/np/sec/pr/2011/pr11190.htm, 06.07.2012

IMF (2012a): "IMF Executive Board Approves €28 Billion Arrangement under the Extended Fund Facility for Greece", in: Press Release No. 12/85, Mar 15, 2012, http://www.imf.org/external/np/sec /pr/2012/pr1285.htm, 06.07.2012

IMF (2012b): "World Economic Outlook Database April 2012", http://www.imf.org/external/pubs/ft/weo /2012/01/weodata/index.aspx, 05.08.2012

Kharas, H./ Pinto, B./ Ulatov, S. (2001): "An Analysis of Russia's 1998 Meltdown: Fundamentals and Market Signals", in: Brookings Papers on Economic Activity, Vol. 2011, No. 1, pp. 1-68

Koen V./ Van den Noord, P. (2005): "Fiscal Gimmickry in Europe – One-off Measures and Creative Accounting", in: OECD Economics Department Working Papers, No. 417

Krugman, P. (1979): "A Model of Balance-of-Payments Crises", in: Journal of Money, Credit and Banking, Vol. 11, No. 3, pp. 311-325

Lequiller, F./ Blades, D. (2007): "Understanding National Accounts", Paris, OECD Publishing

Linzert, T./ Nautz, D./ Bindseil, U. (2004): "The Longer Term Refinancing Operations of the ECB", in: ECB Working Paper Series, No. 359

Lucas, R. E./ Stokey, N. L. (1983): "Optimal Fiscal and Monetary Policy in an Economy without Capital", in: Journal of Monetary Economics, Vol. 12, No. 1, pp. 55-93

Meier, M. D. (2012): "Probleme mit der LTRO-Arznei", in: Never Mind the Markets – Here comes the Economy Blog, http://blog.bazonline.ch/nevermindthemarkets/index.php/7545/probleme-mit-der-ltro-arznei/, 16.07.2012

Morris, S./ Shin, H. S. (1998): “Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks”, in: American Economic Review, Vol. 88, No. 3, pp. 587-596

National Bank van Belgie (2012): “Statistics”, http://www.nbb.be/pub/stats/institutes/institutes.htm?l=en, 16.08.2012

Obstfeld, M. (1986): “Rational and Self-fulfilling Balance-of-Payments Crises”, in: The American Economic Review, Vol. 76, No. 1, pp. 72-81

Obstfeld, M. (1996): “Models of currency crises with self-fulfilling features”, in: European Economic Review, Vol. 40, No. 3-5, pp. 1037-1047

Obstfeld, M./ Rogoff, K. S. (1996): “Foundations of International Macroeconomics”, Cambridge, MIT Press

OECD (2012): “OECD StatExtracts”, http://stats.oecd.org/, 05.08.2012

Pessoa, A. (2011): “The Euro Area sovereign debt crisis: Some implications on its systemic dimension”, in: Munich Personal RePec Archive, No. 35328

Reinhart, C. M./ Rogoff, K. S. (2009): „This Time is Different – Eight Centuries of Financial Follies“, Princeton, Princeton University Press

Sargent, T. J. (1987): “Dynamic Macroeconomic Theory”, Cambridge, Harvard University Press

Schäfer, H. B. (2012): “The Sovereign Debt Crisis in Europe, Save Banks Not States”, http://ssrn.com /abstract=2049299, 25.06.2012

Schuknecht, L./ Moutot, P./ Rother, P./ Stark, J. (2011): “The Stability and Growth Pact – Crisis and Reform”, in: European Central Bank Occasional Paper Series, No. 129

Seidman, L. S. (2011): “Keynesian Fiscal Stimulus – What Have We Learned from the Great Recession?”, in: University of Delaware Working Paper Series, No. 2011-11

Seitz, F./ Jost, T. (2012): „The Role of the IMF in the European Debt Crisis“, in: HAW im Dialog - Weidener Diskussionspapiere, No. 32,

Standard & Poor’s (2012): „Sovereigns Rating List“, http://www.standardandpoors.com/ratings/sovereigns/ratings-list/en/us/?sectorName=Governments&subSectorCode=39&subSectorName=Sovereigns, 19.07.2012

Statista (2012): “Staatsanleihen”, http://de.statista.com/statistik/suche/q/staatsanleihen/kategorie/11/p/1/, 19.08.2012

Statistisches Bundesamt (2012): “Genesis-Online Datenbank”, https://www-genesis.destatis.de/genesis/online, 19.08.2012

Stokey, N. L. (1991): “Credible Public Policy”, in: Journal of Economic Dynamics and Control, Vol. 15, No. 4, pp. 627-656

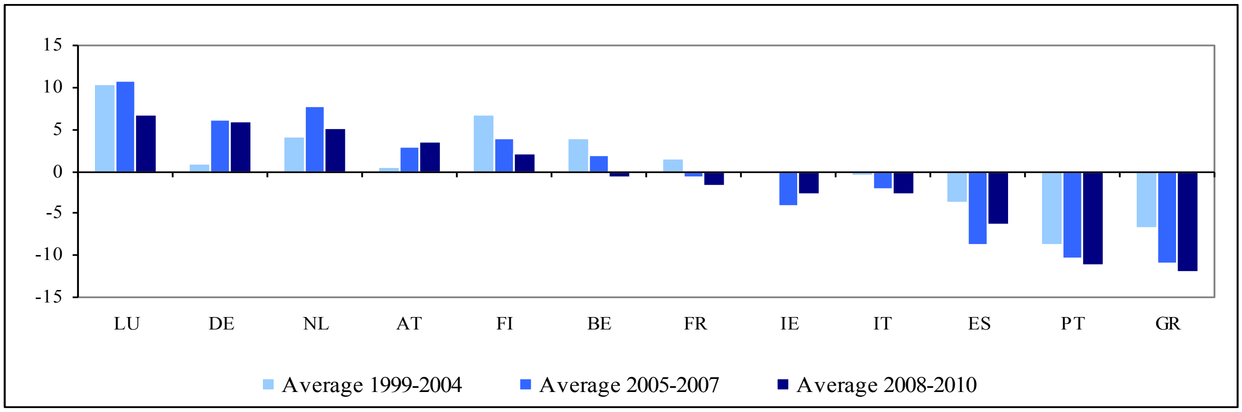
Thompson Reuters (2012): “Datastream”, WWZ-Terminal, 23.07.2012

Vitek, F./ Bayoumi, T. (2011): “Spillovers from the Euro Area Sovereign Debt Crisis: A Macroeconomic Model Based Analysis”, in: CEPR Discussion Paper, No. 8497

Von Hagen, J./ Wolff, G. (2004): “What do Deficits tell us about Debts? – Empirical Evidence on Creative Accounting with Fiscal Rules in the EU”, in: CEPR Discussion Paper, No. 4759

