Master Thesis

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Financial Intermediation and the Welfare Cost of Inflation

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January 31, 2013

Contents

1 Introduction

There is a long line of research concerning the welfare implications associated with inflation. Several issues have been discussed since the mid of the 20^{th} century and remain a hot topic in economic research until today. Two of these topics are especially relevant for this master thesis: First, there is the fundamental question of how to measure the welfare cost of inflation. Second, how should a theoretical money demand function be constructed, such that it provides a good match to real world data? A paper that handles both of these questions is the one of Lagos and Wright (2005). They estimate the costs of inflation by measuring the percentage of consumption agents are willing to give up to reduce the inflation rate from 10% to 0%. I use the same procedure to answer the main question of my master thesis, which is: Does financial intermediation lower the welfare costs of inflation? Concerning this, Lucas (2000) wrote: "In a monetary economy, it is in everyone's private interest to try to get someone else to hold non-interest-bearing cash and reserves. But someone has to hold it all, so all of these efforts must simple cancel out. All of us spend several hours per year in this effort, and we employ thousands of talented and highly-trained people to help us. These persons-hours are simply thrown away, wasted on a task that should not have be performed at all." The view that financial intermediation is useless and even costly, is not shared in the whole scientific community. Berentsen et al (2007) presented a model where financial intermediaries are introduced in context of the Lagos and Wright (2005) framework. They show that financial intermediation improves welfare by paying interests on agents deposits. However, this theoretical approach has not yet been analyzed empirically. My master thesis is about to close this gap by calibrating the theoretical money demand function with financial intermediation. Based on this, welfare cost functions can be constructed which allows to express the costs of inflation quantitatively. Finally, the results are compared to the existing literature where no financial intermediation is included. This is done for the following pricing mechanisms: Nash-, proportional bargaining and constant markup as proposed by Craig and Rocheteau (2008) as well as for competitive pricing as in Berentsen et al (2007). My calibrations indicate that the model with financial intermediation exhibits costs of inflation which are approximately half of the size they would reach without financial intermediation. This result is due to lower calibrated values for the relative risk aversion parameter of the buyers' utility function.

The remainder of the thesis proceeds as follows. In section 2 Bailey's (1956) methodology of measuring the welfare cost of inflation is introduced. Additionally, people's behavior in times of acute hyperinflation is described to show, how the costs of inflation are experienced by societies. Section 3 introduces Meltzer's (1963) definitions of money and money demand. There is a focus towards the question which variables influence the demand for money and about the stability of a log-log money demand function. Section 4 illustrates the approach of Lucas (2000) in measuring the welfare implications associated with inflation. He combines Bailey's (1956) technique of measuring the costs of inflation with Meltzer's (1963) estimated money demand function. The main question of Lucas (2000) is: What are the costs - expressed as a proportion of the nominal GDP - of a 10%- in comparison to a 0% inflation rate? These first chapters provide a short overview about the preliminary economic literature. In section 5 the approach of Lagos and Wright (2005) is shown. Moreover, the considered pricing mechanisms are introduced and the way how to construct the theoretical money demand function is explained. Finally, the methodology of measuring the costs of inflation is described. Section 6 presents the model of Berentsen et al (2007). They use a model that is embedded into the Lagos and Wright (2005) framework, but adds financial intermediation. In section 7 the models presented in the sections 5 and 6 are calibrated or recalibrated, respectively. Finally, the results are compared and interpreted. Section 8 concludes.

2 The Welfare Cost of Inflationary Finance

Bailey (1956) describes the temptation of governments to increase the governmental share of national income by pursuing an inflationary policy. This temptation is called *inflationary finance* and in the past governments often did not resist it. Bailey (1956) distinguished the following three sources which make inflationary finance harmful:

1. Inflation has a *redistributive aspect.* If income and wealth are denominated in money terms, the purchasing power of nominal fixed money balances decreases as the inflation increases.

2. Inflation has a *disruptive aspect.* Uncertainties about future absolute and relative prices or incomplete price adjustments may lead to a misal-

location of resources. Resources do not flow into the direction in which they are needed most and therefore are inefficiently allocated.

3. Inflation acts as a tax on the holdings of real cash balances. Real cash balances bear no interest and larger inflation rates lead to higher nominal interest rates such that the opportunity costs of holding cash balances increase.

Bailey (1956) argues that when governments pursue an openly announced inflationary policy, the costs of inflation, i.e. the redistributive and the disruptive aspect, could be neglected. However, the welfare implication of such a policy on the holding of real cash balances has not already been investigated. To close this scientific gap and to focus on the inflation tax aspect of inflationary finance Bailey (1956) proposed the following two model assumptions:

I. The government announces the expected increase in the money supply. The public believe this announcement and everyone adjusts his inflation expectation to the government's announcement.

II. With respect to inflation expectations, all contracts are rewritten such that their real value keeps constant. In particular this implies that wages, business contracts and pensions keep their purchasing power and no redistribution takes place. If money is injected into the economy, the nominal interest rate increases for the new as well as for the old money loans.

These assumptions guarantee that neither the redistributive nor the disruptive aspect of inflation lead to welfare costs, but inflation still has a negative impact on the holding of real cash balances. Bailey's (1956) argument is that the inflation tax aspect is more fundamental because the welfare costs cannot be avoided even if the inflation rate is fully anticipated.

2.1 Holding of real cash balances under hyperinflation

Bailey (1956) describes the public's behavior concerning the holding of real cash balances if inflation rates achieved tremendous dimensions. He explains how welfare costs are directly experienced by people. The situations described below were observed at hyperinflation which occurred in Europe in the first half of the 20^{th} century. Note that moderate inflation rates do not necessarily have the same consequences. Bailey (1956) argues: "Although no very precise conclusion can be drawn from the available evidence about the real cash balance that people would desire to hold at fully anticipated, but relatively low, rates of inflation, a great deal of precise and useful information about desired cash balances at very high rates of inflation is given in Cagan's study of seven European hyperinflations." Such changes in the desire of holding cash balances are shown in the following examples:

- **People changed their payment behavior**. Let us consider two ways of executing a trade contract:

  I. In a bilateral barter trade both trading partners exchange a real good or a service for another.

  II. In a bilateral trade based on exchange of money, one party provides a service or a real good while the other party settles by paying a certain amount of money.

  Inflation reduces the real value of nominal money balances while barter trades avoid these losses, because exchanging real goods does not require any party to use money. But, barter trades are less convenient than money trades as they are associated with search costs. For example, both parties have expenses to attract someone who is willing to exchange goods. For that reason people prefer to pay with real cash balance if the inflation rate is low, which means that opportunity costs of holding money balances are low while barter trades bear substantial search costs. If the inflation rate increases, people are more likely to engage in barter trades, as the opportunity costs of holding cash balances become more severe than the search efforts associated with barter. The higher the inflation rates, the greater are the costs of holding money balances such that more complicated and therefore more costly barter arrangements get implemented. Hence, Bailey (1956) concludes that an increasing inflation rate is associated with larger welfare costs.

