Uncertainty, credit and distributional effects of monetary policy in a Lagos-Wright framework

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Abstract

This study tries to analytically depict the relation between monetary policy and its distributional effects in an environment of uncertainty and with the presence of a bond market, where cash constrained agents can obtain additional resources via credit. For such purpose, due to its tractability, it is assumed a Lagos-Wright framework with 3 sequential sub-periods: a bond market, a decentralized market and a centralized market. In the first subperiod agents trade money throughout financial intermediation and, in the last 2 markets, agents trade a search and a general good, respectively. Due to the essentiality of fiat money, transfers from the monetary authority are held in the beginning of the last sub-period. These transfers can be, nevertheless, of 2 different types: High (H) - deriving an inflationary environment; and Low (L) - deriving a deflationary environment. In the case of a high transfer and if agents are not able to fully anticipate such event, it is possible to notice, in the short run, the presence of positive effects over wealth distribution between agents and over aggregates of the economy (such as production and amount of credit) which, partially, compensate the loss of welfare arising from an inflationary policy.

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1 Introduction and related literature

1.1 Money

Money and monetary policy have, for a long time, been understood as "neutral", implying that any money-oriented action would not cause any effect over real variables such as real GDP, employment and unemployment and real investment. This idea backs from David Hume and its "Quantitativist" approach during the metalist period.

In Hume's perception, there was an identity between Quantity of Money (money stock; M) times Velocity of Money (the frequency in which one unit of money is exchanged; V) and Level of Price (P) times Production (measured by the real value of final expenditures; Q) – $MV \equiv PQ$ – and whenever there was an increase in the economy's endowment of gold and silver (or any other commodity used for monetary purposes), this would be completely translated into a raise of the price level and, therefore, would have no impact over real production or the velocity in which money is transacted.

This premise is well described in the following passage contained in Robert Lucas's 1996 Nobel Prize Lecture, quoting David Hume: "It is indeed evident, that money is nothing but the representation of labor and commodities, and serves only as a method of rating or estimating them. Where coin is in greater plenty, as a greater quantity of it is required to represent the same quantity of goods, it can have no effect, either good or bad ... any more than it would make an alteration on a merchant's book, if, instead of the Arabian method of notation, which requires few characters, he should make use of the Roman, which requires a great many"[1].

This idea started to be contested by the (New) Keynesian school which, in the middle of the 1930's Great Depression, looked more carefully towards the demand for money (the Quantitativist approach was concerned about the supply of money). Based on the fact that agents might have some reasons and propensity to save (and invest) and that there might be some frictions and imperfections in the markets, what would preclude the fully adjustment of prices (for example sticky prices and menu costs), any given addition or

[1] In this same lecture, Lucas summarize Hume´s points: "These are two of Hume´s statements of what we now call the quantity theory of money: the doctrine that changes in the number of units of money in circulation will have proportional effects on all prices that are stated in money terms, and no effect at all on anything real, on how much people work or on the goods they produce or consume".

retraction of the monetary basis may not be translated solely into changes in the price level but it also might have impact over real aggregate variables such as production, investment, consumption, income and wealth, opening space for a potential monetary policy economic stimulus.

On the other hand, Monetarism, led by Milton Friedman, its keynesianist acceptance and its critics to this school ideas, proposed a new concept: in the short run, with publicly unanticipated actions, money and monetary policy can have real impacts[2]; but on the long run, as people learn and assume the measures taken by the monetary authorities and create, rationally, expectations about future policies, money have only impact over nominal variables[3].

Moreover, Monetarism proposed the fully monetary character of inflation: a raise in an economy's price level is caused uniquely by a raise of the monetary basis. And, therefore, authorities should be concerned with the supply of money and throughout this tool seek for price stability (which, as per Friedman, should be considered the major concern of central banks)[4].

So, with the development of economic theory and the appearance of new schools of thought, money started to gain space in economical discussions, especially after the fully abandonment of the Gold Standard and the rise of fiat money. Moreover, its essentiality was recognized.

Money has 4 main characteristics that make it essential within an economy:

1. It is an instrument that works as a *mean of exchange*, allowing the economy to reach trading allocations which would not be possible in its absence (for example in a barter economy) and solves the problem of double coincidence of interests (once agents have heterogeneity of tastes and goods);

[2] "Unanticipated monetary expansions, on the other hand, can stimulate production as, symmetrically, unanticipated contractions can induce depression" Robert Lucas, Monetary Neutrality, Nobel Prize Lecture, 1996.

[3] Also Baranowski (2012) explores the idea: "Because it takes time for expectations to adjust to unanticipated monetary policy, changes in the money supply have real effects in the short run while satisfying long run neutrality. A monetary injection initially increases the number of expected real balances held by agents, but as agents learn the lower value of money, expected real balances converge to steady state value"

[4] This proposal is successfully corroborated by empirical studies. McCandless and Weber (1995) plotted a 30 years (1960–1990) average annual inflation rate against average annual growth rates of M2 for the same period for 110 countries, finding a simple correlation of 0.95 (Lucas,1995).

2. It is a *reserve of value* as its intrinsic real value is, forward looking, fully recognized ("following Jevons (1875), recognizability is a key property for a good or commodity to be used as money" Nosal and Rocheteau 2011) and it is easily converted into real goods (as per Kiyotaki and Moore 2001 "the dentist accepts money, not because she wants money as such, but because she anticipates that she can use it later to buy what does she want");

3. It is *unity of account* as goods inherent values can be expressed in monetary terms, independently of the nature of the good and;

4. It assumes a technological role of *public record keeping*, fundamental in situations where agents do not trust each other, do not fully compromise and there is no other way of keeping records of previous transactions. As well defended by Kiyotaki and Moore (2001), "distrust is the root of all money".

Once inserted in the middle of a new range of theories and models, money and monetary policy soon extrapolated the spectrum of the "per se" analysis and became the focus of several works which headed to their impacts over real variables.

As explored by the monetarists, monetary policy should look forward to price stability, using as instrument the money supply. Moreover, any inflationary pressure would be the result of too much money inside the economy and, therefore, its adjustment is given via money supply. As on the short run money policy would extrapolate the nominal sphere, analysis started to focus on the supply of money and the size and variations of the monetary basis of the economy.

Following this proposal, many authors have settled different approaches (following the idea of the money supply as the driving force for inflation and deflation) for analyzing how actions taken by the monetary authority can affect the real side of the economy. One of these approaches is regarding how, in a heterogeneous environment, where agents count with different preferences, endowments, utilities and are submitted to idiosyncratic shocks, monetary policy – here represented by lump sum money injections and inflation – can affect the distribution of wealth, income, consumption and welfare between agents.

1.2 Distributional effects of monetary policy

On a short run perspective, quantitative easing has positive effects over output and wealth. A higher quantity of money, in a depressed economy, works as "oil for the engines", allowing agents to reconsider their savings, investments and consumption decisions, lowering possible frictions and asymmetries that might rise when the economy faces a downward perspective and even resurrecting frozen markets[5].

But these effects are temporary and in the long run, as inflation means devaluation of monetary holdings and terms of trade, labor choices, which in most of these models are endogenous, are affected. Changes in labor choices, on the other hand, impact on the total output, which comes back to its natural rate. Moreover, the willingness of agents to hold money balances is reduced, distorting consumption decisions and impacting the utilities of the agents. Therefore, in equilibrium, higher inflation rates generally lead to welfare decrease.

The assumption of heterogeneous agents, though, shed a light over another possible effect arising from monetary policy: the possibility of distributional effects. In such an environment, the optimality of the Friedman rule (setting the nominal interest rate to 0 as the opportunity cost of holding money should be equal to the social cost of creating additional fiat money, which is virtually 0) can be questioned and, "although inflation still serves as a distortionary tax (i.e. the traditional "real balance effect"), there is a potential welfare improving effect of redistributive expansionary monetary policy – it can mitigate the welfare costs of inflation by redistributing liquidity from agents with excess liquidity to agents that are liquidity constrained" (Chiu and Molico, 2008).

In the 2012 paper "Understanding the distributional impact of long run inflation", Camera and Chien argue that, in a money only economy (i.e.

[5] As well described in Chiu and Koeppl (2011): "During the financial crisis of 2007 to 2009, there was a stunning difference in how asset markets were affected according to their infrastructure. Markets with centralized trading functioned rather well. To the contrary, in over the counter markets – where trading takes place on a decentralized and ad hoc basis and where assets are less standardized and arguably more opaque – trading came to a halt. Most prominently, collateralized debt obligations, asset backed securities and commercial paper were traded only sporadically or not at all (see Gorton and Metrick, 2010). This prompted massive government intervention in the form of short term liquidity provision and long term asset purchases which led to a (partial) resurrection of trading in these markets over time".

there are no other kinds of assets, risky or not, used for reserve of value, mean of exchange or public record keeping), monetary policies accomplished via lump sum money injections basically redistribute income in a direct way: for those poor households whose wealth is below the average, they get a net transfer from injections; for the rich households with wealth over the average, they are pushed into a higher level of taxation by these monetary injections, which implies that the money transfer is immediately transformed into the shape of a tax. Thus a faster rate of monetary expansion, which generates a higher inflation rate, primarily lowers income inequality. The redistribution effects over income also work to reduce wealth inequality as rich households intend to save less precautionary money. This implies that the middle class shrinks with inflation and, thus, there is a lower wealth inequality.

Berentsen, Camera and Waller in the 2005 paper "The distribution of money balances and the non-neutrality of money" consider a non-degenerate money distribution and find out that an unanticipated one-time monetary injection, executed at a specific moment of time where agents hold different amounts of money (non-degenerate money distribution) and inflation is not too high, can be used to redistribute liquidity from rich to poor, redirecting consumption to those who most value it and positively affecting welfare. And, by providing consumption insurance, once an asymmetric fully anticipated lump sum transfer is held, it increases aggregate output and welfare in the high inflation economy.