Appendix

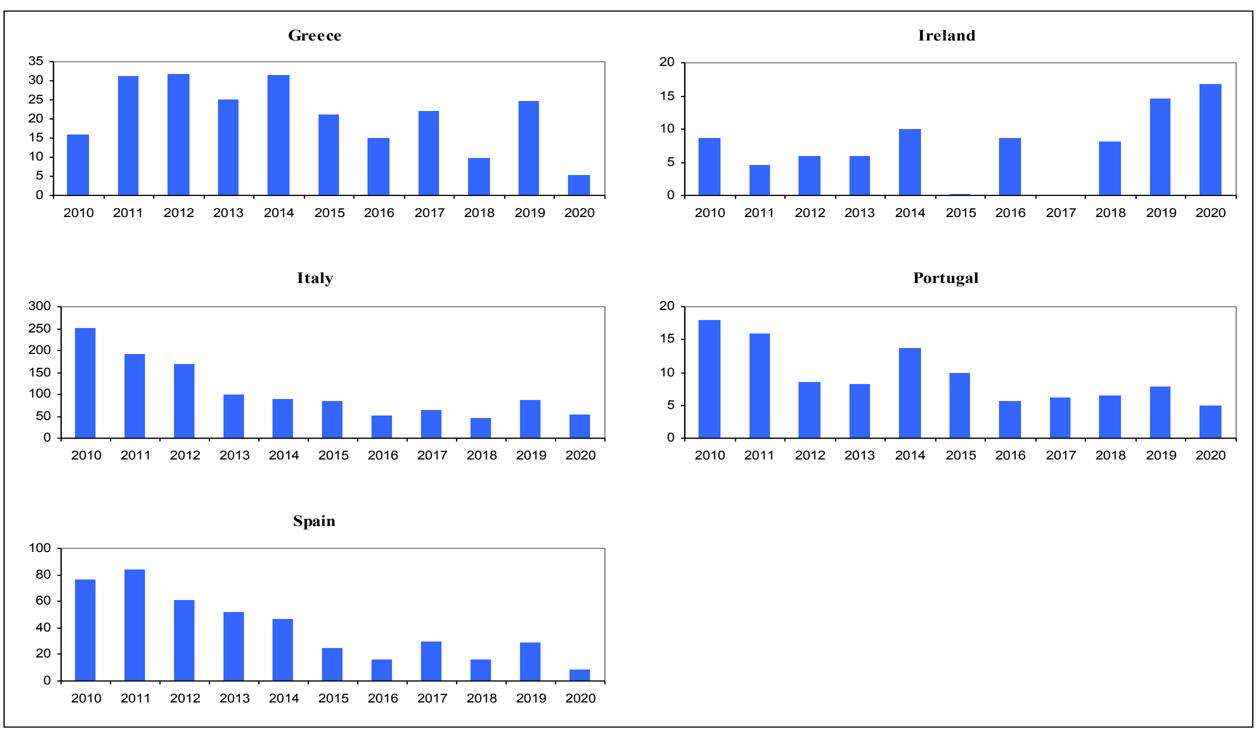
A: Current Account Balance as a percentage of GDP



Idea for this figure provided by Favaro et al. (2011, p.222) / Source Data: (OECD 2012)

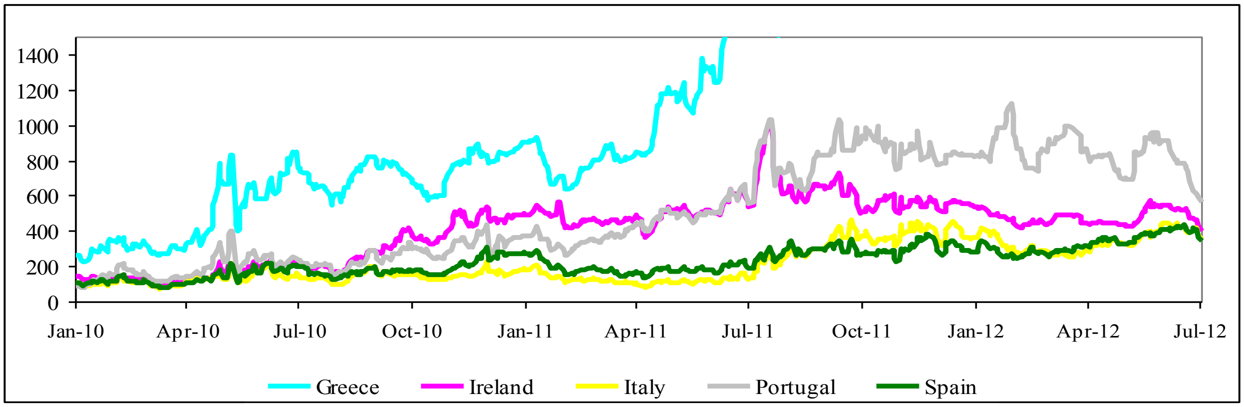
The current account balance is defined as: *Net exports + net factor payments = domestic saving – domestic investment spending*. Is a country running a current account deficit, it consumes more than it produces, forcing the government to run up net exports or otherwise borrow additional debt from abroad to account for the shortfall. Is the contrary case occurring, the country has the ability to send its surplus abroad in order to purchase foreign assets (import) (Abel/ Bernanke/ Croushore 2008, p.40 ff.).

B: Debt Maturity Structure of the GIIPS countries in Bio €: 2010 – 2020



Source Data: Statista (2012)

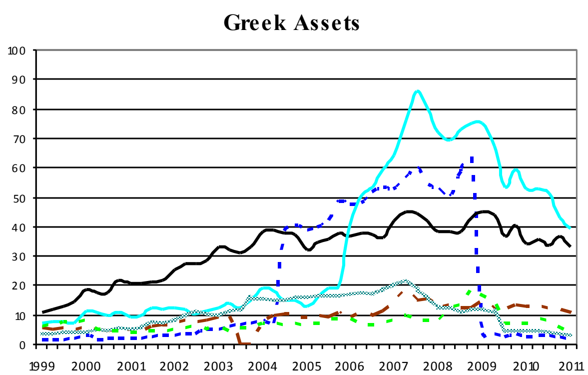
C: 10-year Credit Default Swap Premiums in basis points

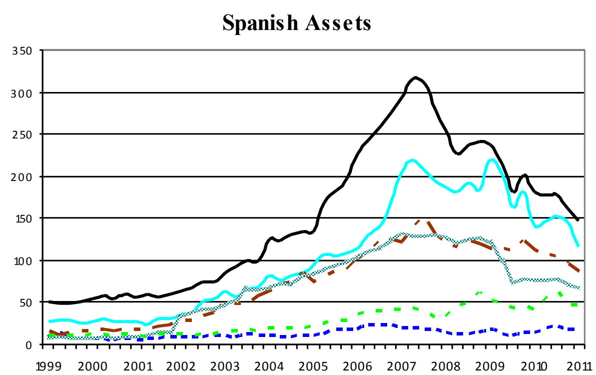


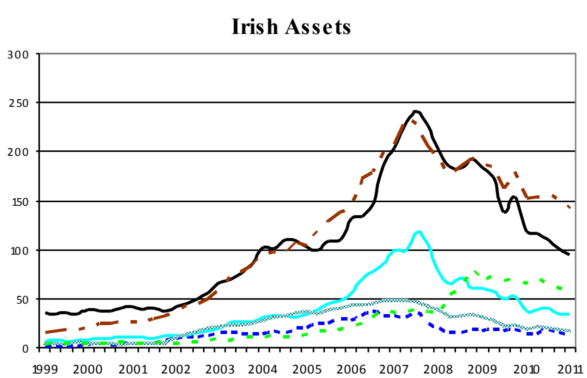
Source Data: Thompson Reuters (2012)

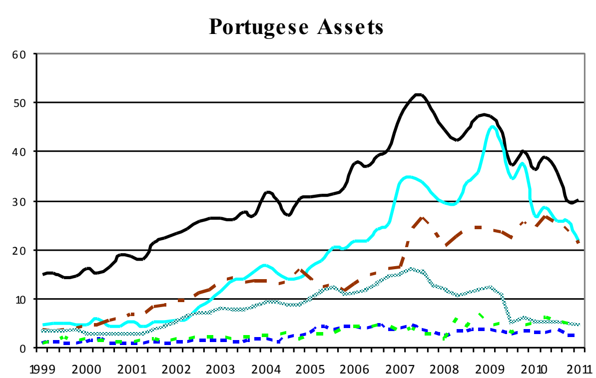
The ordinate shows the CDS prices for the GIIPS countries in basis points. A level of 1’000 basis points means that it costs 1 million € to protect 10 millions of debt for 10 years. Please note that the prices for Greece have been cut off, as the premiums reached levels around 10’000 basis points at the beginning of 2012.