- **Firms had to pay their workers more frequent.** Workers wanted to be protected from the loss of purchasing power associated with inflation. This change in the wage payment procedure of firms gave rise to the following cost sources:

I. More frequent wage payment required a larger administration. For example, additional accounting efforts had to be undertaken.

II. Firms lost bounded working capital. Workers accepted delayed wage payment only as long as inflation was not high.

Some firms even paid their worker in kind to protect them from inflation.

- **Shopkeepers closed their shops earlier.** They wanted to spend their cash balances as fast as possible to acquire new goods and avoid inflation losses, which are a function of accrued time. In situations of acute hyperinflation, shopkeepers even closed after a few sales.

- **Workers rushed to spend their received wages quickly.** They tried to avoid the tax associated with inflation. As firms paid wages more frequently and workers rushed to spend their wages, the volume of an average trade declined while the frequency of trades increased. Consequently, the time spent for trading activities increased as well. As a result, the transaction costs boosted.

All of these examples show the manner in which people and firms changed their payment behavior when the inflation rate increased. The larger the inflation, the less real cash balances persons held to avoid the loss of purchasing power, i.e. to prevent the inflation tax. As a consequence barter trades increased.

2.2 Measuring the welfare cost of inflation

Bailey's (1956) considerations about the measurement of the welfare cost of inflation are visualized based on *Figure 1.* The vertical axis measures the nominal interest rate. The horizontal axis describes the aggregated real cash balances held by the public. The curve represents the society's aggregated money demand at different nominal interest rates. As can be seen, there is a negative relation between the nominal interest rate and the holdings of real cash balances, i.e. the higher the interest rate, the less real cash balances are held by the society. The intuition is, that a larger nominal interest rate increases the opportunity costs of holding money, since cash balances could also be invested in interest bearing assets. If $(M/P)_0 = 1$, Bailey (1956) interprets the horizontal axis of *Figure 1* "as the real value a dollar would have if there were no inflation" while the height of the money demand curve at a specific point on the x-axis indicates "a fraction of such a dollar per unit of time".

The timing of events is as follows:

1. Initially, the nominal interest rate equals r_0, the interest rate consistent with zero inflation. The money demand curve indicates the aggregate real cash balances that are held by the public at a given interest rate. At $r = r_0$, the real cash balances hold by the population are $(M/p)_0$.

2. The government announces to increase the inflation rate by $r_1 - r_0$.

3. Individuals build new inflation expectations in response to this announcement. As a consequence, people want to reduce their real cash balances. They buy interest bearing assets in exchange for cash. The price levels of the assets rise immediately and as a consequence the real cash balances reduce to $(M/p)_1$.

4. After the initial shock, the price level rises at the same rate as the government injects money. The level of real cash balances stays constant at the new level $(M/p)_1$.

For any inflation rate, the individuals form a specific habit of payment. High inflation rates give rise to tighter holdings of real cash balances such that trades will be executed more often on the basis of barter. In contrast, at low inflation rates individuals broad their real cash balances and transact more often on the basis of money. This gauge in the payment procedure indicates that the money demand curve represents the marginal productivity of real cash balances. At a given inflation rate, for example 10%, the marginal value of the last money unit must equal ten pence. Otherwise, a person would not be willing to hold this amount of money and may prefer to trade in a barter fashion. The amount of real cash balances held by the public is exactly the amount that equalizes the opportunity costs of holding money and the marginal productivity of the cash balances. Is the marginal productivity of money bigger (smaller) than the interest rate, people would extend (reduce) their real cash balances. Bailey (1956) concludes: "Hence it follows that the area under the demand curve for real cash balances, over the range of that part of real cash balances which is relinquished because of a

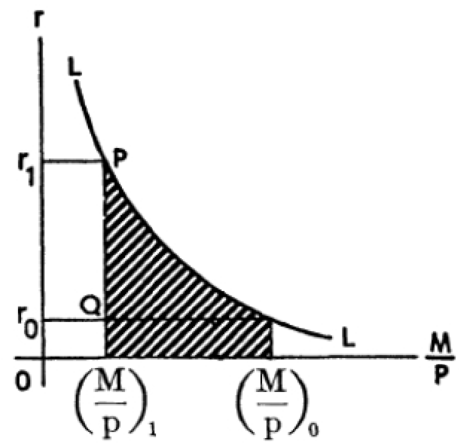

Figure 1: Inverse money demand function. (source: Bailey (1956))

given rate of inflation, measures the costs in loss of convenience, increasingly awkward barter arrangements, and so on, involved in relinquishing those real balances." The shaded area underneath the inverse money demand function in *Figure 1* can be interpreted as the welfare cost of inflation due to the tax imposed by the government on real cash balances if the nominal interest rate is increased from r_0 to r_1.

3 Stability of the Money Demand

According to Bailey (1956), costs of inflation correspond to the area under the inverse money demand function. But how should such a money demand function look like? The definitions of money and its demand are crucial issues in monetary economics. Meltzer (1963) distinguishes between a quantitative and a non-quantitative approach. Quantitative theorists view the demand and the supply of money as a stable macroeconomic relationship. People change their asset composition if there are discrepancies between actual and desired money holdings. For example, if the interest rate rises, the demand for real cash balances decreases and people invest in interest bearing assets. In contrast, the non-quantitative theorists view money as one of numerous financial assets. Capital instead of money is the crucial variable. Meltzer (1963) analyzed the stability of different money demand functions empirically, considering a quantitative approach. A money demand function is stable, if it has existed under different economic, political, social and institutional conditions. Meltzer (1963) poses the following three questions:

1. Which arguments or variables should define the money demand?
2. What are appropriate definitions of money?
3. Do stable money demand functions exist?

3.1 Variables that determine the money demand

Meltzer (1963) suggests the following relationship for the money demand

$$M = f(r^*, \rho, d^*, Y). \tag{1}$$

The aggregated money demand M is a function of the yields of financial assets r^*, physical assets ρ, human wealth δ^* and of human income Y. The derivatives of f with respect to r^*, ρ, d^* are negative. Higher interest rates, irrespective of their type, imply lower money demand since money holdings bear higher opportunity costs. Therefore money, financial and physical assets and human capital compete with each other for a place in the portfolio of a person. Y acts as an income constraint imposed on the money holdings. Meltzer (1963) states three assumptions concerning (1):

1. r^* and ρ are combined in a single variable r. This is justified since physical and financial asset yields report a high covariance.
2. Meltzer (1963) defines d^* as the ratio between human income and human wealth. Multiplying and dividing this ratio by the expected human income, d^* can be re-expressed in two terms:

   First, β is the ratio of actual- to expected income on human wealth. This term should be equal to one in the long run, because expectations should coincide with reality in the long run. Second, d is the ratio of expected human income to human wealth. Meltzer (1963) assumes that d is constant.
3. f is homogenous of degree one in Y.