Chiu and Molico (2008), corroborates the ideas shown above. Dealing with a heterogeneous environment and in the presence of idiosyncratic liquidity risk, it is argued that costs and benefits of inflation are threefold: first, as it is a distortionary tax, it reduces the willingness of agents to hold money balances and distorts consumption decisions. Second, inflation creates "deadweight loss associated with the liquidity management activities". And finally, as monetary expansion relaxes liquidity constraints and acts as a "insurance" mechanism against liquidity shocks risks, monetary policy can, potentially, be welfare increasing, compensating (but not completely) some of its costs - "in the presence of imperfect insurance the estimated long run welfare costs of inflation are on average 40% smaller compared to a complete market, representative agent economy, and inflation induces important redistributive effects across households"[6].

1.3 The financial market channel

The main purpose of this work is to study some of these distributional effects of inflation throughout a financial / credit market channel. Instead of considering a rich / poor agent, it is used a creditor / debtor framework and trades (here characterized by the financial relation lending / borrowing money) between these 2 different types are given within a bond market and intermediated by a financial agency, which is denominated "bank", responsible for receiving deposits from the lenders, issuing loans to borrowers and guaranteeing the repayment of the transaction.

As it is straightforward, considering intertemporal transactions and no-state contingent contracts, one important effect of inflationary monetary policy is the redistribution of wealth from the creditor to the debtor once, in the presence of inflation, the latter repay its debts with money holdings which do not have the same real value as before the money injection.

Literature in this topic runs, basically, in two fronts. The first one considers that financial markets and its institutions play an important role at the real economy and at the economic growth process once "technological developments alone are inadequate to promote growth. Agents are willing to tie up resources in new technologies requiring large scale investments only if the capital markets exist that make these investments sufficiently liquid" (Khan, Senhadji and Smith, 2006 on Hicks, 1969). Moreover, financial institutions are able to address endogenous frictions that are inherently present during the process of allocating credit and investment.

Here, inflation is seen as a misallocating factor that creates informational frictions and asymmetry within financial markets. Basically, as inflation means devaluation of money holdings, the incentives for savers (creditors) to behave as such are decreased. Once there is a higher incentive for agents

[6] Heterogeneous agents, non-degenerate money distribution and idiosyncratic liquidity shocks are not the only channels for positive effects arising from an inflationary monetary policy. For example, Baranowski (2012) makes use of uncertainty and adaptive learning dynamics and reach the conclusion that "the short run benefits of a change in the money growth rate imply a lower social cost to the money growth policy. When taking into account the learning dynamics, the welfare cost of ten percent money growth is much lower than previous estimates and, depending on the learning speed, there can be social benefits. Alternatively, a disinflationary policy of decreasing the money growth rate from 10 percent to 0 percent is costly in the short run".

to act as something that they are not (e.g. lender acting as borrower), the system starts to face adverse selection, moral hazard, costly state verification, increasing risk perception, misleading asset valuations and, ultimately, restriction of capital / credit markets. Accordingly to Khan, Senhadji and Smith (2006), "intermediaries must limit the amount that any agent can borrow because if they did not natural lenders would be tempted to misrepresent their type, obtain borrowed funds, misallocate them and default on loans with high probability".

Still following the ideas of Khan, Senhadji and Smith (2006), these effects would have a negative impact over the financial depth of the economy, here calculated by the sum of bank lending to the private sector, measures of stock market capitalization and trading volume, measures that aggregate both bank lending and stock market activity and measures that aggregate bank lending, stock markets and bond markets. Particularly, higher inflation leads to increasing credit rationing and endogenous volatility, and therefore, to lower investment and slower growth.

This front of models is also marked by the existence of non-linearities and thresholds. Accordingly to Boyd, Choi and Smith (1996) "high rates of inflation tend to depress the real returns received by equity holders and to increase their variability. But in order to adversely affect the financial system, inflation must be high enough".

Corroborating this idea, Khan, Senhadji and Smith (2006) suggest that, empirically based, "increases in the rate of money growth in economies that have initially low rates of money creation appear to increase the long run level of real activity. But permanent increases in the rate of money growth in economies with initially high rates of money growth have detrimental consequences for long run real activity". This same paper provides the range for which inflation changes its impact over the real economy (thresholds): for "developed" countries, it runs from 1-3% inflation; for "developing" countries, it runs from 11-12%. But, still, the transmission channel inflation – real activity is via financial markets and, therefore, for initial rates of inflation below the threshold, any small increase in the inflation rate have no significant or even a positive effect over financial markets conditions. But once the initial inflation is above the threshold, any small increase on it has strongly negative effects on financial conditions.

The second front of models is marked by the insertion of micro-foundations and intertemporally optimizing agents. Here it is possible to find the "Lagos-Wright" framework. This set up tries to study the search for money based on explicit micro-foundations, quasi-linear preferences and an initially degenerate money holdings distribution.

As per Nosal and Rocheteau (2011), "Lagos and Wright (2005) introduced competitive markets that operate periodically and quasi linear preferences. The competitive markets allow agents to adjust their money holdings following random-matching shocks. Since quasi-linear preferences eliminate wealth effects, all agents will make the same choices in the competitive market, except for the choice of the "quasi-linear good". The Lagos-Wright environment can accommodate different pricing mechanisms in the decentralized exchange market (Rocheteau and Wright 2005), such as bargaining, price posting and Walrasian pricing. Moreover the existence of periodic competitive markets allows for the reintroduction of Arrow-Debreu-type general equilibrium apparatus, such as state contingent commodities".

Although Lagos-Wright is a useful tool for inserting money in monetary policy models, in its original proposition it precludes the discussion of the impacts of policies over the real economy and its distributional effects. This is due to the sum of the 2 main assumptions of the model: 1) between each of the decentralized markets exist centralized markets where agents are able to rebalance their money holdings, what results on degenerate money distribution and; 2) preferences are quasi-linear, meaning that there are no wealth effects in the demand for money. Therefore authors who make use of this model often have to loosen or even abandon some of its assumptions[7].

One possible path to study the impacts of inflation on real variables throughout the financial sector in a micro-founded macro framework is allowing for borrowing constraints (impossibility of insuring against productivity shocks), whose presence can amplify the effect of shocks when compared to a complete markets benchmark.

As per Scheinkman and Weiss (1986) and Lippi and Trachter (2012), when agents face the inability to fully insure themselves against adverse productivity shocks, they become motivate to hoard some quantity of financial assets for intermediating intertemporal trade. This is the "precautionary savings" motive and has "implications for both asset pricing and aggregate

[7]This is the case of, for example, Berentsen, Camera and Waller (2005) and Chiu and Molico (2008). The first, in order to study symmetric / asymmetric money injections and their effects over real aggregates "assume two rounds of trade before agents can rebalance their money holdings". The second drops the assumption of quasi-linearity of preferences, allowing for diminishing marginal value of money and a non-degenerating monetary distribution.

employment and output".

Therefore, once individuals are not able to issue private debt (they can not borrow), there is space for analyzing the planner, the one that provides the necessary liquidity for the system, and which economical features impact and are impacted by the planner's decisions. So, here, the question is: what is the optimal money (liquidity) supply and how to regulate it?

Lippi and Trachter in the 2012 paper "On the optimal supply of liquidity with borrowing constraints" allow for the presence of heterogeneity between agents, uncertainty and borrowing constraints. These features of the economy have impacts over the best monetary policies to be taken by the planner, which, in this specific case, is the government controlling the system's liquidity through lump sum transfers. More importantly, such characteristics of the economy drive monetary policy away from the Friedman's rule.

Accordingly to the paper, there is a large mass of infinitely lived rich and poor agents, and any agent is unable to infer the other agent's type. Both types are equally submitted to productivity shocks (random oscillations) that might turn them into productive, who transforms labor into consumption one to one; or unproductive agents, who can not produce (the model assumes that "types are perfectly negatively correlated so in each instant only one type is productive"). These shocks happen with a determined frequency and therefore the inverse of this frequency can be denominated as "productivity spells" or the mean duration of each production period.

The planner is able to observe the wealth distribution of the economy between productive and unproductive agents (but not between rich and poor agents). Due to borrowing constraints, he has space to work through liquidity injections / contractions which will depend on the aggregate state of the economy (expansion or recession), economical information availability, productivity spells and money distribution, in order to achieve better allocations and satisfy insurance motives via money injection (when the unproductive agents have a lower wealth share, insuring them for intertemporal consumption and tax payments) or production motives via money retraction (when the unproductive agents have a higher wealth share, increasing their incentive to produce as liquidity contraction increase money return).

So the authors were able to: endogeneize money supply decisions (the planner has to provide liquidity to the system based on the system's necessities) and wealth distribution ("dynamics of wealth distribution as a function of liquid asset growth and the history of shocks"), determine a relation between these two features[8] and study impacts over real aggregates[9].

Another possible exercise for analyzing effects of monetary policy throughout a credit channel taking into account micro-foundations and agents' behavior is to assume the Lagos-Wright model with the insertion of a third sub-period, a centralized market, where money trading is given following a Walrasian price taking protocol - bond market. This market takes place before the periods of goods trading (DM and CM) and trades are intermediated by a financial agency (bank), who is able to assume liquidity from those who do not need it immediately and provide it to those who are cash constrained and, more importantly, keep record of each transaction. This was the procedure taken in Berentsen, Camera and Waller (2007) and it is here expanded by the insertion of uncertainty.

In this set up, agents are, in the beginning of each period, submitted to a preference shock defining which agents are willing to consume the search good and which are going to produce it. Both of the types bring the same amount of money from the previous period (degenerate money distribution). However the total amount brought does not fully compensate the cost of production of the search good.