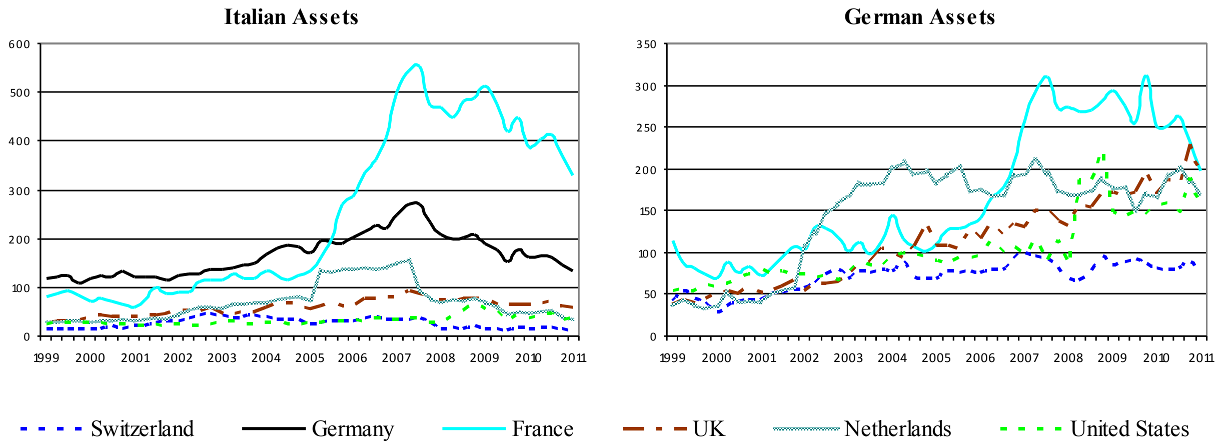
D: Foreign Bank Claims in Mio US $





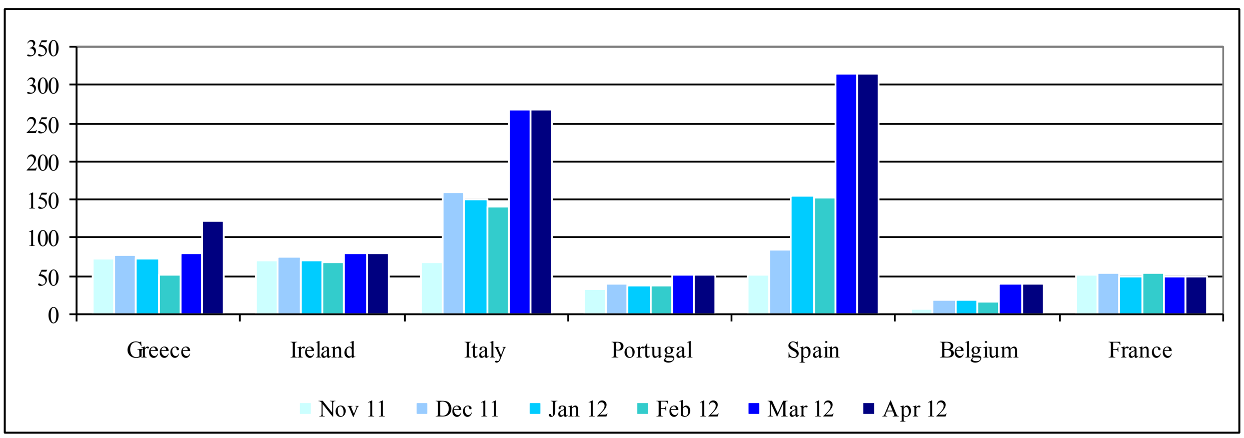






Source Data: BIS (2012c)

E: Total use of the ECB Longer Term Refinancing Operations in Bio €



Data Source: Bank of Greece (2012), Central Bank of Ireland (2012), Banca d'Italia (2012), Banco de Portugal (2012), Banco de Espana (2012), National Bank van Belgie (2012) and Banque de France (2012)

F: Determination of Bond Prices for $\bar{b}(1;p,\pi) > \bar{B}(0;p,\pi)$ and $\bar{b}(1;p,\pi) = \bar{B}(0;p,\pi)$

Changing the sequence of the respective thresholds has a direct effect on the zones of the framework. However, by using the basic equation defined in section 3.1.3, the bond prices for these particular cases can as well be defined.

This case refers to the situation, where $\bar{b}(1;p,\pi) > \bar{B}(0;p,\pi)$, resulting in the following pricing functions:

$$q(B',(B,0,1,\zeta);p,\pi)=\begin{cases}\beta & \text{if } B' \leq \bar{b}(0;p,\pi)\\ \beta(p+(1-p)(1-\pi)) & \text{if } \bar{b}(0;p,\pi) < B' \leq \bar{B}(0;p,\pi)\\ \beta p & \text{if } \bar{B}(0;p,\pi) < B' \leq \bar{b}(1;p,\pi)\\ \beta p(1-\pi) & \text{if } \bar{b}(1;p,\pi) < B' \leq \bar{B}(1;p,\pi)\\ 0 & \text{if } B' > \bar{B}(1;p,\pi)\end{cases}$$

Considering the case where $\bar{b}(1;p,\pi)$ equals $\bar{B}(0;p,\pi)$, the following bond prices result (Conesa / Kehoe 2011, p.11):

$$q(B',(B,0,1,\zeta);p,\pi)=\begin{cases}\beta & \text{if } B' \leq \bar{b}(0;p,\pi)\\ \beta(p+(1-p)(1-\pi)) & \text{if } \bar{b}(0;p,\pi) < B' \leq \bar{b}(1;p,\pi)\\ \beta p(1-\pi) & \text{if } \bar{b}(1;p,\pi) < B' \leq \bar{B}(1;p,\pi)\\ 0 & \text{if } B' > \bar{B}(1;p,\pi)\end{cases}$$

G: Derivation of the upper stationary debt limit $\bar{B}^s(1;p,0)$

Let's assume that the government finds itself in a situation where the current holdings of public debt exceed the upper stationary debt limit: $B_0 > \bar{B}^s(1;p,0)$. In this situation, it cannot be optimal for the country to run down its expenditures within one period by setting $g_0=\theta\bar{y}-B_0+\beta B_1$ in $t=0$ and then $g_t=\theta\bar{y}-(1-\beta)B_1$ for all periods following. In order to demonstrate this, let's assume the contrary case and refer to the participation constraint, stating that the utility of not defaulting is higher than the utility of defaulting, as long as new debt can be sold at a positive price, e.g. $q=\beta$:

$$u((1-\theta)\bar{y},\theta\bar{y}-B_0+\beta B_1)+\frac{\beta u((1-\theta)\bar{y},\theta\bar{y}-(1-\beta)B_1}{1-\beta}$$
$$\geq u((1-\theta)Z\bar{y},\theta Z\bar{y}+\beta B_0)+\frac{\beta u((1-\theta)Z\bar{y},\theta Z\bar{y}}{1-\beta}$$