These assumptions allow to define the money demand function as follow

$$M = g(r; d, \beta)Y = g(r)Y. \tag{2}$$

The aggregated money demand depends on two variables. First, it depends linearly on income. If income is zero, there is no money demand. Second, it depends on r. There are substitution effects between money holdings and the nominal interest rate. These are represented in the function $g(r)$.

Meltzer (1963) did not only consider an income, but also a wealth constraint. His regression results even suggest that assuming a wealth constraint is more reasonable, since its estimated money demand function is more stable. Nevertheless, I replicate Meltzer's (1963) money demand function with respect to an income constraint solely. This is done for three reasons:

1. Empirically, the results do not vary much using either of the two constraints. Both money demand functions are stable, but using Y is not as stable as using W.

2. Further studies presented in this thesis are based on an income constraint.

3. A direct connection between income and the money demand is consistent with the view that money is held to implement a desired transaction volume. In contrast, a wealth constraint leads to the interpretation of money as "an asset held for the services which it provides" (Meltzer (1963)). This is a much broader definition of money, since implementing a desired transaction volume is one of several services which money provides. The former interpretation of money is more appropriate for this thesis, because it corresponds to the interpretation of the search theoretic models presented in the sections 5 and 6.

3.2 Definition of money and stability of the money demand

Meltzer (1963) uses the following three money definitions:

1. M_1 is the measure for currency plus demand deposits.

2. M_2 contains M_1 plus time deposits at commercial banks.

3. M_3 includes M_2 plus saving deposits.

Meltzer (1963) tested the stability of the money demand functions using each of these three money definitions separately. One needs to specify $g(r)$ to analyze the money demand functions empirically. Meltzer (1963) assumes $g(r) = r^\eta$. Inserting this into (2), dividing both sides by the implicit price deflator P and taking logs yield

$$\ln \frac{M}{P} = a + \eta \ln r + \delta \ln \frac{Y}{P}. \tag{3}$$

The logarithm of the aggregated money holdings (M/P) equals the logarithms of the real GDP (Y/P) times δ plus the logarithm of the interest rate r multiplied by η. η and δ are the interest respectively income elasticity of the money demand. (3) can be re-expressed to the regression equation

$$\ln \frac{M_t}{P_t} = a + \eta \ln r_t + \delta \ln \frac{Y_t}{P_t} + u_t. \tag{4}$$

The subscript t refers to a particular time period t. If the money demand is homogenous of degree one in income, δ should be equal to one. Meltzer (1963) regressed (4) with respect to US data ranging from 1900 to 1958 and concludes the following:

1. The long run money demand function is stable. The interest rate and income elasticity estimates are both significant. These two variables explain most of the variance in the money demand. They have a multiple correlation of 0.98.

2. δ is approximately equal to one. This confirms the assumption that money demand depends linearly on income. The estimated value of η is approximately -0.8. Hence, a one percent point increase in the interest rate leads to a decrease in the money demand of 0.8 percent.

3. Using M_1 yields money demand functions which are stable. One should not broad the definition of money since broader measures bear the risk that substitution effects between times, saving, demand deposits and money lead to a bias in the estimated interest elasticity coefficient. M_1 is appropriate as it measures the direct effect of a changing interest rate on the money demand.

4 Inflation and Welfare

In Lucas (2000) the empirical findings from Meltzer (1963) are combined with the methodology used by Bailey (1956). In particular, Lucas (2000) replicates Meltzer's (1963) estimated money demand function with respect to an income constraint. Moreover, he reuses Bailey's methodology of measuring the costs of inflation. These correspond to the area below the inverse money demand function or equivalently stated to the fraction of income that people are willing to give up to reduce the inflation rate. In contrast to Bailey (1956), Lucas (2000) is interested in the welfare implications associated with intermediate inflation rates, while Bailey considered cases of acute hyperinflation. In particular, Lucas (2000) asks: How much income are people willing to give up to reduce the inflation rate from 10% to 0%? In the remainder of this chapter, Lucas' (2000) methodology is described and his results are compared to the results of Craig and Rocheteau (2008) and my own.

4.1 Theoretical money demand

In Lucas (2000), the money demand is a function of the nominal interest rate r and the real income y

$$\frac{M}{P} = L(r, y) = m(r)y. \tag{5}$$

Since Lucas (2000) assumes that the theoretical money demand function is homogenous of degree one in real income, the second equality of (5) holds. M/P depends linearly on y. Meltzer's (1963) findings, presented in the previous section, support this view. Lucas' (2000) money demand function can be rewritten as follows

$$m(r) = \frac{M/P}{y}. \tag{6}$$

$m(r)$ is specified in two ways

$$m(r) = Ar^{-\eta}, \tag{7}$$

$$m(r) = Be^{-\xi r}. \tag{8}$$

Inserting (7) into (6) and taking logs reveals the log-log relationship found in (3). Plugging (8) into (6) and taking logs results in a semi-log relationship. The logarithm of the real money holdings is linearly related to the interest rate and logarithmically related to real income.

Lucas (2000) collected two different time series which indicate the money demand for the United States ranging from $1900 - 1994$. One time series are the short-term nominal interest rates r_t, measured in the short-term commercial paper rate. Another time series report the public's money demand $m = \frac{M_t}{P_t y_t}$. Lucas (2000) uses M_1, $P_t y_t$ where y_t is the real GDP and P_t the GDP deflator.

Now, three approaches of fitting the theoretical money demand function to the real world data are presented.

4.1.1 Lucas' approach

Lucas (2000) concretely defines η (ξ) for the log-log (semi-log) case. Thereafter, A (log-log case) respectively B (semi-log case) are choosen such that the money demand curve passes through the geometric mean of the data pairs. Lucas (2000) computes money demand curves based on different values of η and chooses the money demand function which visually best matches the data. The interest elasticity parameter that gives the best visual fit in the log-log case is $\eta = 0.5$. The respective value in the semi-log case is $\xi = 7$. The calibrated parameter values are $A = 0.0488$ and $B = 0.3548$. These values are reported in Ireland (2009). Lucas (2000) computes the money demand by varying the nominal interest rate in a range between 0% and 16%. This allows to study the welfare cost of 10% in comparison to 0% inflation. Lucas' (2000) theoretical money demand functions for the log-log- and the semi-log case are plotted in *Figures (2)* and *(3)*. The vertical axis represents the real cash balances m. The horizontal axis measure the nominal interest rate r. The money demand functions indicate the amount of real cash balances that people are willing to hold at a given interest rate. *Figure (2)* (*Figure (3)*) shows the money demand functions for the log-log (semi-log) specification where the interest elasticity parameters are set to $\eta = 0.3, 0.5, 0.7$ ($\xi = 5, 7, 9$). Each data point in the figures represent a pair of the two time series for a given year.