After the shock, agents look for the financial intermediary. The ones who want to consume (are cash constrained), look forward to obtain loans (borrow money). The ones who hold idle balances (do not want to consume), on the other hand, look forward to deposit the excess liquidity. The intermediary is the "middle man". It provides money to the first type agents, throughout the issuance of an IOU contract, certifying the repayment of the loan; and promises the second type agents their deposits in the end of the period with a premium over it. In this sense it is possible to see the creation of a bond market, making possible a better final allocation of the economy by providing liquidity to who values it the most and hold a higher marginal value for consumption.

[8] "Since the liquid asset growth rate affects the distribution of wealth, policy has real effects which involve two important margins: the first one is that an increase in the supply of liquidity provides insurance to agents who incur in a long spell of unproductive periods (who end up having low liquidity and low consumption). The second margin is the classic cost of inflation: an expansionary policy lowers the return on liquid assets, lowering productive agents´ incentives to save and produce" (Lippi and Trachter, 2012).

[9] "It follows that trade volumes, aggregate production, the asset price and the risk premium depend on the distribution of wealth, which evolves through time following the history of shocks" (Lippi and Trachter, 2012).

By assumption, goods trading takes place only by the exchange of fiat money. This comes from the fact that agents are anonymous and there is no other technology for keeping track of their goods trading histories. Therefore, by the beginning of each last market, the monetary authority acts via lump sum money injections (or contractions). So it is possible to notice the presence of both money and credit, what would not be possible in the original Lagos-Wright framework.

A novel in this study is that agents are uncertain about the type of policy that is going to be held in the end of the period. This uncertainty is going to rule trades in both two first sub-periods for which equilibrium and optimality conditions are going to be based on the expected value of money.

Due to the laws of motion that money transfers, money growth rate and price of money follow, exists a high inter-temporal correlation between monetary policy in actual period and trades in the next period. Moreover, monetary policy also impacts actual period's last market, when it is held (by assumption, money transfers are held by the beginning of the centralized market). Therefore money policy, in the presence of uncertainty, is a useful tool for achieving results over real aggregates and wealth distribution. However, agents are assumed to be rational and, moreover, to face an adaptive learning process. Therefore, in the long run, monetary policy loses its efficiency as agents are no longer going to be "surprised", meaning that they are able to anticipate the policy, based on last period's state, and create correct expectations.

1.4 Overview of the analysis

The work proceeds as follows: In section 2 the environment, laws of motion of money transfers and money growth rate, instantaneous utility functions, agents' value functions, maximization problems and equilibrium conditions for each sub-market are defined.

In section 3 it is given the discussion between uncertainty, discretionary monetary policy and aggregate and distributional effects. For this instance, it is traced an inter-temporal comparison between both equilibrium and optimality conditions in period t and period $t+1$, considering the 2 possible cases: when the state in period t is high (H), reflecting a "higher than the previous period" money growth rate, and when it is low (L), reflecting a "lower than the previous period" money growth rate.

The last section concludes the study.

2 The model

2.1 The environment

As described above, the main motivation of this model is to study how distributional effects can arise from monetary policies (lump sum money injections) throughout a loan channel in a model where future monetary policy is uncertain.

Following Berentsen, Camera and Waller (2007), the first difficulty to be faced is to address the coexistence of both credit and money in the same environment. The essentiality of money requires the inexistence of a public record keeping technology in the economy, however, by definition, for credit to exist it is necessary the existence of record keeping.

So, to surpass this difficulty, it is assumed that credit trades in the first sub-period, the bond market, are intermediated by an institution capable of issuing and holding 2 different no-state-contigent debt contracts: one, an IOU, is signed by the borrower and determines the conditions of the loan transaction and enforces the repayment of the amount borrowed; the other one is a "demand deposit contract", signed by the depositor and defines the obligation of the intermediary to repay, by the end of the period, the total amount deposited plus a premium over it.

However, the existence of these contracts do not preclude money essentiality in the goods trade. Once goods exchanges happen between anonymous independently matched agents, who do not have any access to commodity trades history among themselves, it is necessary a mean of exchange surpassing trusting issues - money.

Due to the essentiality of money in this set up and the possibility of credit, it is possible to affirm that the model here developed surpass the difficulty of summing credit and money in the same framework.

As already discussed, the work evolves based on Lagos-Wright model: it counts with explicit micro-foundations and leaves behind extreme restrictions on how much cash agents can hold. It also has as basic groundwork, as per Nosal and Rocheteau (2011), periodically competitive markets and quasi-linear preferences. The existence of competitive markets allows agents to adjust money holdings and, due to quasi-linear preferences, which eliminate wealth effects, agents make the same choices in the competitive market (except for the choice of the “quasi-linear good”). These attributes of the model bring easier tractability and the possibility to quantify macro and monetary policies.

The model counts with a continuum of long lived agents divided into buyers (borrowers) and sellers (lenders). These agents discount the future at rate $\beta \in (0, 1)$, trade fiat money (divisible and storable) through financial intermediation, a search good and a general good (both divisible and non-storable) in, respectively, 3 different and sequential sub-periods: a Bond Market (BM); a Decentralized Market (DM); and a Centralized Market (CM):

Bond market: at the beginning of the bond market agents receive a preference shock such that, with probability η, they can consume but can not produce in the DM. On the other hand, with probability $1 - \eta$ they are not willing to consume but have the resources and conditions to produce. We refer to consumers as "buyers" and to producers as "sellers".

Buyers and sellers begin the period with m money holdings brought from period $t - 1$ (at the beginning of the period, money is degenerate). By assumption, a seller holds idle balances (as he is not spending it in the DM) and a buyer is resource constraint (the amount of money with which he starts the period, m, does not satisfies his demand for the search good). So, as the buyer is the one that most value consumption in next sub-period, allowing for bond trades improves the obtained allocation of the economy by redistributing liquidity to the one that needs it the most.

Trades are intermediated by a financial agency, a "perfectly competitive firm who accept nominal deposits and make nominal loans" (Berentsen, Camera, Waller 2007) throughout the price r (interest rate), who holds a record keeping system - a bank[10].

Once an agent recognizes himself as a seller, he looks for the bank to deposit his idle balances. His incentives to do it so are the following: as he has no value for consumption in DM, he owns additional liquidity which is not going to be used until the last sub-period, CM; in an inflationary environment, keeping this liquidity "under the mattress" does not protect him from money devaluation as does the demand deposit contract. In a non-inflationary environment, the demand deposit contract provides him a premium for postponing liquidity, what the "mattress" does not provide. Moreover, the bank is a trustworthy agent, with no possibility of default.

On the other hand, a buyer looks for the bank to obtain liquidity: he

[10] As in Berentsen, Camera and Waller (2007) this agency is a "bank" – "financial intermediaries who perform these activities – taking deposits, making loans, keeping track of credit histories – are classified as "banks" by regulators around the world".

receives from the agency the seller's deposited money - a loan (l_t) - and gives in exchange a one period no-state-contingent IOU contract, which is redeemable by the end of the last sub-period CM (due to simplicity reasons, it can not be rolled over). It is important to highlight that these loans have an upper bound: d_t, the amount of money deposited by the seller, with $d_t \leq m$. And by assumption this IOU is non-tradable, leaving no space for inside money in the system (the only mean of exchange used is fiat money, coming from monetary injections done by the authority at each CM).

It is also assumed that agents are long-lived, have a high-enough β (they have high valuation of the future in comparison to the present) and the payoff of acting correctly (repaying the assumed loan) is higher than the one of not doing it so. Therefore they don't have incentives to default (what would mean being out of the system).

Moreover the quantity of loans required by the buyer (and therefore his consumption in the DM) is ruled by his expectations regarding the money growth of the economy, which can be high (H), with probability π, and low (L), with probability $1 - \pi$.

Decentralized market: agents meet anonymously in large groups and take price as given (Walrasian price taking protocol). A seller is able to produce a search good which is going to be bought by a buyer in a spot monetary transaction by the nominal price p. His problem is to choose optimally the quantity q_s to supply. Sellers derive a disutility $c(q_s)$ for the production of the good.

A buyer, who derives utility $u(q)$ from the consumption of q units of the good, pay pq units of fiat money (with $pq \leq m + l_t$). His problem is to define how much money to borrow in the BM, or, equivalently, how much of the search good to consume.

As in Berentsen, Camera and Waller (2007) there is a motivation for the use of fiat money coming from the fact that all good trades are anonymous. So the "trading histories of agents are private information and sellers require immediate compensation so buyers must pay with money".

Centralized market: in the last market every agent can apply y units of labor into a production technology that transforms labor in product in a proportion $1 : 1$, resulting in y units of general good, and consume x units of this good.

The monetary authority makes equally distributed lump sum money injections τ in the beginning of every CM, deriving a real value ϕ for money (ϕ can also be thought as the price of money and $\frac{1}{\phi}$ is the nominal price of general goods).

After the injection, buyers pay $(1+r)\,l_t$ to the financial intermediary for the l_t units of loans acquired and the bank pays $(1+r)d_t$ over the deposits d_t made by the sellers at the beginning of the period.

Agents are able to rebalance and decide how much money holdings m' they take to next period $t+1$. The motivation for taking some money holdings to next period is based on the fact that agents are not sure about the type they are assuming in the next period. In matter of fact, such uncertainty is one of the main reasons for the existence of a bond market: if agents continue with the same type assumed in a determined period forever, liquidity needs for upcoming DM sub-periods are satisfied via production in the CM.

Each agent derives the following instantaneous utility in period t.