In order to maximize the left hand side of the equation, the utility of not defaulting, a stationary debt policy is applied by setting $B_0=B_1$. Having done this, the following inequality results:

$$u((1-\theta)\bar{y},\theta\bar{y}-B_0+\beta B_0)+\frac{\beta u((1-\theta)\bar{y},\theta\bar{y}-(1-\beta)B_0}{1-\beta}$$
$$\geq u((1-\theta)\bar{y},\theta\bar{y}-B_0+\beta B_1)+\frac{\beta u((1-\theta)\bar{y},\theta\bar{y}-(1-\beta)B_1}{1-\beta}$$

This resulting expression shows that as long as $\pi=0$, the utility of keeping debt high is bigger than the utility of running debt up or down for any $B_0 \neq B_1$. This in turn contradicts the assumption that the initial debt level was above the upper stationary debt limit, as in such a situation, immediate default would have been the dominant strategy (Conesa/ Kehoe 2011, p.16).

H: Determination of government spending in $\bar{b}(1;p,\pi)<B\leq\bar{B}(1;p,\pi)$ with $\pi>0$

Let's suppose that the government finds itself confronted to a debt level higher than the upper safe debt limit ($B_0>\bar{b}(1;p,\pi)$) and decides to reduce its debt positions to the same in T periods. Considering the first order conditions derived for the case where $\pi=0$, the following expression for government expenditures results: $g_t=g^T(B_0;\pi)$.

Applying this expression to the governments' budget constraints, under the assumption that $B_t\neq B_{t-1}$, the following constraints result:

$$
\begin{aligned}
g^T(B_0;\pi)+B_0 &= \theta\bar{y}+\beta(1-\pi)B_1\\
g^T(B_0;\pi)+B_1 &= \theta\bar{y}+\beta(1-\pi)B_2\\
&\cdots\\
g^T(B_0;\pi)+B_{T-2} &= \theta\bar{y}+\beta(1-\pi)B_{T-1}\\
g^T(B_0;\pi)+B_{T-1} &= \theta\bar{y}+\beta\bar{b}(1;p,\pi)
\end{aligned}
$$

Multiplying each of these equations by $(\beta(1-\pi))^t$ and adding them, the following expression is obtained:

$$\sum_{t=0}^{T-1}(\beta(1-\pi))^t g^T(B_0;\pi)+B_0=\sum_{t=0}^{T-1}(\beta(1-\pi))^t\theta\bar{y}+(\beta(1-\pi))^{T-1}\beta\bar{b}(1;p,\pi)$$

which results in (Conesa/ Kehoe 2011, p.17):

$$g^T(B_0;\pi)=\theta\bar{y}-\frac{1-\beta(1-\pi)}{1-(\beta(1-\pi))^T}(B_0-(\beta(1-\pi))^{T-1}\beta\bar{b}(1;p,\pi)).$$

I: Backward Induction in $\bar{b}(1;p,\pi)<B\leq\bar{B}(1;p,\pi)$ with $\pi>0$

In this section, the value functions $V^T(B_0;\pi)$ for each of the policies of running down the public debt positions with $T=1,2,\ldots,\infty$ is compued. Thereby, $V_t^T(B_0;\pi)$ denotes the value of the policy, for which there are still t periods in running down debt to go. The resulting value functions are the following:

$$
\begin{aligned}
V_T^T(B_0;\pi) &= u((1-\theta)\bar{y},g^T(B_0;\pi))+\beta(1-\pi)V_{T-1}^T(B_0;\pi)+\frac{\beta\pi u((1-\theta)Z\bar{y},\theta Z\bar{y}}{1-\beta}\\
V_{T-1}^T(B_0;\pi) &= u((1-\theta)\bar{y},g^T(B_0;\pi))+\beta(1-\pi)V_{T-2}^T(B_0;\pi)+\frac{\beta\pi u((1-\theta)Z\bar{y},\theta Z\bar{y}}{1-\beta}\\
&\cdots\\
V_2^T(B_0;\pi) &= u((1-\theta)\bar{y},g^T(B_0;\pi))+\beta(1-\pi)V_1^T(B_0;\pi)+\frac{\beta\pi u((1-\theta)Z\bar{y},\theta Z\bar{y}}{1-\beta}\\
V_1^T(B_0;\pi) &= u((1-\theta)\bar{y},g^T(B_0;\pi))+\frac{\beta u((1-\theta)\bar{y},\theta\bar{y}}{1-\beta}
\end{aligned}
$$

As visible in the last equation above, the level of government spending in period T increases from $g^T(B_0;\pi)$ to $\theta\bar{y}$. In order to calculate now the value function $V^T(B_0;\pi)$, a backward induction is being applied:

$$V_2^T(B_0;\pi)=(1+\beta(1-\pi))u((1-\theta)\bar{y},g^T(B_0;\pi))+\frac{\beta\pi u((1-\theta)Z\bar{y},\theta Z\bar{y})}{1-\beta}+\beta(1-\pi)\frac{\beta u((1-\theta)\bar{y},\theta\bar{y})}{1-\beta}$$

$$V_3^T(B_0;\pi) = (1+\beta(1-\pi)+(\beta(1-\pi))^2)u((1-\theta)\bar{y}, g^T(B_0;\pi)) + (1+\beta(1-\pi))\frac{\beta\pi u((1-\theta)Z\bar{y}, \theta Z\bar{y})}{1-\beta}$$
$$+(\beta(1-\pi))^2\frac{\beta u((1-\theta)\bar{y}, \theta\bar{y})}{1-\beta}$$

$$V_T^T(B_0;\pi) = (1+\beta(1-\pi)+(\beta(1-\pi))^2+...+(\beta(1-\pi))^{T-1})u((1-\theta)\bar{y}, g^T(B_0;\pi))$$
$$+(1+\beta(1-\pi)+(\beta(1-\pi))^2+...+(\beta(1-\pi))^{T-2})\frac{\beta\pi u((1-\theta)Z\bar{y}, \theta Z\bar{y})}{1-\beta}+(\beta(1-\pi))^{T-2}\frac{\beta u((1-\theta)\bar{y}, \theta\bar{y})}{1-\beta}$$

As per definition, $V^T(B_0;\pi) = V^T(B_0;\pi)$, resulting in the following expression:

$$V^T(B_0;\pi) = \frac{1-(\beta(1-\pi))^T}{1+\beta(1-\pi)}u((1-\theta)\bar{y}, g^T(B_0;\pi)) + \frac{1-(\beta(1-\pi))^{T-1}}{1+\beta(1-\pi)}\frac{\beta\pi u((1-\theta)Z\bar{y}, \theta Z\bar{y})}{1-\beta}$$
$$+(\beta(1-\pi))^{T-2}\frac{\beta u((1-\theta)\bar{y}, \theta\bar{y})}{1-\beta}$$