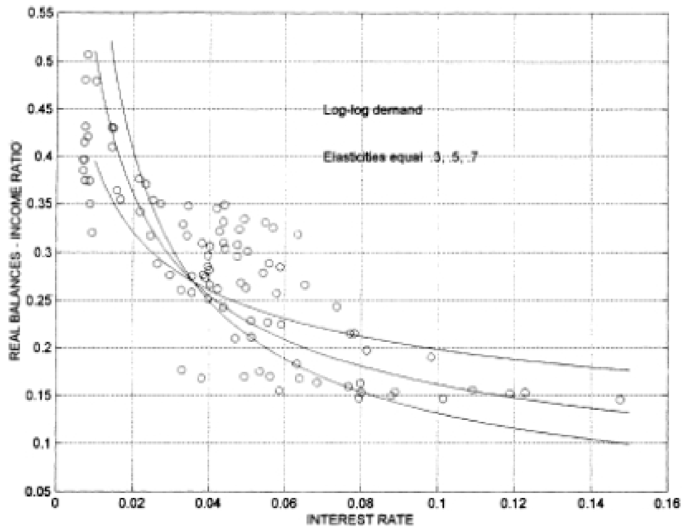

Figure 2: U.S. money demand, 1900-1994. (source: Lucas (2000))

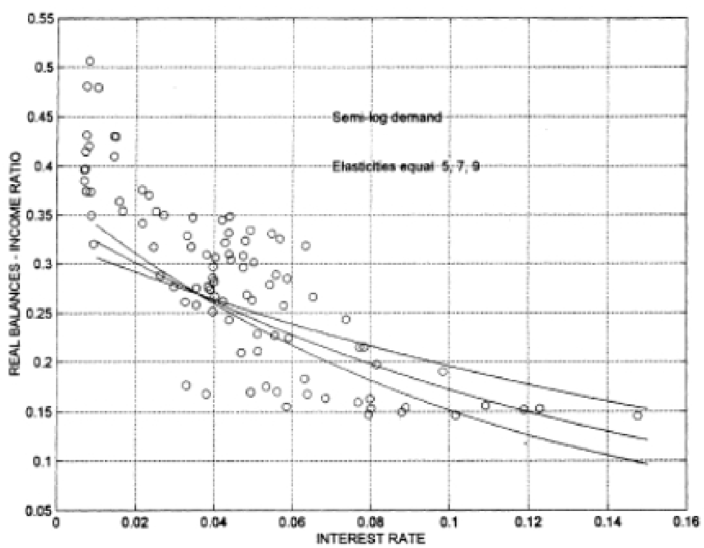

Figure 3: U.S. money demand, 1900-1994. (source: Lucas (2000))

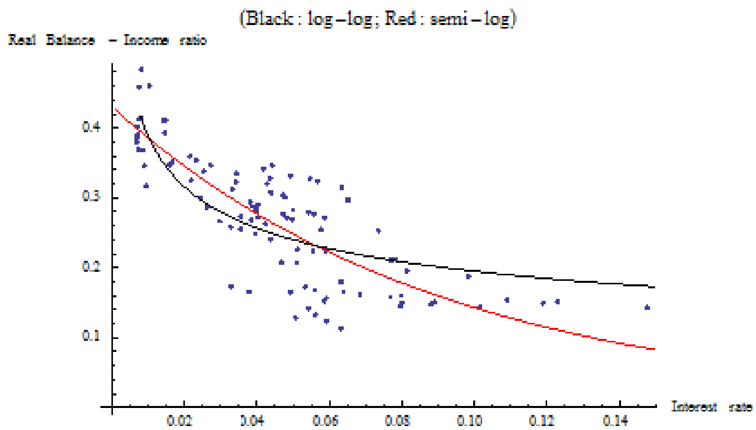

Figure 4: US money demand, 1900-2000.

4.1.2 Craig and Rocheteau's approach

Craig and Rocheteau (2008) applied a different approach. They calibrate the parameters A and η for the log-log- and B and ξ for the semi-log specification. Plugging in the calibrated parameter values into (7) and (8) the relationship between money demand and the nominal interest rate is obtained. My recalibration of Craig and Rocheteau (2008) yields the parameter values $A = 0.0978$, $\eta = 0.2995$ for the log-log- and $B = 0.4306$, $\xi = 11.0277$ for the semi-log case. Using this values the theoretical money demand function (5) can be constructed and compared to the actual data. The log-log- and semi-log money demand functions are computed in *Figure (4)*.

The money demand curve is computed for interest rates between 0% and 16%. The black (red) curve exibits the log-log (semi-log) money demand function.

4.1.3 Own approach

The third method is as a mixture between the first two approaches. It is introduced to build a bridge between their results. As in Lucas (2000), I assume $\eta = 0.5$ for the log-log- and $\xi = 7$ for the semi-log case. In contrast to Lucas (2000), A and B are calibrated as in Craig and Rocheteau (2008). The log-log- (black) and semi-log (red) money demand functions are computed in *Figure (5)*.

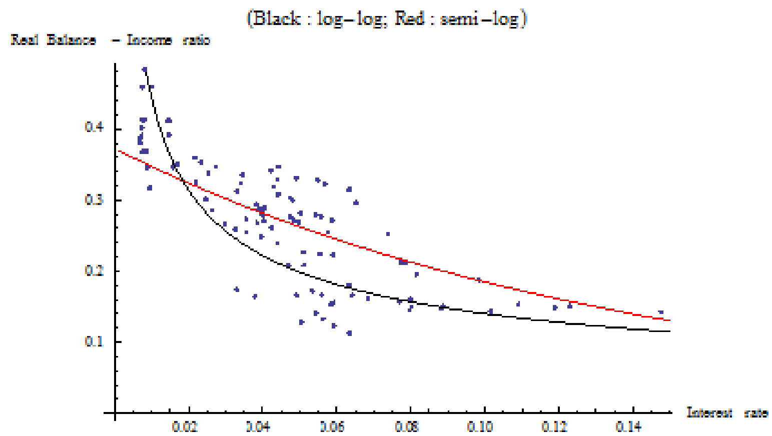

Figure 5: US money demand, 1900-2000.