The Buyer's instantaneous utility is:

$$v\,(l_t) + u\,(q) + x - y \tag{1}$$

Seller's instantaneous utility:

$$-\psi\,(d_t) - c\,(q_s) + x - y \tag{2}$$

where $v\,(l_t)$ and $-\psi\,(d_t)$ stands, respectively, for the utility of borrowing l_t (basically, the possibility to consume the search good) and the disutility of depositing d_t units of money (giving up d_t units of liquid assets)[11]. We assume that there are solutions l_t^* and q^* for which, respectively, $l_t = d_t = l_t^*$, $v'\,(l_t) = \psi'(d_t)$ and $q = q_s = q^*$, $u'\,(q^*) = c'(q^*)$. Or, in words, there are optimal quantities of loans and search good for which BM and DM are in equilibrium and the sets of incentive feasible allocations are maximized (the marginal benefit of acquiring one more unit of loan / search good is equal to the marginal cost of depositing one more unit of money / producing another search good).

2.2 Expectations and laws of motion

Agents in the beginning of each period create expectations regarding money transfers (τ_t), money growth rate (γ_t) and price of money (ϕ_t) to be held in

[11] By assumption, $v\,(.)$, $\psi\,(.)$, $u\,(.)$and $c(.)$ are twice continuously differentiable with $v'\,(.) > 0$, $\psi'\,(.) > 0, u'\,(.) > 0, c'\,(.) > 0, v''\,(.) < 0, \psi''\,(.) \geq 0, u''\,(.) < 0, c''\,(.) \geq 0$.We also assume that $v\,(0) = \psi\,(0) = u\,(0) = c\,(0) = 0$.

the end of the period, with M_t standing for money basis of the economy in period t. These expectations are going to rule equilibrium in the two first sub-periods of the economy.

Agents believe that, with probability π the monetary authority is going to make a positive injection of money (H), resulting in a total supply of money $M_t > M_{t-1}$ (or in a money growth rate $\gamma_t > 1$); with probability $1-\pi$ agents believe that the monetary authority is withdrawing liquidity from the system (L), resulting in a total supply of money $M_t < M_{t-1}$ (or in a money growth rate $0 < \gamma_t < 1$).

So, by definition[12]:

Expected money transfers held at period t:

$$E_t(\tau_t) = \pi\tau_t^H + (1-\pi)\,\tau_t^L, \text{ with } \tau_t = \frac{M_t - M_{t-1}}{M_{t-1}} \tag{3}$$

$$\tau_t^H > 0 \text{ and } \tau_t^L < 0$$

Expected money growth rate at period t:

$$E_t(\gamma_t) = \pi\gamma_t^H + (1-\pi)\,\gamma_t^L, \text{ with } \gamma_t = \frac{M_t}{M_{t-1}} \tag{4}$$

$$\gamma_t^H > 1 \text{ and } 0 < \gamma_t^L < 1$$

Expected price of money at period t :

$$E_t(\phi_t \mid H, L) = \pi\phi_t^H + (1-\pi)\,\phi_t^L \tag{5}$$

$$\phi_t^H < \phi_{t-1} \text{ and } \phi_t^L > \phi_{t-1}$$

for which a high/low money growth rate gives, respectively, a low/high price of money.

Also, expectations for price of money are conditioned to the state that the economy is living. This comes from a sequence of events: first money

[12] It is important to define that, respectively, τ_t^H and γ_t^H and τ_t^L and γ_t^L are money transfers and money growth rate in the H and L state. Moreover, $\tau_t^L \leq E_t(\tau_t) \leq \tau_t^H$ and $\gamma_t^L \leq E_t(\gamma_t) \leq \gamma_t^H$. Regarding the price of money, due to its inverse relation to the money growth rate: $\gamma_t = \frac{M_t}{M_{t-1}} = \frac{\phi_t}{\phi_{t-1}}$, we have $\phi_t^H \leq E_t(\phi_t) \leq \phi_t^L$.

is injected and it creates the state of the economy. Then, price of money is shaped accordingly to which state is realized[13].

It is important to define the law of motion for money growth rate is submitted. From (4), $M_t = \gamma_t M_{t-1}$ where γ_t is a random variable that follows the process below (as in Berentsen, Camera and Waller 2005):

$$\gamma_t = \begin{cases} \gamma_t^H = (1+\varepsilon^H) \\ \gamma_t^L = (1-\varepsilon^L) \end{cases} \text{with probabilities} \begin{cases} \pi \\ 1-\pi \end{cases} \tag{6}$$

By assumption,

$$\varepsilon^H \text{ and } \varepsilon^L > 0 \text{ and } \pi = \frac{\varepsilon^L}{\varepsilon^L + \varepsilon^H}$$

therefore:

$$\begin{aligned} E_t(\gamma_t) &= 1; \\ E_t(\tau_t) &= 0 \\ \text{and } E_t(\phi_t) &= \phi_{t-1} \end{aligned}$$

following that the high state is inflationary and the low state is deflationary.

Attention is focused on stationary equilibrium, where by the end of each period, real money balances are the same (time invariant):

$$\phi_t M_t = \phi_{t-1} M_{t-1}$$

Next it is given a detailed description of each sub-period, deriving also its equilibrium conditions. For tractability the analysis starts with the last sub-period, the centralized market.

2.3 The centralized market

During the CM both agents are able to produce and consume a general good. It is assumed that, due to the necessity of fiat money in this economy, at the beginning of the market the monetary authority injects money in the system via lump-sum transfers that follow the process referred before.

After the transfers are held, IOU´s and demand deposit contracts signed in the BM are redeemed throughout the payment of interests at rate r. Also,

[13] One could condition price of money on transfers instead of states. It provides the same results as transfers define the state.

agents are able to adjust their money holdings and decide how much to take to the next period.

For a buyer and a seller entering, respectively, the CM with $z^b = (m + l_t - pq)$ and $z^s = (m - d_t + pq_s)$ units of money and planning to take $m^{'}$ units of money to the next period, we have:

Buyer´s program is:

$$W_{b,t}\left(z^b, l_t\right) = \max_{x,y,m'} \left[x - y + \beta B^e_{b,t+1}\left(m^{'}\right)\right] \quad (7)$$

$$s.t.x + (1 + r_t)\,\phi^i_t l_t + \phi^i_t m' = y + \phi^i_t z^b + \phi^i_t \tau^i_t M_{t-1} \quad (8)$$

where $\beta B^e_{b,t+1}\left(m^{'}\right)$ stands for the present value of his expected value function in the next sub-period, BM, for the next period $t+1$, once the agent brings $m^{'}$ money units to the next period.

So, the buyer finances his general good consumption, the payment of the loan assumed during the BM and his decision of how much real money holdings to take to the next period $\left(x, (1 + r_t)\,\phi^i_t l_t, \phi^i_t m^{'}\right)$ by the production of the general good, how much real holdings does he take to CM after DM and the real value of the received transfers $(y, \phi^i_t z^b, \phi^i_t \tau^i_t M_{t-1})$[14]. It is important to highlight that transfers, money growth rate and price of money are indexed by i, representing the two possible states of the economy, high or low $(i \in (H, L))$.

Rewriting the constraint in terms of $(x - y)$ and substituting it into the maximization function (7) we have that the buyer's problem reduces to:

$$W_{b,t}\left(z^b, l_t\right) = \phi^i_t z^b + \phi^i_t \tau^i_t M_{t-1} - (1 + r_t)\,\phi^i_t l_t + \max_{m^{'}} \left[-\phi^i_t m^{'} + \beta B^e_{b,t+1}\left(m^{'}\right)\right] \quad (9)$$

for which the maximizing term refers now only to how much money holdings is the buyer willing to take to next period $m^{'}$.

Not so different, the seller's problem is also to maximize his lifetime utility subject to his budget constraint. But, here, he is receiving the payment of the interest rate over his deposits signed in the BM, what relax his constraint:

$$\begin{aligned} W_{s,t}\left(z^s, d_t\right) &= \max_{x,y,m^{'}} [x - y + \beta B^e_{s,t+1}\left(m^{'}\right)] && (10) \\ s.t.\ x + \phi^i_t m^{'} &= y + \phi^i_t z^s + \phi^i_t \tau^i_t M_{t-1} + (1 + r_t)\,\phi^i_t d_t && (11) \end{aligned}$$

[14] As transfers are equally distributed, for notation purposes, instead of considering $\frac{\tau}{2}$ for the transfer to each agent, they receive $\tau_b = \tau_s = \tau$ and $\tau_b + \tau_s = \tau_t$

So, he finances his general good consumption and his decision of how much real money holdings to take to the next period $(x, \phi_t^i m^{'})$ by the production of the general good, how much real holdings he takes to CM, real transfers and the real value of the payment of interest over its deposits.

Again, rewriting the constraint in terms of $(x - y)$ and substituting it in equation (10) the seller's problem reduces to:

$$W_{s,t}\left(z^s, d_t\right) = \phi_t^i z^s + \phi_t^i \tau_t^i M_{t-1} + \left(1 + r_t\right) \phi_t^i d_t + \max_{m^{'}} \left[-\phi_t^i m^{'} + \beta B_{s,t+1}^e \left(m^{'}\right)\right] \tag{12}$$

Taking FOCs for equations (9) and (12) for $m^{'}$:

Buyer $m^{'}$ FOC:

$$B_{b,\ t+1}^{'e}\left(m^{'}\right) = \frac{\phi_t^i}{\beta} \tag{13}$$

Seller $m^{'}$ FOC:

$$B_{s,t+1}^{'e}\left(m^{'}\right) = \frac{\phi_t^i}{\beta} \tag{14}$$

So, the agents' optimal decisions of how much money holdings to take to the next period in order to maximize bond market's expected value function is independent of the agent's current type and portfolio. But it is still state dependent: agents' maximization problem still depends on the period's price of money ϕ_t^i which, on the other hand, is state dependent.

So, regardless the type of the agent, given uncertainty about the preference shock and liquidity necessities for next period, both have incentives to bring the same amount of money, originating a degenerate money distribution by the beginning of BM in $t+1$. Once the preference shock is given, those with purchasing constraints search for the financial intermediary to obtain money deposited by those with idle balances.