Being aware of this procedure, please notice that (Conesa/ Kehoe 2011, p. 17 f.):

$$V^\infty(B_0;\pi) = \frac{u(1-\theta)\bar{y}, \theta\bar{y} - (1-\beta(1-\pi))B_0}{1+\beta(1-\pi)} + \frac{\beta\pi u((1-\theta)Z\bar{y}, \theta Z\bar{y})}{(1-\beta)(1+\beta(1-\pi))}$$

J: Determination of the upper sustainable debt limit $\bar{B}(0;p,0)$ in a recession

To determine the upper sustainable debt limit in a prevailing recession, we follow the same procedure as in section 3.2.2 by equalizing the value functions of the participation constraint:

$$V_n^g(\bar{B}(0;p,0),0,1;p,0) = V_d^g(\bar{B}(0;p,0),0,1;p,0)$$

The strategic assumption is that the government increases its foreign borrowing up to a point, where it is less than or equal the mentioned upper threshold. Then, it again heightens its debt burden to $\bar{B}(0;p,0)<B\leq \bar{B}(1;p,0)$ at a bond price $q=\beta p$, speculating on a recovery of the economy.[57] The value of borrowing additional debt $\bar{B}(1;p,0)$ at the price $q=\beta p$, then repaying the current debt positions and again repaying the incorporated debt in the forthcoming period if the private sector recovers, or defaulting otherwise, is:

$$V_n^g(B,0,1;p,0) = u((1-\theta)A\bar{y}, \theta A\bar{y}+\beta p\bar{B}(1;p,0)-B)$$
$$+\beta(1-p)\left(\frac{u((1-\theta)AZ\bar{y}, \theta AZ\bar{y})}{1-\beta(1-p)}+\frac{\beta p u((1-\theta)Z\bar{y}, \theta Z\bar{y})}{(1-\beta)(1-\beta(1-p))}\right)$$
$$+\frac{\beta p}{1-\beta}u((1-\theta)\bar{y}, \theta\bar{y}-(1-\beta)\bar{B}(1;p,0))$$

[57] Please notice that the price for a bond has changed from $q=\beta p(1-\pi)$ to $q=\beta p$, as the probability for a future crisis to occur was set to $\pi=0$.

Contrary to that, the value of borrowing additional debt $\overline{B}(1;p,0)$ at the price $q=\beta p$ and directly defaulting is:

$$V_d^g(B,0,1;p,0) = u((1-\theta)AZ\overline{y}, \theta AZ\overline{y} + \beta p\overline{B}(1;p,0))$$
$$+\beta(1-p)\left(\frac{u((1-\theta)AZ\overline{y}, \theta AZ\overline{y})}{1-\beta(1-p)} + \frac{\beta p u((1-\theta)Z\overline{y}, \theta Z\overline{y})}{(1-\beta)(1-\beta(1-p))}\right)$$
$$+\frac{\beta p}{1-\beta}u((1-\theta)Z\overline{y}, \theta Z\overline{y})$$

Equalizing those two equations and solving them brings us then to the upper sustainable debt limit in a recession $\overline{B}(0;p,0)$ (Conesa/ Kehoe 2011, p.21):

$$u((1-\theta)A\overline{y}, \theta A\overline{y} + \beta p\overline{B}(1;p,0) - \overline{B}(0;p,0) + \frac{\beta p}{1-\beta}u((1-\theta)\overline{y}, \theta\overline{y} - (1-\beta)\overline{B}(1;p,0))$$
$$= u((1-\theta)AZ\overline{y}, \theta AZ\overline{y} + \beta p\overline{B}(1;p,0)) + \frac{\beta p}{1-\beta}u((1-\theta)Z\overline{y}, \theta Z\overline{y})$$

K: Gambling for Redemption – Equilibrium without Default

The government's expected payoff of holding a debt level at the upper sustainable debt limit in a recession needs to be higher than the payoff of defaulting, after the international bankers have rolled over the bonds $B'=\overline{B}(0;p,0)$ at the price $q=\beta(1-\pi)$.

$$u((1-\theta)A\overline{y}, \theta A\overline{y} - (1-\beta)\overline{B}(0;p,0))$$
$$+\beta(1-p)\left(\frac{u((1-\theta)A\overline{y}, \theta A\overline{y} - (1-\beta)\overline{B}(0;p,0))}{1-\beta(1-p)} + \frac{\beta p u((1-\theta)\overline{y}, \theta\overline{y} - (1-\beta)\overline{B}(0;p,0))}{(1-\beta)(1-\beta(1-p))}\right)$$
$$+\frac{\beta p}{1-\beta}u((1-\theta)\overline{y}, \theta\overline{y} - (1-\beta)\overline{B}(0;p,0))$$
$$\geq$$
$$u((1-\theta)AZ\overline{y}, \theta AZ\overline{y} + \beta\overline{B}(0;p,0))$$
$$+\beta(1-p)\left(\frac{u((1-\theta)AZ\overline{y}, \theta AZ\overline{y})}{1-\beta(1-p)} + \frac{\beta p u((1-\theta)Z\overline{y}, \theta Z\overline{y})}{(1-\beta)(1-\beta(1-p))}\right) + \frac{\beta p}{1-\beta}u((1-\theta)Z\overline{y}, \theta Z\overline{y})$$

The decision to violate the constraint $B \leq \overline{B}(0;p,0)$, by buying additional debt at the price $q=\beta p(1-\pi)$ and repaying it in case of a recovery or defaulting otherwise, does not lead to a higher expected payoff than holding the debt level constant at $B'=\overline{B}(0;p,0)$.

$$u((1-\theta)A\bar{y},\theta A\bar{y}-(1-\beta)\overline{B}(0;p,0))$$

$$+\beta(1-p)\left(\frac{u((1-\theta)A\bar{y},\theta A\bar{y}-(1-\beta)\overline{B}(0;p,0))}{1-\beta(1-p)}+\frac{\beta pu((1-\theta)\bar{y},\theta\bar{y}-(1-\beta)\overline{B}(0;p,0))}{(1-\beta)(1-\beta(1-p))}\right)$$

$$+\frac{\beta p}{1-\beta}u((1-\theta)\bar{y},\theta\bar{y}-(1-\beta)\overline{B}(0;p,0))$$

$$\geq$$

$$u((1-\theta)A\bar{y},\theta A\bar{y}+\beta pB'(\overline{B}(0;p,0))-\overline{B}(0;p,0))$$

$$+\beta(1-p)\left(\frac{u((1-\theta)AZ\bar{y},\theta AZ\bar{y})}{1-\beta(1-p)}+\frac{\beta pu((1-\theta)Z\bar{y},\theta Z\bar{y})}{(1-\beta)(1-\beta(1-p))}\right)+\frac{\beta p}{1-\beta}u((1-\theta)\bar{y},\theta\bar{y}-(1-\beta)B'(\overline{B}(0;p,0)))$$

with $B'(B)=min[\hat{B}'(B), \overline{B}(1;p,0)]$.