4.2 Costs of inflation

All approaches presented in the previous section use Bailey's (1956) methodology to measure the costs of inflation. The theoretical money demand function $m(r)$ and its inverse $\psi(m)$ can be translated into a welfare cost measure by quantifying the area under the inverse money demand function. Lucas (2000) defines the welfare costs of inflation $w(r)$ as the "fraction of income people would require as compensation in order to make them indifferent between living in a steady state with an interest rate constant at r and an otherwise identical steady state with an interest rate of zero". This statement is captured in the formula

$$w(r) = \int_{m(r)}^{m(0)} \psi(x)dx = \int_{0}^{r} m(x)dx - rm(r). \tag{9}$$

The first equality of (9) indicates that the area under the inverse money demand function equals the costs of inflation. The bounds of the integral are given by the people's money demand $m(r)$ at the nominal interest rate r and the money demand $m(0)$ at the nominal interest rate 0. At higher nominal interest rates, the real cash holdings decrease as there is a negative relationship between money demand and the nominal interest rate. The term on the right hand side of (9) measure the area below the money demand function within bounds r and 0, minus the inflation tax effect which equals the seignorage revenue of the central bank. The inflation tax is the product of the nominal interest rate and the real cash balances demanded at this interest rate. Moreover, the nominal interest rate linearly depends on the inflation rate, as the inflation rate is perfectly anticipated. The log-log- and semi-log welfare cost function are as follows (proofs are provided in Appendix 9.1)

$$w(r) = A\frac{\eta}{1-\eta}r^{1-\eta}, \tag{10}$$

$$w(r) = \frac{B}{\xi}\left[1-(1+\xi r)\,e^{-\xi r}\right]. \tag{11}$$

Lucas (2000) estimates the welfare costs associated with 10% compared to 0% inflation. He shows that the costs of inflation for the semi-log- and the log-log money demand curve vary considerably at low levels of the nominal interest rate, but do not differ much at rates above 3%. Because of this, Lucas (2000) assumes that a nominal interest rate of 3% is associated with zero inflation. Thus, the welfare costs associated with a rise in the inflation rate from 0% to 10% is given by $w(0.13) - w(0.03)$. The three approaches yield different estimates for the parameters A and η (B and ξ) in the log-log (semi-log) case. Since these parameters define the welfare costs of inflation, the considered approaches yield different estimates. The following sections summarize the results.

4.2.1 Results Lucas

Lucas' (2000) approach using $\eta = 0.5$ and $A = 0.0488$ ($\xi = 7$ and $B = 0.3548$) report welfare gains of 0.91% (1.07%) in the log-log- (semi-log) case when inflation is reduced from 10% to 0%. Lucas (2000) log-log and semi-log welfare cost functions are plotted in *Figure (6).*

The horizontal axis measures the nominal interest rate. The vertical axis reports the fraction of income that people are willing to give up to reduce (increase) the nominal interest rate from r to 3%. The black (red) curve exhibits the log-log (semi-log) welfare cost function. A nominal interest rate above 3% leads to inflation while a nominal interest rate below 3% leads to deflation. Interestingly, *Figure 6* reveals that implementing a deflation rate improves welfare much more in the log-log- than in the semi-log case. Further, in both cases the *Friedman rule* is the optimal monetary policy. This

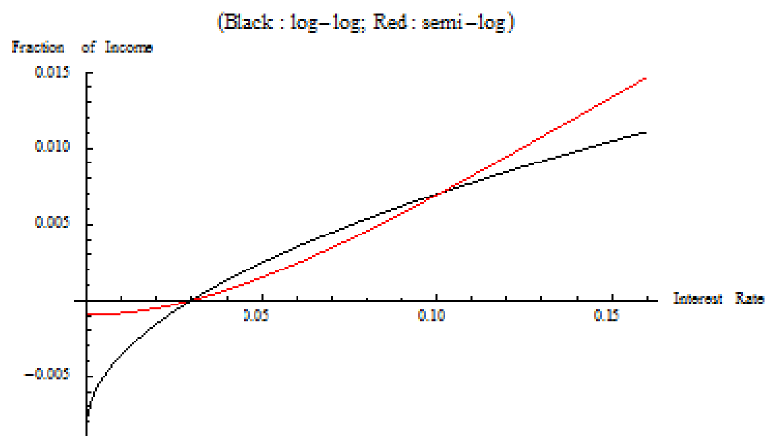

Figure 6: Welfare costs relative to 3% interest.

implies that the nominal interest rate should be set equal 0. This correspond to a deflation rate of 3%.

4.2.2 Results Craig and Rocheteau

Inserting the calibrated parameter values from Craig and Rocheteau (2008) into the welfare cost functions (10) and (11) yield different welfare cost estimates. Reducing the inflation rate from 10% to 0% is associated with a 0.64% (1.47%) gain in real income for the log-log (semi-log) specification. *Figure (7)* shows the log-log (black) and semi-log (red) welfare cost functions.

A comparison of *Figures (6)* and *(7)* reveals that Craig and Rocheteau's (2008) model exhibits smaller (larger) welfare costs for the log-log (semi-log) specification than in Lucas' (2000) model. At nominal interest rates higher than 3%, the semi-log welfare cost function reports higher costs of inflation than the log-log welfare cost function. Similar to Lucas (2000), a nominal interest rate below 3% implies that the welfare gains in the semi-log model is smaller than in the log-log model. There are two reasons that might explain the differences in the estimated welfare cost functions: First, in Craig and Rocheteau (2008) the parameter values for A respectively B are calibrated. But the money demand functions do not need to pass through the geometric mean of the data pairs as in Lucas (2000). Second, Lucas (2000) chooses values for the parameters η and ξ while Craig and Rocheteau (2008) calibrate these parameters as well. The differences in these parameter

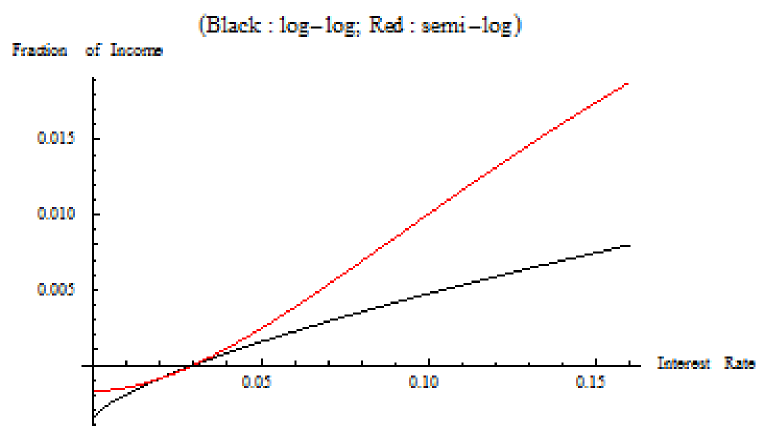

Figure 7: Welfare costs relative to 3% interest.

values are considerable: Lucas (2000) used $\eta = 0.5$, $A = 0.0488$ and $\xi = 7$, $B = 0.3548$ for the log-log- respectively semi-log specification, while my recalibration of Craig and Rocheteau's (2008) yield $\eta = 0.2995$, $A = 0.0978$ and $\xi = 11.0277$, $B = 0.4306$.