Different from the original Lagos-Wright proposal, where the seller has no incentive to bring any money holding to the next period as he does not derive utility from consumption in the DM, in this particular framework the seller today is encouraged to bring money holdings for not knowing his necessities and constraints in the future.

Moreover, agents face a trade-off regarding consumption today and the devaluation of money (coming from the monetary basis growth). Agents are indifferent between bringing one additional unit of money to the next period or spending it in consumption in current's period CM when today's expected marginal utility provided by one additional unit of money in the next period

$(\beta B'^{e}_{b,s,\ t+1}\left(m'\right))$ is equal to the value of today's money holdings, indexed by the 2 possible states (ϕ^i_t).

Following Berentsen, Camera and Waller (2007), in order to ensure that all loans can be repaid in the end of the centralized market, it is defined:

$$\omega_t = \frac{\eta(1+r)l_t}{M_t}$$

where ω_t is the ratio between the aggregate nominal loans repayments, $\eta(1+r)l_t$, and the aggregate money supply M_t.

If $\omega_t \leq 1$ then borrowers' cash nominal demand for repayment of loans is less than the money supply. However if $\omega_t > 1$ then buyers "can´t acquire sufficient balances in the aggregate to repay loans at once. This implies that they repay part of their loans which is then used to settle deposit claims and the cash reenters the goods markets as depositors use the cash to acquire more goods. This recycling of cash occurs until all claims are settled" (Berentsen, Camera and Waller 2007).

2.4 The decentralized market

After the preference shock and the trade of loans, agents start production and consumption of goods. In the DM, buyers and sellers, who are anonymous, meet in large groups in a competitive market with a Walrasian price taking pricing protocol. Since agents are anonymous, credit is not possible and, therefore, money is essential. The nominal price of the search good is defined as p.

So, a quantity q of search good is traded by pq money units, and by feasibility, buyer has to carry enough money holdings to pay for its consumption:

$$pq \leq m + l_t \tag{15}$$

As already explored, trade is spot and uses fiat money as mean of exchange for solving the double coincidence problem.

Assuming no informational asymmetry, it is known by all agents that in the end of CM there will be money injection. So, in the BM of the period t, agents create expectations over transferences and the money growth rate (and price of money) in period t. These expectations are going to rule trades in this economy. For analyzing optimality in the DM, it is considered that the main gear for uncertainty comes from the planner's transfers in the two different states (and, therefore, value functions are conditioned on τ_t^i).

For a buyer entering the DM with m money holdings and l_t units of loans, the expected DM value function, conditioned on the end of period transfer τ_t^i is:

$$\begin{aligned} V_{b,t}^e\left(m, l_t, \tau_t^i\right) &= \pi[u\left(q\right) + W_{b,t}(m + l_t - pq, \tau_t^H)] + \\ & (1-\pi)\left\{[u\left(q\right) + W_{b,t}(m + l_t - pq, \tau_t^L)]\right. \end{aligned} \tag{16}$$

which, applying expectations, resumes to:

$$V_{b,t}^e(m, l_t, \tau_t^i) = u(q) + W_{b,t}^e(m + l_t - pq) \tag{17}$$

It is important to highlight that the equation above considers the expected CM value function $W_{b,t}^e(.)$ instead of the value function in CM $W_{b,t}(.)$. Such feature arise from the insertion of uncertainty into agents' optimality conditions.

Similarly, the sellers' expected value function is:

$$\begin{aligned} V_{s,t}^e\left(m, d_t, \tau_t^i\right) &= \pi[-c\left(q_s\right) + W_{s,t}(m - d_t + pq_s, \tau_t^H)] + \\ & (1-\pi)\left[-c\left(q_s\right) + W_{s,t}(m - d_t + pq_s, \tau_t^L)\right] \end{aligned} \tag{18}$$

Following the same process applied to the buyers, equation (18) resumes to:

$$V_{s,t}^e\left(m, d_t, \tau_t^i\right) = -c\left(q_s\right) + W_{s,t}^e\left(m - d_t + pq_s\right) \tag{19}$$

Again,value function for the CM turns to expected value function.

Sellers want to solve the following problem:

$$\max_{q_s}\{-c(q_s) + W_{s,t}^e\left(m - d_t + pq_s\right)\} \tag{20}$$

which, by linearity[15] and expectations regarding price of money, turns into:

$$\max_{q_s}\{-c(q_s) + W_{s,t}^e(0) + [E_t(\phi_t)](m - d_t) + [E_t(\phi_t)](pq_s)\} \tag{21}$$

and, simplifying,

$$W_{s,t}^e(0) + [E_t(\phi_t)](m - d_t) + \max_{q_s}\{-c(q_s) + [E_t(\phi_t)](pq_s)\} \tag{22}$$

[15] As per Nosal and Rocheteau (2011), the CM value function is linear on money holdings: $W(m) = W(0) + \phi m$

Taking first order conditions of the Lagrangian function, we have:

$$c^{'}(q_s) = p[E_t(\phi_t)] \tag{23}$$

meaning that sellers supply search good until the marginal disutility of production is equal to the expected real price of the DM good. Or as in Berentsen, Camera and Waller (2007), "sellers produce such that the ratio of marginal costs across (goods) markets $\left(\frac{c^{'}(q_s)}{1}\right)$ is equal to the (expected) relative price $p[E_t(\phi_t)]$ of goods across (goods) markets". And the decision of how much to produce is the same for each seller and it is independent of their money holdings.

On the other hand, the buyer's problem is:

$$\max_q \{u(q) + W^e_{b,t}(m + l_t - pq)\} \tag{24}$$

subject to the feasibility condition for payments (equation 15) and the constraint over the loans size:

$$\max_q \{u(q) + W^e_{b,t}(m + l_t - pq)\} \tag{25}$$

$$s.t. pq \leq m + l_t$$

$$l_t \leq \ell_t$$

where ℓ_t is the maximum amount to be obtained in loans such that the repayment is still compatible with its incentives. As per Berentsen, Camera and Waller (2007) this constraint is "intended to capture the fact that there is a threshold ℓ_t above which an agent would choose to default". For simplicity, it is assumed that $\ell_t \to \infty$, therefore the second constraint is always satisfied.

Applying linearity and expectations, equation (25) becomes:

$$W^e_{s,t}(0) + [E_t(\phi_t)](m + l_t) + \max_q \{u(q) - [E_t(\phi_t)](pq)\} \tag{26}$$

Taking first order conditions:

FOC for q :

$$u^{'}(q) - p[E_t(\phi_t)] - \lambda p = 0$$

FOC for λ :

$$pq = m + l_t$$

and simplifying FOC for q:

$$u^{'}(q) = p[E_t(\phi_t) + \lambda] \tag{27}$$

from where it is possible to notice that buyers consume until the marginal utility of the search good is equal to the real payment for it. It is possible also to notice the presence of a wedge λ between the buyer's marginal utility of consumption of search good and the real expected price of the search good $p[E_t(\phi_t)]$.

Considering equation (23), (27) is:

$$u^{'}(q) = c^{'}(q_s)[1 + \frac{\lambda}{E_t(\phi_t)}] \tag{28}$$

The clearing condition for the DM requires

$$q = q_s$$

Dividing (27) by (23) and assuming clearing condition,

$$\frac{u^{'}(q)}{c^{'}(q)} = \frac{p[E_t(\phi_t) + \lambda]}{p[E_t(\phi_t)]}$$

which turns to:

$$\frac{u^{'}(q)}{c^{'}(q)} = 1 + \frac{\lambda}{E_t(\phi_t)} \tag{29}$$

For a $\lambda = 0$ and no inflation, the first best allocation is achieved by the Friedman rule $r_t = 0$. Applying this policy brings efficiency in trades, for which it is given $q = q^*$ and

$$u^{'}(q^*) = c^{'}(q^*) \tag{30}$$

It is important to highlight that "at the Friedman rule random monetary injections are neutral. Under this rule, holding money is costless so agents are never cash constrained no matter what money shock prevails" (Berentsen, Camera and Waller 2005).

However if λ is different from 0 and the system faces inflation, it is necessary to set $r \neq 0$ in order to insure agents against the devaluation of money. This implies in deviation from Friedman rule and drive trades away of efficiency. So:

$$u^{'}(q) > c^{'}(q_s) \tag{31}$$

In this specific situation, buyers are constrained, it is costly to hold money, the demanded quantity of search good is bigger than the supply and the buyer holds a higher marginal value for consumption than the marginal cost of production. Buyers spend all money holdings and obtain:

$$q = \frac{(m + l_t)}{p} < q^* \tag{32}$$

It is important to notice the relation between agents optimal choices and the state of the economy here represented by $E_t(\phi_t)$. As has been showed before, the expectation for price of money in period t is simply the price of money in period $t-1$. However, in period $t-1$ the economy also suffered an aggregate shock by the beginning of its CM and, therefore, ϕ_{t-1} is also state contingent.

As will be shown later, on an inter-temporal analysis about monetary policy and different states of the economy, in the presence of a high money injection, trades in the DM of the next period will be high. Nevertheless, if the monetary authority chooses for a contractionary policy, then trades in the next period DM will be low. In the framework here developed, this is the main characteristic of an economy submitted to uncertainty regarding money policy. And this characteristic is also surrounding trades in the previous market, the BM.

Considering that there are η buyers and $1-\eta$ sellers in this economy, q^* solves:

$$u^{'}(q^*) = c^{'}(q^* \frac{\eta}{1-\eta}). \tag{33}$$

As per assumption, at the beginning of period t, buyers are resource constrained. This implies that, without the bond market, the efficient allocation q^* would never be achieved as the buyer would never be able to pay enough pq money holdings to offset the seller's costs for producing the demanded quantity of search goods (buyers always face $([E_t(\phi_t)]m < [E_t(\phi_t)]pq)$.