Is the satisfaction of those two conditions assured, the problem function leading to this optimal government policy takes the following form (Conesa/ Kehoe 2011, p. 22 f.):

$$V^g(B,a)=\max_{B,B'}((1-\theta)A^{1-a}\bar{y},\theta A^{1-a}\bar{y}+\beta B'-B)+\beta EV(B',a')$$

$$s.t. B\leq\overline{B}(0;p,0)$$

L: Gambling for Redemption – Equilibrium with Default – Algorithm

In order to calculate the optimal policy for a country willing to gamble for redemption, a policy function iteration needs to be applied. The underlying principle is a backward induction starting in the period, where the government exceeds the upper sustainable threshold and borrows at the bond price $q=\beta p(1-\pi)$, leading to a default of the country, unless the private sector of the economy recovers. Considering this way of proceeding, the labelling of the value functions is reversed the following way:

$$V(B,0)=\max_{B_{T+1}}u((1-\theta)A\bar{y},\theta A\bar{y}+\beta pB'-B)+\beta(1-p)\frac{u((1-\theta)Z\bar{y},\theta Z\bar{y})}{1-\beta}+\beta p\frac{u((1-\theta)\bar{y},\theta\bar{y}+(1-\beta)B')}{1-\beta}$$

subject to:

$$.\overline{B}(0;p,0)\leq B\leq\overline{B}(1;p,0).$$

The detailed steps of the algorithm are thereby the following:

1. First, the lower debt limit $\underline{B}$ is set, determining the interval of possible debt levels [$\underline{B}$, $\overline{B}(0;p,0)$]. Having defined this grid of bonds, the value function $V_0(B,0)$ and the policy function $B_0'(B)$ are then being solved. The analytical solution to this problem has already been provided in section 3.3.1 with $B'(B)=min[\hat{B}'(B), \overline{B}(0;p,0)]$, unless $B'(B)<\overline{B}(0;p,0)$, in which case $B_0'(B)=\overline{B}(0;p,0)$. As a result:

   $$B_0'(B)=max[\overline{B}(0;p,0),min[\hat{B}'(B), \overline{B}(1;p,0)]]$$

   The solution of this policy function shows that for future debt levels below the upper sustainable debt limit, it is not optimal to set the number of periods T before crossing the threshold to zero.

2. Set t=0 and let $\widetilde{B}_0 = \overline{B}(0; p, 0)$

3. Use the bellman's equation to solve the value $V_{t+1}(B,0)$ and the policy function $B_{t+1}'(B)$: and let thereby $\tilde{B}_t$ be the largest amount of debt, for which the future value is bigger or equal to the current value $V_{t+1}(B,0) \geq V_t(B,0)$:

$$V_{t+1}(B_t, 0) = \max u((1-\theta)A\bar{y}, \theta A\bar{y} + \beta B' - B) + \beta(1-p)V_t(B', 0) + \beta p \frac{u((1-\theta)\bar{y}, \theta\bar{y} + (1-\beta)B)}{1-\beta}$$

4. This step is repeated until $\tilde{B}_t = \underline{B}$, whereas the number of periods until crossing the upper threshold is determined by $\tilde{B}_T = \underline{B}$. It can be proven that $\underline{B} < \tilde{B}_{T-1} < \tilde{B}_{T-2} < \ldots < \tilde{B}_1 < \overline{B}(0;p,0)$. We the use of this algorithm, the interval $[\underline{B}, \overline{B}(0;p,0)]$ is subdivided in to the following subintervals: $[\underline{B}, \tilde{B}_{T-1})$, $[\tilde{B}_{T-1}, \tilde{B}_{T-2})$, …, $[\tilde{B}_1, \overline{B}(0;p,0)]$. If the initial debt level B is in the subinterval $[\tilde{B}_t, \tilde{B}_{t-1}]$, then the optimal debt policy involves gambling for redemption by selling additional debt contracts, $\overline{B}(0;p,0) < B < \overline{B}(1;p,0)$, in period $t-1$, and defaulting in period t unless the private sector recovers. The optimal sequence of debt is thereby B_0, $B_{t-1}'(B_0)$, $B_{t-2}'(B_{t-1}'(B_0))$, …, $B_0'(B_1'(\ldots(B_{t-1}'(B_0))))$ (Conesa/ Kehoe 2012, p.27 f.).

M: Code for the Calibration of the General Model in FORTRAN

```
program values


implicit none

integer,parameter:: prec=selected_real_kind(15,307)

! PARAMETERS

integer,parameter:: ndebt=1000 ! Size of debt grid

! Tecnhology and Policy Parameters
real(prec),parameter:: A=0.90 ! Output in bad (relative to good) times
real(prec),parameter:: Z=0.95 ! Output loss upon default
real(prec),parameter:: y=100.0  ! Output in good times and no default
real(prec),parameter:: theta=0.4041  ! Tax rate
real(prec),parameter:: Btop=50.0 ! Maximum level of debt in the grid

! Preference Parameters
real(prec),parameter:: gamma=0.5  ! Share of consumption in utility function
real(prec),parameter:: beta=0.97 !0.832972=0.97**6   ! Individual discount factor (in
year terms)
real(prec),parameter:: rhoc=-1.0  ! Curvature in C
real(prec),parameter:: rhog=-1.0  ! Curvature in G
real(prec),parameter:: gbar=28.0  ! Reference G

! Crisis Parameters
real(prec),parameter::  p=0.20 !0.737856=1-(1-0.20)**6   ! Probability of a recovery
real(prec),parameter:: pi=0.03 !0.167028=1-(1-0.03)**6  ! Probability of a panic
real(prec),parameter:: pi2=0.03 ! Premium charged on interest rate


! VARIABLES AND FUNCTIONS
real(prec):: BLowW,BHighW ! Thresholds in good times
```

```
real(prec):: BLow,BHigh ! Thresholds in bad times

real(prec),dimension(ndebt):: B,Bopt,BoptW ! Level of debt today and policy functions
integer,dimension(ndebt):: posBopt,posBoptW !position in the grid of policy functions


real(prec):: Vdef,Wdef       ! Value of default in bad and good times
real(prec),dimension(ndebt):: Wfun,WfunAux       ! Value function in good times
real(prec),dimension(ndebt):: Vfun,VfunAux       ! Value function in good times


! COUNTERS

integer:: cdebt,cdebt2
real(prec):: bb


real(prec):: tol=1.0e-4,val,q(ndebt)
real(prec)::  BLowNew,BLowWNew,BHighNew,BHighWNew
integer:: iter,posBLowNew


! Grid on the level of debt

B(1)=0.0
do  cdebt = 2,ndebt
  B(cdebt) = B(cdebt-1)+Btop/(ndebt-1.0)
end do

print*, 'Grid debt ', B

! Compute the values upon default
Wdef=(1.0/(1.0-beta))*U((1.0-theta)*Z*y,theta*Z*y) !In good times
Vdef=(1.0/(1.0-beta*(1.0-p)))*U((1.0-theta)*A*Z*y,theta*A*Z*y)+(beta*p/((1.0-
beta)*(1.0-beta*(1.0-p))))*U((1.0-theta)*Z*y,theta*Z*y) !In bad times
print*, ' Wdef,Vdef ', Wdef,Vdef
pause

! Initial guess of the thresholds
BLow=10.0
BLowW=15.0
BHigh=25.0
BHighW=30.0

! Compute the value function in good times and we do not default (value of the sunspot
is small)

10 WfunAux=0.0 ! Initial guess of value function

! Do interations
do iter=1,1000

 do cdebt=1,ndebt

      Wfun(cdebt)=-1.0e5
   do cdebt2=1,ndebt

    ! ZONE 1: Choose low debt tomorrow
      if ( B(cdebt2)<= BLowW ) then
      val=U((1.0-theta)*y,theta*y+beta*B(cdebt2)-B(cdebt))+beta*WfunAux(cdebt2)
        if ( val>=Wfun(cdebt) ) then
             Wfun(cdebt)=val
             posBoptW(cdebt)=cdebt2
             BoptW(cdebt)=B(cdebt2)
        else
```

```
        end if

    ! ZONE 2: Choose debt tomorrow in crisis zone
      else if ( B(cdebt2)> BLowW .and. B(cdebt2)<= BHighW ) then
!             q(cdebt2)=beta*(1.0-pi)  ! Accurate pricing of bonds
              q(cdebt2)=beta*(1.0-pi2) ! Subsidized interest rates
      val=U((1.0-theta)*y,theta*y+q(cdebt2)*B(cdebt2)-B(cdebt))+beta*(1.0-
pi)*WfunAux(cdebt2)+beta*pi*Wdef
        if ( val>=Wfun(cdebt) ) then
              Wfun(cdebt)=val
              posBoptW(cdebt)=cdebt2
              BoptW(cdebt)=B(cdebt2)
        else
        end if

      else
      end if

   end do

 end do


!      print*, 'Max error in value function iteration ',iter,maxval(abs(Wfun-WfunAux))
  if ( maxval(abs(Wfun-WfunAux))<tol ) then
  go to 50
  else
      WfunAux=Wfun
  end if

end do

50 print*, 'Convergence achieved in Wfun in iteration ',iter


! Compute the value function in bad times and we do not default (value of the sunspot
is small)
VfunAux=0.0 ! Initial guess of value function

! Do interations
do iter=1,1000

 do cdebt=1,ndebt

      Vfun(cdebt)=-1.0e5
   do cdebt2=1,ndebt

    ! ZONE 1: Choose low debt tomorrow
      if ( B(cdebt2)<= BLow ) then
      val=U((1.0-theta)*A*y,theta*A*y+beta*B(cdebt2)-B(cdebt))+&
       beta*p*Wfun(cdebt2)+beta*(1.0-p)*VfunAux(cdebt2)
       if ( val>=Vfun(cdebt) ) then
              Vfun(cdebt)=val
              posBopt(cdebt)=cdebt2
              Bopt(cdebt)=B(cdebt2)
        else
        end if

    ! ZONE 2: Choose debt tomorrow in crisis zone if no recovery
      else if ( B(cdebt2)> BLow .and. B(cdebt2)<= BLowW ) then
!             q(cdebt2)=beta*(p+(1.0-p)*(1.0-pi))  ! Accurate pricing of bonds
              q(cdebt2)=beta*(p+(1.0-p)*(1.0-pi2)) ! Subsidized interest rates
      val=U((1.0-theta)*A*y,theta*A*y+q(cdebt2)*B(cdebt2)-B(cdebt))+&
       beta*p*Wfun(cdebt2)+beta*(1.0-p)*(1.0-pi)*VfunAux(cdebt2)+beta*(1.0-p)*pi*Vdef
        if ( val>=Vfun(cdebt) ) then
```