4.2.3 Own Results

The third method is introduced to understand the source which is responsible for the differences in the costs of inflation between the first two approaches. I calibrate the parameters A and B while setting the parameters $\eta = 0.5$ and $\xi = 7$. The calibration yields $A = 0.0444$ and $B = 0.3724$. Inserting these values into the log-log- (10) and semi-log (11) welfare cost function to calculate the costs of inflation. A reduction of the inflation rate from 10% to 0% is associated with a gain in real income of 0.83% (1.13%) for the log-log (semi-log) case. The estimated welfare costs for the log-log specification are higher than in the previous section and approximately equal as in section 4.2.1. The semi-log welfare cost function reports larger (lower) costs of inflation than in section 4.2.2 (section 4.2.1). *Figure (8)* exhibits the log-log (black) and the semi-log (red) welfare cost functions.

Comparing *Figures (8)* and *(6)* reveal similar patterns for the log-log- and the semi-log welfare cost functions concerning the two approaches. Nevertheless, there are differences. Here, the log-log specification reports higher costs of inflation than the semi-log specification for nominal interest rates

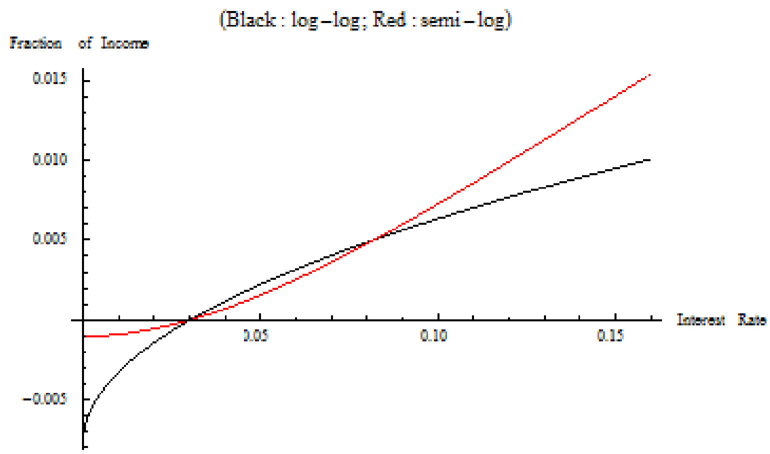

Figure 8: Welfare costs relative to 3% interest.

lower than 8.2%. For higher values, the semi-log specification reports higher costs of inflation. Lucas' (2000) respective threshold value is at the 10% nominal interest rate. In section 4.2.1 the semi-log specification always indicates higher costs of inflation. The different calibration technique used in Lucas (2000) and the third approach seem not to lead to major distinctions in the calibrated parameter values of A and B. Hence, I identify the second difference, calibration vs. parameterization of η and ξ, as the major source which leads to the differences in the calculated costs of inflation.

Considering these three approaches, the question of the most reliable parameter estimates arises. They are especially important, since they are needed to calculate the money demand and the welfare cost function. I argue that the second approach leads to the most reliable estimates, because it calibrates both parameters. Fixing the parameter η (ξ) as in Lucas (2000) and the third approach does not explain the money demand curve satisfactorily. Thus, a reduction of the inflation rate from 10% to 0% is most reliably estimated as a gain in real income of 0.64% (1.47%) for the log-log (semi-log) specification.

5 The Lagos and Wright Framework

In many macro models, the role of money in trading activities is not modeled explicitly. Instead, they use cash-in-advance constraints or include money into the utility function. Lagos and Wright (2005) pick a different approach. They use search theory to make the role of money explicit by describing the frictions of the markets in which agents meet for trading activities. Additionally, Lagos and Wright (2005) model the preferences of the consumers and the production technology of the sellers. Using these ingredients allow to micro found macro models. Before Lagos and Wright (2005), search theoretic models suffer from a lack of tractability such that they were inappropriate to study monetary policy devices. The Lagos and Wright (2005) model overcomes this weakness by making the agents money holdings degenerate. That implies all agents carry the same amount of money. In contrast to other frameworks that use numerical methods, the model presented here yields analytical results. Lagos and Wright (2005) explain the advantages of analytical models: "For one thing, simple models with sharp results are often a better vehicle than computer output for developing economic understanding." I now proceed by describing the Lagos and Wright (2005) framework. Thereafter, pricing mechanisms are presented.

5.1 Set up of Lagos and Wright

A continuum $[0,1]$ of agents live forever. Time is discrete and a period is divided into the two sub-periods day and night. Sequentially, a market opens in each sub-period. Agents can produce and consume in each of them. Production leads to costs whereas agents derive a utility gain from consuming a good. The utility gain depends on the quantity consumed. Goods cannot be stored and have to be consumed in the respective sub-period. Time periods are discounted by the time preference parameter $\beta = \frac{1}{1+r}$.

5.1.1 Day market

In the day market (DM) trades are executed bilaterally and anonymously on decentralised market places. Since there is anonymity, record keeping of agent's past trading behavior is not possible such that credit contracts can not be implemented. Trades base on *single coincidence of wants*. This means, agent 1 likes the good agent 2 produces but agent 2 does not like the good of agent 1 or vice versa. The agent that likes the good produced is called the buyer, while his trading partner is called the seller. Because there are no double coincidence of wants, barter trades are not feasible. Hence, trades are executed by exchanging goods in compensation for money. However, there are *search frictions* in the DM. They are modeled as the frequency of single coincidence of wants meetings, denoted as σ. Because both agents may like the good their opponent produces, the probability that there will be no trade is $1-2\sigma$. Since there are frictions in the DM, the good is called the search good.

5.1.2 Night market

There are no frictions in the night market. Trades are executed on a centralized *Walrasian* market (CM). The good produced and consumed is a general good. In the DM trades are executed with money. Since buyers consume but do not sell in the day market, they need to carry real cash balances at the beginning of the DM in order to attract a trading partner. In the CM, buyers and sellers produce and consume goods. All agents adjust their real cash balances to the level which they wish to bring into the next DM. If buyers want to increase their money holdings, they have to produce more goods than they consume. Sellers also use the night market to spend real cash balances which they have received by trades in the previous DM. Agents discount future periods and prefer immediate to future consumption. Bringing money into the next period bears cost if the nominal interest rate is larger than zero. At the end of the CM the next period begins with a new DM.