Here lies the importance of the BM: although agents are not able to pay spot for their consumption in the DM or can not insure themselves against idiosyncratic shocks, once there are idle resources available within the economy, the existence of a financial system satisfies the constrained agents' needs by allocating liquidity to who values it the most, throughout the payment of an interest rate.

Next it is analyzed the features and equilibrium of this credit market.

2.5 The bond market

The Bond Market is the first sub-period of any period t where $t \to \infty$.

By assumption, at the beginning of the period, agents suffer a preference shock. With probability η they can not produce but want to consume the next sub-period good, the search good, although they do not hold enough money for it. With probability $1-\eta$ they do not want to consume but can produce such good. After the shock agents continue with the assumed type until the end of the period and will be submitted to a similar shock in period $t+1$, and so on infinitely.

As already described, buyers are "poor" in the sense that they do not own enough resources to purchase the search good. On the other hand, sellers are "rich" as they own more resources than they actually need (as they are not consuming in the DM). Moreover, once matched, sellers are going to be monetarily recompensed by the disutility of the production of the search good, what enables the consumption of the general good in the CM and the accumulation of money holdings (precautionary savings, for example, once the agent is not sure about what type is he assuming in the next period).

Following this idea, allowing for reallocation of liquidity via credit is a feasible measure for achieving better final goods placements and increasing welfare. As commented in Berentsen, Camera and Waller (2007) financial intermediation executed by a bank able to match agents' desires and to keep record of transactions (guaranteeing the feasibility of credit) reduces borrowers constraints and provide a profitable use for the lenders' idle holdings. Moreover, throughout the payment of interests over the sellers' deposits, financial intermediation is able to provide a gain on welfare[16].

As in the DM, we assume that trades are given throughout a Walrasian price taking mechanism. After the preference shock, agents who are cash constrained look for the financial intermediary to obtain loans l_t and agents who hold idle balances deposit a share of this amount into the bank $d_t = \delta m$ (where $\delta \in (0,1]$ represents a share of its money holdings).

The bank, who is a more trustworthy agent than the buyer / borrower, take the deposit from the seller and lends it to the buyer. By the end of the period, the bank receives back the loan amount with a premium over it $(1+r_t)l_t$ and pays interests on the deposits $(1+r_t)\,d_t$. An important assumption

[16] As per Berentsen, Camera and Waller (2007), "welfare gain is not due to relaxing buyers' cash constraints but comes from generating positive rates of returns on idle cash balances", showing that "being constrained is not per se a source of inefficiency".

for the study is that the financial intermediary does not make any profit. Therefore the interest rate paid by the buyer and received by the seller are the same.

Exists an upper bound for the loans obtained by the buyer. It is the seller´s initial money holdings[17]:

$$0 < l_t \leq d_t \leq m$$

In a non-uncertainty inflationary environment, sellers would deposit all their money holdings ($\delta = 1$) given that the interest rate r_t is bigger than the actual inflation γ_t (interests compensate the devaluation of money).

Inserting uncertainty into the system does not change this result: everytime the bank offers a positive interest rate, the deposit constraint binds and the seller deposits all his money holdings[18].The buyer, on the other hand, considers uncertainty when looking for loans - his demand for loan is going to be state contingent.

Moreover, it is straightforward to understand that, as the seller derives no utility from consumption in the DM and he is receiving back the loan in the end of CM, in a no- inflation economy, r_t should be equal 0, corroborating Friedman rule as the best policy and illustrating a simple problem of money transferences in a no-default environment. But in the presence of uncertainty regarding monetary policy, allocation moves away from Friedman rule and it is charged (received) a price for obtainment (provision) of liquidity.

So, for a representative agent starting the period with m money holdings, his expected lifetime utility in BM is going to be:

$$\begin{aligned} B_t^e\left(m, \tau_t^i\right) &= \eta \left\{ \begin{array}{c} \pi\{v\left(l_t\right) + V_{b,t}\left(m + l_t, \tau_t^H\right) - \left[\left(1 + r_t\right) l_t\right]\} + \\ \left(1 - \pi\right)\{v\left(l_t\right) + V_{b,t}\left(m + l_t, \tau_t^L\right) - \left[\left(1 + r_t\right) l_t\right]\} \end{array} \right\} + \\ & \left(1 - \eta\right) \left\{ \begin{array}{c} \pi\{-\psi\left(d_t\right) + V_{s,t}\left(m - d_t, \tau_t^H\right) + \left[\left(1 + r_t\right) d_t\right]\} + \\ \left(1 - \pi\right)\{-\psi\left(d_t\right) + V_{s,t}\left(m - d_t, \tau_t^L\right) + \left[\left(1 + r_t\right) d_t\right]\} \end{array} \right\} \end{aligned} \tag{34}$$

[17] As already analyzed, due to the possibility of an adverse preference shock, sellers are encouraged to not spend all their money in the CM and bring some to the next period's BM (precautionary reasons).

[18] Even if the actual inflation is bigger than the offered interest rate, for the seller it is better to deposit entirely his money holdings. This comes from the fact that he is, by definition, not spending his money holdings. Therefore keeping it "under the mattress" would bring a higher real devaluation when in face of inflation than applying it at the bank and earning some interests that compensate, at least partly, the devaluation over money holdings arising from inflation.

The equation above means that with probability η the agent is going to be a buyer - resource constrained - and, hence, is looking forward to obtain l_t money holdings in the form of loans, what would provide him the utility $v(l_t)$. The amount of loans he is looking for, on the other hand, is submitted to his uncertainty regarding the amount of money to be injected in the economy in the last sub-period, CM. Therefore, his expected value function is conditioned to the possible H and L states of the economy.

The analysis is similar for the seller: the amount of money he is willing to deposit (and therefore face the disutility $\psi(d_t)$ of giving up, momentarily, d_t units of liquidity) depends on his expectation regarding the state the economy is assuming after the money injection.

If an agent ends up to be a seller after the preference shock, his value function turns to:

$$\begin{aligned} B^e_{s,t}\left(m, \tau^i_t\right) &= \pi\{-\psi(d_t) + V_{s,t}\left(m - d_t, \tau^H_t\right) + [(1+r_t)\,d_t]\} + \\ &\quad (1-\pi)\{-\psi(d_t) + V_{s,t}\left(m - d_t, \tau^L_t\right) + [(1+r_t)\,d_t]\} \end{aligned} \tag{35}$$

Applying expectations:

$$B^e_{s,t}\left(m, \tau^i_t\right) = -\psi(d_t) + V^e_{s,t}(m - d_t) + (1+r_t)\,d_t \tag{36}$$

In this sense, the seller's problem during the BM is:

$$\max_{d_t}\{-\psi(d_t) + V^e_{s,t}(m - d_t) + [(1+r_t)\,d_t]\} \tag{37}$$

$$s.t.\ d_t \leq m \tag{38}$$

where the constraint stands for the impossibility of depositing a higher amount than the amount of money holdings he owns in the beginning of the period.

Substituting the expected value function of the seller in the DM (19)

$$\max_{d_t}\left\{-\psi(d_t) - c(q_s) + W^e_{s,t}(m - d_t + pq_s) + [(1+r_t)d_t]\right\} \tag{39}$$

$$s.t.\ d_t \leq m$$

which, applying linearity and simplifying, becomes:

$$-c(q_s) + W^e_{s,t}(0) + [E_t(\phi_t)](m + pq_s) + \max_{d_t}\{-\psi(d_t) - [E_t(\phi_t)]d_t + [(1+r_t)d_t]\} \tag{40}$$

$$s.t.\ d_t \leq m$$

Taking first order conditions of the Lagrangian function in respect to d_t and λ_d:

FOC to d_t :

$$\psi^{'}(d_t) = (1 + r_t) - E_t(\phi_t) - \lambda_d \tag{41}$$

FOC to λ_d :

$$d_t = m$$

Equation (41) means that sellers continue depositing until the point where the marginal disutility of giving up one additional unit of liquid holdings is equal to prize received over one more unit of deposit net from the expected cost of holding money and the multiplier for the deposit constraint.

In a non-inflation environment and counting with $\lambda_d = 0$, the best policy is the Friedman rule with $r_t = 0$ (agents do not need insurance against money devaluation once there is none).

However marginal disutility of depositing is increasing in money growth. This comes straightforward from the proposition that, on an inflationary environment, holding money is costly. In "high enough" inflationary states, the incentives for saving disappear and the system starts to face adverse selection issues. Therefore, driving away from Friedman rule in order to offer incentives for savings, in such situation, is the best policy. It is also important to highlight that at any $r_t > 0$ the deposit constraint binds and sellers deposit all their money holdings.

Now we turn attention to the buyer. His expected value function in the beginning of the BM is:

$$B^e_{b,t}\left(m, \tau^i_t\right) = \begin{array}{c} \pi\{v(l_t) + V_{b,t}\left(m + l_t, \tau^H_L\right) - [(1 + r_t)\, l_t]\} + \\ (1 - \pi)\{v(l_t) + V_{b,t}\left(m + l_t, \tau^L_t\right) - [(1 + r_t)\, l_t]\} \end{array} \tag{42}$$

which applying expectations turn into:

$$B^e_{b,t}\left(m, \tau^i_t\right) = v(l_t) + V^e_{b,t}(m + l_t) - (1 + r_t)\, l_t \tag{43}$$

In this sense, the buyer's problem during the BM is to:

$$\max_{l_t}\{v(l_t) + V^e_{b,t}(m + l_t) - [(1 + r_t)\, l_t]\} \tag{44}$$

$$s.t.\ d_t \leq m$$

$$l_t \leq d_t$$

$$(1 + r_t)\, l_t \leq y - x - m^{'} + z^b + \tau_t^i M_{t-1}$$

The 2 first constraints stand for the lenders deposit constraint and the total amount of resources available for loans (addressing feasibility) and the third constraint accounts for the buyer conditions to honor its debt (this is the same constraint he faces in his CM maximization problem).