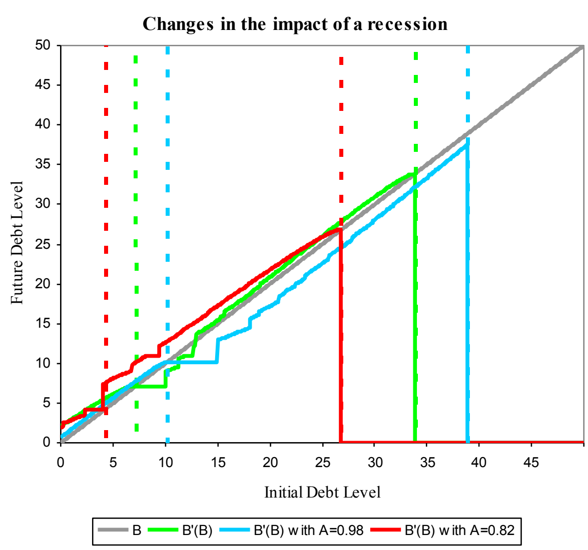
```
             Vfun(cdebt)=val
             posBopt(cdebt)=cdebt2
             Bopt(cdebt)=B(cdebt2)
         else
         end if

    ! ZONE 3: Choose debt tomorrow in crisis zone for sure
       else if ( B(cdebt2)> BLowW .and. B(cdebt2)<= BHigh ) then
!            q(cdebt2)=beta*(1.0-pi)  ! Accurate pricing of bonds
             q(cdebt2)=beta*(1.0-pi2) ! Subsidized interest rates
       val=U((1.0-theta)*A*y,theta*A*y+q(cdebt2)*B(cdebt2)-B(cdebt))+&
        beta*p*(1.0-pi)*Wfun(cdebt2)+beta*p*pi*Wdef+&
        beta*(1.0-p)*(1.0-pi)*VfunAux(cdebt2)+beta*(1.0-p)*pi*Vdef
         if ( val>=Vfun(cdebt) ) then
             Vfun(cdebt)=val
             posBopt(cdebt)=cdebt2
             Bopt(cdebt)=B(cdebt2)
         else
         end if

    ! ZONE 4: Choose debt tomorrow in crisis zone if there is a recovery and default
zone if no recovery
       else if ( B(cdebt2)> BHigh .and. B(cdebt2)<= BHighW ) then
!            q(cdebt2)=beta*p*(1.0-pi)  ! Accurate pricing of bonds
             q(cdebt2)=beta*p*(1.0-pi2) ! Subsidized interest rates
       val=U((1.0-theta)*A*y,theta*A*y+q(cdebt2)*B(cdebt2)-B(cdebt))+&
        beta*p*(1.0-pi)*Wfun(cdebt2)+beta*p*pi*Wdef+beta*(1.0-p)*Vdef
         if ( val>=Vfun(cdebt) ) then
             Vfun(cdebt)=val
             posBopt(cdebt)=cdebt2
             Bopt(cdebt)=B(cdebt2)
         else
         end if

       else
       end if

   end do

 end do

  if ( maxval(abs(Vfun-VfunAux))<tol ) then
  go to 100
  else
       VfunAux=Vfun
  end if

end do


100 print*, 'Convergence achieved in Vfun in iteration ',iter


! UPDATE THRESHOLDS

do cdebt=1,ndebt

 if ( U((1.0-theta)*A*y,theta*A*y-B(cdebt))+beta*p*Wfun(1)+beta*(1.0-p)*Vfun(1)>Vdef )
then
 BLowNew=B(cdebt)
 posBLowNew=cdebt
 else
 end if

 if ( U((1.0-theta)*y,theta*y-B(cdebt))+beta*Wfun(1)>Wdef ) then
```