5.1.3 Preferences and welfare

The expected lifetime welfare of the society is given by

$$(1-\beta)\,V(q_b,q_s,X)=\sigma\left[u\left(q_b\right)-c\left(q_s\right)\right]+U(X)-C(X). \tag{12}$$

$V(\cdot)$ denotes the societies welfare at a given period. Welfare depends on the quantity consumed in the DM by the buyers q_b, the quantity produced by the sellers q_s and the aggregated consumption (production) of the general good X in the CM. The first term on the right hand side of (12) is the difference between buyers' utility of consumption and sellers costs of production multiplied by the frequency of trades. This term corresponds to the surplus in the DM. The second term is the difference between the agents utility of consumption and costs of production in the CM. This term corresponds to the surplus in the CM. Lagos and Wright (2005) use the following specifications:

1. Market clearing in the DM $q_b = q_s = q$.
2. Frequency of bilateral trades in the DM $\sigma = \frac{1}{2}$.
3. Buyers DM utility function $u(q) = \frac{q^{1-\eta}}{1-\eta}$.
4. Sellers DM cost function $c(q) = q$.
5. Agents CM utility function $U(X) = A \ln(X)$.
6. Agents CM cost function $C(X) = X$.

The DM and the CM utility functions satisfy $u'(q) > 0$; $u''(q) < 0$ given $q > 0$ and $U'(X) > 0$; $U''(X) < 0$ given $X > 0$, i.e the utility functions are increasing and concave if consumption is larger than zero. The cost functions are assumed to be linear with respect to the produced quantity. This linearity assumption implies that money holdings at the beginning of the DM are degenerate which means that all agents accumulate the same amount of money in the CM. This makes the model tractable and eliminates income effects. Given the specifications, the optimal quantity traded in the DM and CM can be calculated. The optimal quantity in the DM satisfies $c'(q^*) = u'(q^*)$ which yields $q^* = 1$. The optimal quantity in the CM satisfies $U'(X^*) = C'(X^*)$ which yields $X^* = A$. The maximal lifetime welfare of the society can be expressed as

$$(1-\beta) V(q^*, X^*) = \sigma [u(q^*) - c(q^*)] + U(X^*) - C(X^*). \tag{13}$$

5.1.4 Central bank

The value of money is determined through its role as media of exchange. The money supply varies over time due to central bank interventions. In a steady state, money is injected (withdrawn) in a lump-sum fashion at the beginning of the CM at a constant rate equal to τ. Hence, $M_{t+1} = M_t(1+\tau)$ characterizes the evolution of the money supply over time. In equilibrium τ equals π, i.e. the money growth rate correspond to the inflation rate. If $0 < 1+\tau < 1$, the monetary authority withdraws money from the market and implements a deflation rate of $\tau < 0$ by levying nominal taxes. Hence, the central bank is able to enforce agents to trade. If $1+\tau > 1$, the central bank injects money into the market by giving agents lump-sum transfers τM_{t-1} at the cost of a positive inflation τ. The parameter ϕ_t denotes the price of money in period t and thus equals $1/p_t$ where p_t is the nominal price of the general good. In a steady state, the money growth rate equals the change in the price level of the general good, i.e. $\frac{M_{t+1}}{M_t} = \frac{\phi_t}{\phi_{t+1}} = \frac{p_{t+1}}{p_t}$.

5.1.5 Equilibrium allocation

Lagos and Wright (2005) established that a monetary equilibrium satisfies the differential equation

$$\frac{z(q_t)}{M_t} = \beta \frac{z(q_{t+1})}{M_{t+1}} \left[\sigma \frac{u'(q_{t+1})}{c'(q_{t+1})} + 1 - \sigma \right]. \tag{14}$$

$z(q_t) = \phi_t M$ corresponds to the real balances an agent has to bring into the DM to buy a quantity q of the search good. To analyze the welfare consequences of inflation, it is necessary to focus on steady states. In a steady state, $z(q)$ is constant since the money growth rate equals the inverse growth rate of its price. Agents have incentives to accumulate money balances in the CM because they may want to consume in the next DM. Craig and Rocheteau (2008) have proven that the optimal quantity buyers want to consume in the next DM is given by

$$q = \arg\max \left[-iz(q) + \sigma \left[u(q) - z(q) \right] \right]. \tag{15}$$

Bringing money into the next period is associated with costs $-iz(q)$ such that agents suffer a loss from accumulating real cash balances as long as the central bank implements a nominal interest rate larger than zero. Solving (15) or considering (14) to be in a steady state give rise to the following relationship

$$\frac{u'(q)}{z'(q)} = 1 + \frac{i}{\sigma}. \tag{16}$$

(16) indicates that q and the nominal interest rate i are related to each other. The efficient outcome $q^* = 1$ is attained if two conditions are met: (i) the nominal interest rate equals zero and (ii) $z'(q) = c'(q)$. The first condition is fulfilled if the central bank implements the Friedman rule. The

second condition holds for special cases of the different pricing mechanisms. *Fisher's equation* states the relationship between the nominal interest rate, the real interest rate and the inflation rate

$$(1+i) = (1+r)(1+\pi). \tag{17}$$

For low values of r and π, $i \approx r + \pi$. Due to the direct relationship between the nominal interest rate and the inflation rate, (16) provides a direct link between the trading volume in the DM and the inflation rate π. This relationship is important to estimate the welfare cost of inflation.

5.2 Pricing mechanisms

The explicit form of $z(q)$ depends on the assumed pricing mechanism. To recalibrate the results of Craig and Rocheteau (2008), I use the same three pricing mechanisms. These are (i) Nash bargaining, (ii) proportional bargaining and (iii) markup.