Proceeding the same way as done for the sellers: the buyers' DM expected value function (17) is inserted into equation (44). So, applying linearity, the buyer's problem turns to:

$$u(q)+W^e_{b,t}(0)+[E_t(\phi_t)](m-pq)+\max_{l_t}\{v(l_t) + [E_t(\phi_t)]l_t - [(1+r_t)\, l_t]\} \quad (45)$$

$$s.t.\ d_t \leq m$$

$$l_t \leq d_t$$

$$(1 + r_t)\, l_t \leq y - x - m^{'} + z^b + \tau_t^i M_{t-1}$$

Now, taking first order conditions for loans and constraints:

FOC to l_t:

$$v^{'}(l_t) = (1 + r_t)(1 + \lambda) - E_t(\phi_t) - \lambda_l \quad (46)$$

FOC to deposits constraint multiplier λ_d:

$$d_t = m$$

FOC to loans constraint multiplier λ_l:

$$d_t = l_t$$

and FOC to repayment constraint multiplier λ:

$$(1 + r_t)\, l_t = y - x - m^{'b} + z^b + \tau_t^i M_{t-1}$$

Equation (46) shows that buyers keep borrowing until the point where the marginal utility of one more unity of loan is equal to upcoming interest payment summed to the payment constraint multiplier net from the cost of holding money and the loans constraint multiplier.

It is possible to notice the positive relation between marginal utility of borrowing and the money growth rate, as happens for the marginal disutility of depositing. As it is costly to hold money, agents look forward to obtain liquidity via financial markets instead of store money (which is submitted to the inflation tax). However this is the procedure adopted by all agents and, as already mentioned, for high enough inflation rates, the system might start to face adverse selection. In such situation, for the monetary authority is advisable to drive away from the Friedman rule and offer high enough compensations for the devaluation of money in order to diminish the incentives to borrow and increase incentives to save.

The clearing condition for the bond market requires the amount of money demanded in the form of loans to be equal to the amount deposited. Therefore

$$l_t = d_t (= m)$$

Dividing equation (46) for equation (41):

$$\frac{v^{'}(l_t)}{\psi^{'}(d_t)} = \frac{(1+r_t)(1+\lambda) - E_t(\phi_t) - \lambda_l}{(1+r_t) - E_t(\phi_t) - \lambda_d} \tag{47}$$

For $\lambda_d = \lambda_l = \lambda = 0$:

$$\frac{v^{'}(l_t)}{\psi^{'}(d_t)} = \frac{(1+r_t) - E_t(\phi_t)}{(1+r_t) - E_t(\phi_t)} = 1 \tag{48}$$

In this specific situation, given no inflation, the best policy to be applied is setting $r_t = 0$ (Friedman rule is the first best) and therefore approach efficiency with

$$d_t = m_t = l_t = l_t^*$$

for which, considering η buyers and $1-\eta$ sellers, l^* solves:

$$v^{'}(l_t^*) = \psi^{'}(l_t^* \frac{\eta}{1-\eta}) \tag{49}$$

For the case where any multiplier is > 0, the marginal utility of one additional loan is higher than the marginal disutility of depositing one additional unit of money holding:

$$\frac{v^{'}(l_t)}{\psi^{'}(d_t)} > 1$$

what implies in $l_t < d_t$ and consequently a lower than optimal consumption of the search good once the buyer is not able to obtain enough resources to fully compensate the seller for his costs of production. One of the reasons for such situation may be a higher than optimal interest rate charged over loans.

So, in an inflationary environment, which allows deviation from the Friedman rule in order to insure agents against the real devaluation of money, the monetary authority can enhance the obtained allocation of loans and goods by forcing banks to lower the interest rates charged on loans (but is important to keep it on a baseline which still provides insurance against inflation).

From equation (47) on it is possible to see the insurance effect that a credit market and a financial intermediary provide for the economy in an inflationary environment. The provision of a premium over deposits offsets (at least partly) the devaluation effects of inflation. And this is the main reason why introducing a financial market into the system provides gains on welfare. However it is important to highlight that inflation is a necessary condition for equilibrium with credit once, in a non-inflationary environment, "agents are able to self-insure at low cost, thus having access to financial markets is of little value" (Berentsen, Camera and Waller 2007).

The next section brings up the relation between unanticipated monetary policy and distributional effects. The main idea here is that the monetary authority, responsible for issuing money, has a useful tool in its hands: the discretionary policy, which, once not correctly anticipated by the agents ($E_t(\gamma_t) \neq \gamma_t$) can possibly address inter-temporal social and distributional effects.

For such procedure, the analysis start on a period where efficiency is achieved, the amount of demanded loans is equal to the amount of deposits (loans supply) and, therefore, buyers hold enough money holdings to satisfy the demand for the search good.

However, the non-inflation environment is left behind and the upcoming period must be characterized as inflationary or deflationary (by assumption, the monetary authority can take liquidity out of the system via taxes). Moreover, to address effects of uncertainty, expectations regarding policies are going to be different from the actual policies.

3 Monetary policy and distributional effects

3.1 Expected monetary policy

As mentioned before, this section tries to depict the relation between an unanticipated monetary policy and the effects it brings over aggregate variables and the distribution of the monetary wealth of the economy in the short run[19].

Several papers, such as Berentsen, Camera and Waller (2005), Baranowski (2012) and Chiu and Molico (2008), have described the necessary caveats (respectively the addition of another round of trades before agents are able to re-balance their money holdings, borrowing constraints and dropping out quasi-linearity and complete markets premises) to study distributional effects from monetary policy in a micro-founded macro model. They have also proposed results, mainly numerical and related to the welfare function of the economy.

The study here developed proposes a different groundwork. As the model is mainly driven by uncertainty and expectations regarding transfers, money growth and price of money, the main results and equilibrium conditions are all state dependent - how much loans an agent is able to contract, how much does he deposits, how many search and general goods is he able to consume, how much money holdings does he take to the next period - these are all state contingent variables with dependence on the $E_t(\phi_t)$ or on the ϕ_t^i (note that the former represents the expectation agents hold for the price of money and the latter is the price of money conditioned on the state of the economy). Therefore the main question to be analyzed is: what happens with these features of the model if agents are not able to fully anticipate monetary policy and expectations turn out to not realize?

It is important to highlight that this model already counts with a liquidity redistribution scheme, namely the bond market and the issuance of debt contracts. Such scheme is responsible for allocating liquidity to those agents who values it the most (due to resource constraints and consumption needs), by means of the payment of an interest rate. As per Berentsen, Camera and Waller (2007): "debt contracts play a similar role as Kocherlakota's (2003)

[19] As in Baranowski (2012), it is assumed that agents expectations go through a process of "adaptive learning". This "learning process takes time, however, so a change in the money supply has real effects in the short run, but it is neutral in the long run because agents eventually learn the lower steady state value of money".

"illiquid bonds" - they transfer money at a price from those with low marginal value of money to those with a high one".

However, this scheme does not allow for wealth redistribution once borrowers are paying the borrowed value back to the bank. Here lies one of the main points about monetary policy: if it is expansionary, via lump sum money transfers, resulting in inflation, agents holding idle balances are going to be taxed by the inflation tax (devaluation of money). And agents with resource constraints are going to promptly satisfy their consumption needs more easily, impacting on a lower wealth inequality.

On the other hand, if monetary policy is contractionary, via taxes, ending up in deflation, those with idle balances are going to see their initial money holdings gain real value and those ones initially constrained are going to suffer more to support their consumption needs. As argumented in Berentsen, Camera and Waller (2005) "away from the Friedman rule, random monetary injections can be non-neutral eventhough all prices change proportionately. In particular, an unexpectedly high money growth rate can cause aggregate output to increase. These results occur eventhough injections are symmetric across agents and prices are fully flexible".

Though, it is important to highlight that in order to achieve social features of the economy, the monetary policy must be executed in a specific moment where money distribution is non-degenerate (in the model here developed, in the beginning of the centralized market).

Heading back to the expected money transfers, money growth and price of money: from equations (3), (4) and (5) it is possible to notice the positive relation between money transfers and money growth rate - a higher rate of transfers impact on a higher rate of money growth; and the inverse relation between the 2 variables and the price of money - a higher injection of money in the economy ends up in a real devaluation of money. So:

$$\frac{\phi_t^H}{\phi_{t-1}} = \frac{1}{\gamma^H} \; and \; \frac{\phi_t^L}{\phi_{t-1}} = \frac{1}{\gamma^L}$$

for which

$$\gamma^H > \gamma^L \; and \; \phi_t^L > \phi_t^H$$

As exposed before, in a non-inflationary environment there is no place for credit as agents can costlessly insure themselves against the devaluation of money. Moreover, applying the Friedman rule as policy brings first best allocation.

However, as already demonstrated, for the existence of equilibrium with credit it is necessary inflation - $\gamma > \beta$. And inflation, by definition, imposes deviation from the Friedman rule and loss on efficiency. So, agents are no longer able to trade the optimal quantities (the amounts traded l and q are smaller than l^* and q^*). However, such loss can be partially compensated by positive effects over production and agents' income, what results in a smaller total loss in the economy´s welfare.

3.1.1 High state (H)

The first approach is to consider $\gamma_t > E_t(\gamma_t)$. So, $\gamma_t > 1$, the environment becomes inflationary and $\phi_t < E_t(\phi_t)$ (or $\phi_t < \phi_{t-1}$). As trades in BM and DM in period t happen before the money injection, there will not be any direct impact over these markets equilibrium conditions during period t.