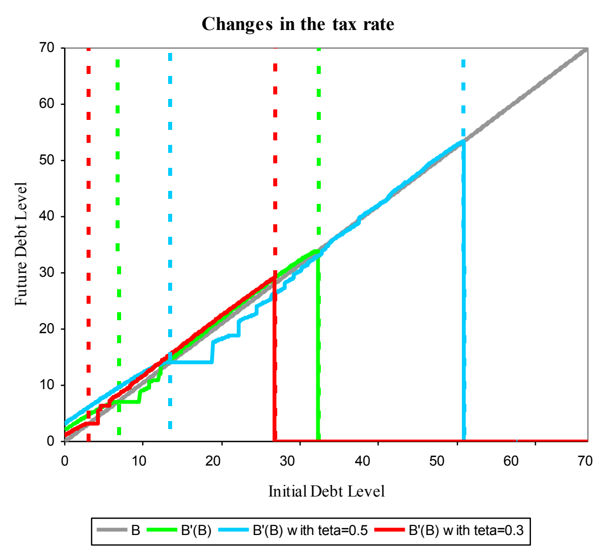
```
 BLowWNew=B(cdebt)
 else
 end if

if ( Bopt(cdebt)<=BHigh ) then
!  q(cdebt)=beta*(1.0-pi)
   q(cdebt)=beta*(1.0-pi2)   ! Subsidized interest rates
else if ( Bopt(cdebt)>BHigh ) then
!  q(cdebt)=beta*p*(1.0-pi)
  q(cdebt)=beta*p*(1.0-pi2)   ! Subsidized interest rates
end if

 if ( Vfun(cdebt)>U((1.0-
theta)*Z*A*y,theta*Z*A*y+q(cdebt)*Bopt(cdebt))+beta*p*Wdef+beta*(1.0-p)*Vdef ) then
 BHighNew=B(cdebt)
 else
 Vfun(cdebt)=Vdef
 Bopt(cdebt)=0.0d0
 end if

 if ( Wfun(cdebt)>U((1.0-theta)*Z*y,theta*Z*y+q(cdebt)*BoptW(cdebt))+beta*Wdef ) then
 BHighWNew=B(cdebt)
 else
 Wfun(cdebt)=Wdef
 BoptW(cdebt)=0.0d0
 end if

end do

! Check convergence in Thresholds
print*, 'Old Thresholds ', BLow,BLowW,BHigh,BHighW
print*, 'New Thresholds ', BLowNew,BLowWNew,BHighNew,BHighWNew

!print*, 'No Default ', U((1.0-theta)*A*y,theta*A*y)+beta*p*Wfun(1)+beta*(1.0-
p)*Vfun(1)
!print*, 'Default ', Vdef

if ( abs(BLowNew-BLow)>0.0001 .or. &
abs(BLowWNew-BLowW)>0.0001 .or. &
abs(BHighNew-BHigh)>0.0001 .or. &
abs(BHighWNew-BHighW)>0.0001 ) then
BLow=BLowNew
BLowW=BLowWNew
BHigh=BHighNew
BHighW=BHighWNew
go to 10
else
print*, 'Thresholds converged! '
end if

!print*, 'W(0),V(0) ',Wfun(1),Vfun(1)

if ( Bopt(posBLowNew)>BLowNew ) then
print*, ' YES gambling for redemption '
else
print*, ' NO gambling for redemption '
end if


! Save results to files
open(unit=10,file='Res.txt')

  do cdebt=1,ndebt
            write(10,'(5f20.4)')
B(cdebt),Wfun(cdebt),BoptW(cdebt),Vfun(cdebt),Bopt(cdebt)
```

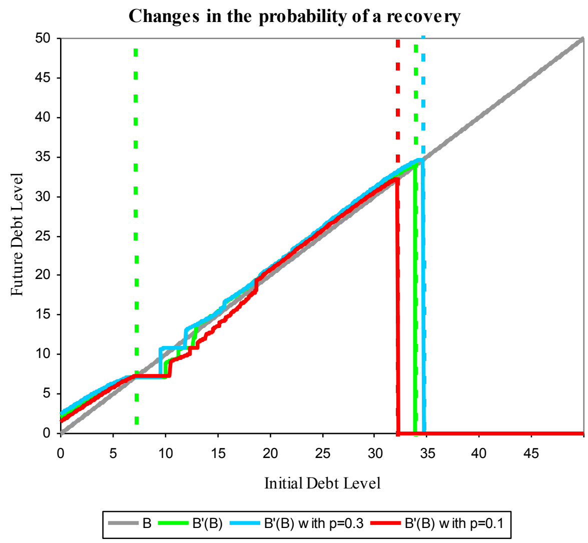
```
  end do

       write(10,*) BLow
       write(10,*) BLowW
       write(10,*) BHigh
       write(10,*) BHighW
       write(10,*) Wdef
       write(10,*) Vdef

close(10)


contains

function U(c,g)
implicit none
real(prec),intent(in):: c,g
real(prec):: U

       if (c<=0.000001) then
              U=-1.0e8
              else if ((g-gbar)<=0.000001) then
              U=-1.0e8
       else

!             U= gamma*log(c)+(1.0-gamma)*log(g)
!             U= gamma*(-c**(-2.0))+(1.0-gamma)*(-(g-0.1)**(-2.0))
              U= (c**rhoc)/rhoc+gamma*((g-gbar)**rhog)/rhog
!             U= log(c+g)
       end if

end function U

end program values
```

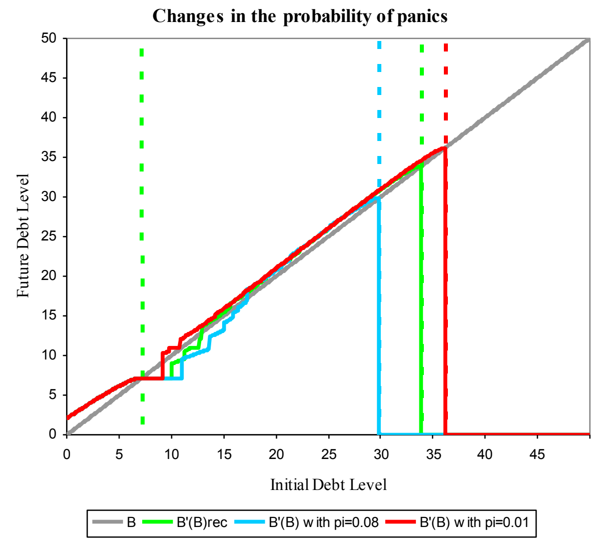
N: Sensitivity Analysis

The sensitivity analysis involves changes in the revenue loss incurred through the recession A, the probability of a crisis π, the probability of a recovery of the private sector p and the tax rate θ. All calibrations are thereby provided for the benchmark economy, suffering from a recession.









Considering first the impact of the recession on a country's revenue positions, one can see that the height of this variable is crucial for the decision over the future debt positions. The incentive to gamble for redemption is only given, when the revenue loss takes on a substantial amount. This is intuitively understandable, as a slight drop in government revenue can be counteracted through a decrease in expenditures. Is this the case, the optimal policy includes running down the debt positions and exiting the crisis zone, as the costs of suffering from self-fulfilling debt crisis want to be avoided by all means.

Another significant effect is provided through a change in the tax rate. Is the revenue share generated through taxation high, the incentive to run down the future debt dominates even for high levels of indebtedness. However, is the dependency on debt comparably high, gambling for redemption turns out to be the optimal strategy.

The influence of the probability for a panic and on the probability of a recovery in the future provides less significant results. A change in these variables indeed determines at which point in time it is optimal to follow martingale gambling strategies, but not whether to follow this strategy at all.

Basel, den 23. August 2012

Plagiatserklärung

Ich bezeuge mit meiner Unterschrift, dass meine Angaben über die bei der Abfassung meiner Arbeit benützten Hilfsmittel sowie über die mir zuteil gewordenen Hilfe in jeder Hinsicht der Wahrheit entsprechen und vollständig sind. Ich habe das Merkblatt zu Plagiat und Betrug vom 23.11.05 gelesen und bin mir den Konsequenzen eines solchen Handelns bewusst.

________________ ________________________________

(Ort), (Datum) (Unterschrift)