However, this aggregate shock is going to rule expectations over price of money and, consequently, rule equilibrium conditions in both bond market and decentralized market in next period $t+1$. This comes from the fact that, as in period t, $E_t(\phi_t) = \phi_{t-1}$,

$$E_{t+1}(\phi_{t+1}) = \phi_t \tag{50}$$

Considering the equilibrium conditions for both agents in DM and BM in period $t+1$ (from equations 23,27,41,46):

$$c^{'}(q_s)_{t+1} = p[E_{t+1}(\phi_{t+1})] \tag{51}$$

$$u^{'}(q)_{t+1} = p[E_{t+1}(\phi_{t+1}) + \lambda] \tag{52}$$

$$\psi^{'}(d_t)_{t+1} = (1 + r_{t+1}) - E_{t+1}(\phi_{t+1}) - \lambda_d \tag{53}$$

$$v^{'}(l_t)_{t+1} = (1 + r_{t+1})(1 + \lambda) - E_{t+1}(\phi_{t+1}) - \lambda_l \tag{54}$$

From equation (50) and everything else constant it is possible to notice that, as $E_{t+1}(\phi_{t+1}) = \phi_t$ and $\phi_t < E_t(\phi_t)$:

$$c^{'}(q_s)_{t+1} < c^{'}(q_s)_t \text{ and } u^{'}(q)_{t+1} < u^{'}(q)_t \tag{55}$$

and

$$\psi^{'}(d_t)_{t+1} > \psi^{'}(d_t)_t \text{ and } v^{'}(l_t)_{t+1} > v^{'}(l_t)_t \tag{56}$$

Therefore, if the monetary authority injects a high quantity of money into the economy, resulting in a low ϕ_t, the marginal utility and disutility of consuming and producing the search good falls. So, the quantity of search goods demanded by the buyers increases, but the quantity supplied by the sellers decreases. This means that each trade will be marked by a lower quantity of search good traded but there will be more trades in order to satisfy buyers demand, clearly showing the "hot potato" effect that inflation brings to the economy, raising the extensive margin and lowering the intensive margin of trades. Overall, production in the DM increases in the short run.

Trades in the BM, on the other hand, suffer the opposite effect. In the presence of inflation, the disutility of depositing increases. Therefore there will be less resources available for loans and borrowers are going to suffer from a deficit of loans. This result corroborates the proposition that inflation brings agents incentives to not behave accordingly to their types: sellers with less incentives to save (due to the devaluation of money) might start to demand borrowings, increasing the number of buyers in the economy and bringing disequilibrium to the system.

Transfers in period t, nevertheless, promote effects over the CM in period t, when agents money holdings are heterogeneous. As in Berentsen, Camera and Waller (2005), "when money is higher than expected, the price increase reduces the real balances of every agent, acting as a proportional tax on their money holdings. ... those who hold less cash are taxed less than those who hold more". This would allow buyers to increase consumption in the CM once the transfers more than offset the inflation tax. Moreover, given that loans and deposits contracts are no state contingent and are redeemed after the money injection, buyers have to produce less to pay their debts (with ϕ_t falling, the price of the general good, $\frac{1}{\phi_t}$ increases), what relax their constraints.

For the sellers the result is the opposite. The bank pays their deposits back with r_t interests over it. However, as r_t is also no state contingent (during one specific period), the value received, in real terms, depending on the level of inflation, might not fully satisfy the devaluation of money (sellers suffer more with the inflationary tax). Sellers might lose real wealth even receiving transfers.

Therefore the system suffers a real wealth distribution, lowering the differences between those who are cash constrained and those holding idle balances.

Following this arrange, it is possible to notice a short run Phillips curve, allying higher production in the DM and higher inflation. This effect, summed to the real wealth distribution, partially compensates the welfare loss that the economy is subject to once in an inflationary environment. However, on the long term, as agents are able to learn the lower money value steady state and the aggregate shocks and due to the randomness of the preferences shocks, the positive effects of inflation decease.

3.1.2 Low state (L)

For obtaining a low state of the economy, it is considered that the monetary authority is able to withdraw liquidity from the system trough a taxing policy. So, $0 < \gamma_t < 1$, $\gamma_t < E_t(\gamma_t)$ and $\phi_t > E_t(\phi_t)$.

Again, trades in period t for both BM and DM are not affected by money policy in period t. However, as trades in period $t+1$ are conditioned on the $E_{t+1}(\phi_{t+1})$, which, on the other hand, is equal to ϕ_t, the state the economy assumes today is going to rule trades tomorrow.

So, going back to equations (51), (52), (53) and (54), assuming the same procedure as for the H state and studying the impacts of deflationary policy on the economy's features in a analytical intertemporal set up:

$$c'(q_s)_{t+1} > c'(q_s)_t \text{ and } u'(q)_{t+1} > u'(q)_t \tag{57}$$

and

$$\psi'(d_t)_{t+1} < \psi'(d_t)_t \text{ and } v'(l_t)_{t+1} < v'(l_t)_t \tag{58}$$

A monetary policy that withdraws liquidity from the system, at the same time that provides a higher valuation for money holdings, discourages production and trades in the DM.

Such result comes from the fact that the cost of holding money is now very low. So agents have incentives to postpone spending decisions and hold money instead of disburse it on consumption today. If shocks are recurrently deflationary then the trading system might come to even lower levels. This corroborates the proposition that, in the short run, deflationary policies have a depressing effect over total production.

During the bond market, on the other hand, as the disutility of depositing lowers with the decrease of the monetary basis, sellers have now more incentives to save. On the other hand, the marginal utility provided by one unit of loans decreases. This comes straightforward from the fact that trades in DM diminish: as agents are holding money instead of spending it, there are no reasons for borrowing. Again, as in the high state, the economy might face a problem of adverse selection where buyers start acting as sellers (depositing instead of borrowing).

In the CM, the effect is the opposite from the one seen in the high state: due to the withdraw of money from the system on a period where money distribution is non-degenerate, those with less money holdings suffer more than those with more money holdings. Moreover, as debts and deposits are state contingent, with the reduction of prices, buyers have to work more to pay their debts and sellers see a real appreciation of the amount the deposited. Therefore deflationary policies, on the short run, impact on a worsening real wealth distribution.

4 Conclusion

This paper tried to depict the relation between monetary policy and its distributional effects in an uncertainty environment and with the presence of a bond market (credit market). It was proposed an analytical framework and, therefore, the paper does not hinge on numerical analysis.

For such procedure, it was developed a "Lagos-Wright based" model with 3 sequential markets: a *bond market*, a *decentralized market* and a *centralized market*.

It was also assumed that agents are recurrently exposed to preference shocks that determine their types. Agents can assume 2 different types: buyers, resource constrained, with high valuation for money and wanting to consume in the DM; and sellers, holders of idle balances, with low valuation for money and without any consumption needs during the DM.

Allowing for financial trades between both agents enhance the obtained allocation of the economy. Therefore it was inserted a third agent: a financial intermediary (or, simply, a bank) responsible for receiving deposits from sellers and lending loans to buyers, throughout interests. Such procedure, also, positively impacts the welfare of this economy by providing interests on the deposits made by the sellers (as in Berentsen, Camera and Waller 2007 "gains in welfare arise from paying interest on deposits and not from relaxing borrowers´ liquidity constraints").

Nevertheless, during trades of goods, agents meet anonymously, what requires for an instrument capable of surpassing trust issues and assume a public record keeping role. This instrument is fiat money, injected in the economy by the monetary authority.

Therefore, the framework here proposed was able to ally the presence of credit and money, surpassing the difficulties regarding inserting money in credit models or allowing for credits in models where money is essential.

Agents are aware of the presence of the monetary authority and the existence of upcoming money policies. However they do not know which policy is being taken. So, in the beginning of each period, agents create expectations regarding the monetary policy to be realized in the last market of that same period: they believe that it could be a high (H) money injection, deriving an inflationary environment; or a low (L) money injection, deriving a deflationary environment. Each state is accomplished with certain probability and, therefore, the expected state is simply the average state. Expected states are directly connected to expected price of money, the driving force of uncertainty in this model.

Trades, optimality and equilibrium conditions in all markets are going to be state dependent. Bond market and decentralized market optimal conditions are ruled by the expected price of money ($E(\phi)$) and centralized market optimal conditions are ruled by the price of money conditioned on the state of the economy assumed after the money injection (ϕ^i with $i \in (H, L)$). And once expectations differ from the actual policy there is space for intertemporal aggregate and wealth distribution effects arising from monetary policy.

In a non-inflationary environment, applying the Friedman rule is achieving the first best allocation on the economy for both search goods and loans.

However once the economy is submitted to inflation, then monetary policy must drive away from Friedman rule in order to insure agents against money devaluation. And this is the environment where an economy with credit effectively achieves better allocations than an economy with no financial market (without inflation there is no necessity for credit markets as insuring against money devaluation is costless).

Assuming expectations different from monetary policy, it was analyzed impacts over economy in the 2 possible states. It is important to highlight, also, that such effects are only possible when money injection is done during a non-degenerate money distribution period the economy is passing through (in this model, the beginning of the CM).

An inflationary policy brings higher production and less financial trades (depending on the level might even result in credit ration due to adverse selection) and improves the wealth distribution.

The model was also able to corroborate the idea of the "hot potato effect" of inflation: with a losing value money, agents decrease the amount of goods traded however the transactions happen with a higher velocity: there is a gain in the extensive margin in detriment of the intensive margin.

On the other hand, a deflationary policy brings lower production and less financial trades (with a high valued money buyers do not face resource constraints and therefore do not have to borrow) and deteriorates wealth distribution.

This work is still open for further studies. One possible matter to be analyzed would be how does monetary policy affect the economy in an increasing uncertainty environment. Another possible field of studies would be to quantitatively analyze the effects of money policy in the credit market in what concerns amount, size and frequency of trades assuming the Lagos-

Wright framework. Such kind of research could give to issues as financial market frictions, investment capital provision and allocation and variability in returns (addressed by, for example, Boyd, Choi and Smith 1996) a new perspective.

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