5.2.1 Nash bargaining

In the Nash bargaining mechanism, buyers bargain with sellers bilaterally at trading conditions (q, d). q is the quantity traded, d the money transferred from buyers to sellers. The buyers' bargaining power is captured by the parameter θ. If θ equals 1, buyers have full bargaining power and obtain the entire surplus of the trade which is equivalent to $u(q) - c(q)$. This corresponds for example to a game where buyers make take it or leave it offers to sellers. According to Lagos and Wright (2005), the Nash bargaining solution is determined through

$$[q(m), d(m)] = \arg\max_{q,d\leq m} [u(q) - \phi d]^{\theta} [-c(q) + \phi d]^{1-\theta}. \tag{18}$$

Buyers spend all of their m money balances in the DM to buy the search good if the nominal interest rate is larger than zero, since accumulating cash is then associated with costs. If $i > 0$, $d = m$ and solving (18) yields

$$\phi m \equiv z_{\theta}(q) = \frac{\theta c(q) u'(q) + (1-\theta) u(q) c'(q)}{\theta u'(q) + (1-\theta) c'(q)}. \tag{19}$$

The buyers' share of the trade surplus $u(q) - z(q)$ is defined as

$$\Theta(q) = \frac{\theta u'(q)}{\theta u'(q) + (1-\theta)\, c'(q)}. \tag{20}$$

The buyers' share of the aggregated surplus decrease in quantity, since $\Theta'(q) < 0$ for any q. Craig and Rocheteau (2008) call this fact the *non-monotonicity inefficiency* of the Nash bargaining solution.

5.2.2 Proportional bargaining

In the proportional bargaining mechanism, buyers bilaterally bargain with sellers about the terms of trade in the DM. Again, θ captures the buyers' bargaining power. However, buyers are now able to extract a constant fraction of the total surplus

$$u(q) - z(q) = \theta\,(u(q) - c(q))\,. \tag{21}$$

Under these circumstances, the non-monotonicity inefficiency is obviously avoided because the buyers' share of the aggregated trade surplus does not depend on q. Solving (21) with respect to $z(q)$ yields

$$\phi m \equiv z_{\theta}(q) = \theta c(q) + (1-\theta) u(q). \tag{22}$$

5.2.3 Markup

I distinguish two concepts to implement a markup. The first is the one proposed by Craig and Rocheteau (2008). Buyers receive the quantity q of the search good and compensate sellers by paying them the production costs plus an additional constant markup μ. Therefore, the amount of money buyers have to accumulate in the CM in order to buy the quantity q in the DM equals

$$\phi m \equiv z(q) = (1+\mu)\, c(q). \tag{23}$$

Another, more sophisticated way to construct a markup is proposed by Lagos and Wright (2005). They make use of the Nash bargaining protocol described in section 5.2.1. The buyers' bargaining power θ is constructed such that it generates a markup μ that corresponds to the aggregated markup over the weighted sum of specific markups in the DM and the CM (for details, see the Appendix 9.2.2). This is different to the approach of Craig and Rocheteau

(2008) which only considers a markup in the DM. In both approaches $\mu = 0.1$. For a justification of this value see Basu and Fernald (1997).

5.3 Money demand in Lagos and Wright

Lagos and Wright (2005) define the money demand as follows

$$L(i) = \frac{M/P}{Y}. \tag{24}$$

Aggregated real balances M/P are proportional to real GDP multiplied by the inverse velocity $L(i)$ that depends on the nominal interest rate. Recall that the money holdings of agents are degenerate such that they accumulate the same amount of money in the CM. Agents carry $\phi M = M/P = z(q)$ units of money at the beginning of the DM. Buyers spend all their money in the DM if the nominal interest rate is larger than zero. Hence, the nominal GDP in the DM equals the frequency of bilateral trades multiplied by the aggregated money holdings, i.e. σM. In the CM there are no frictions and all agents consume the optimal quantity A. This quantity multiplied by the price of the general good correspond to the nominal GDP in the CM, i.e. $PX^* = PA$. Using this information, (24) can be rewritten as

$$L(i) = \frac{z(q)}{A + \sigma z(q)}. \tag{25}$$

(25) is the theoretical money demand function. It depicts the relationship between DM- and CM- consumption and the nominal interest rate. The quantity traded in the DM depends on the nominal interest rate and on the search frictions. Solving (16) with respect to q and using the specifications reported in section 5.1.3, q can be plugged into $z(q)$. Now, (25) is a function depending on the parameters A and η. These parameters are calibrated with nonlinear regression such that they fit the US money demand data between 1900 and 2000. Using the calibrated values of A and η allow to compute the theoretical money demand function. Thereafter, the money demand curve can be compared to the data.

5.4 Compensated welfare cost of inflation

Lagos and Wright (2005) determine the society's lifetime welfare at a steady state inflation rate of τ as follows

$$(1-\beta)\, V(\tau) \equiv U(X^*) - X^* + \sigma\left[u(q_\tau) - q_\tau\right]. \tag{26}$$

If the inflation rate is reduced (increased) from τ to 0 and the DM-, CM-consumption is reduced (increased) by a factor $(1-\Delta)$ at the same time, the society's welfare is given by

$$(1-\beta)\, V_{(1-\Delta)}(0) \equiv U\left[X^*(1-\Delta)\right] - X^* + \sigma\left[u\left(q_0(1-\Delta)\right) - q_0\right]. \tag{27}$$

q_τ (q_0) represents the consumed quantity of the search good if the inflation rate is equal to τ (0). Δ reflects the percentage of GDP that people are willing to give up to have an inflation rate of 0 instead of τ. Hence, to measure the welfare costs of inflation, Lagos and Wright (2005) propose to solve (28) for Δ

$$V_{(1-\Delta)}(0) = V(\tau). \tag{28}$$

Using the functional forms from section 4.1.3, (28) can be reformulated to

$$\frac{(q_0)^{1-\eta}}{1-\eta}(1-\Delta)^{1-\eta} - \frac{A}{\sigma}\Delta = \frac{(q_\tau)^{1-\eta}}{1-\eta} + q_0 - q_\tau. \tag{29}$$

(29) corresponds to the compensated welfare cost measure of Craig and Rocheteau (2008). Since (28) only resembles (29) for low values of Δ, it is called a compensate welfare cost measure. For details, see the Appendix 9.4.

6 Financial Intermediation

Financial intermediaries are institutes deal in with agents that have idle money balances and agents that have a lack of funds. They take deposits from the former and make loans to the latter. There is only a role for financial intermediation if some agents wish to get rid of their idle cash balances while others want to relax their cash constraint. In the model of Berentsen et al (2007) agents experience a preference shock at the beginning of the DM. Because of this, some agents hold idle cash balances which creates a role for financial intermediation. The differences of Berentsen et al (2007) to Lagos and Wright (2005) are shortly explained in the following sections. The main question is, if the introduction of financial intermediation leads to a reduction of the welfare costs of inflation.

6.1 Set up of Berentsen et al

Berentsen et al (2007) introduce financial intermediaries into the Lagos and Wright (2005) framework such that agents have the possibility to lend or borrow money from financial institutions. These institutions are modelled as banks. Agents experience a preference shock at the beginning of the DM. More precisely, a proportion $(1-n)$ (n) of all agents become buyers (sellers). Buyers (sellers) are able to consume (produce), but not to produce (consume). Since sellers do not intend to consume in the DM, they have no use for their money balances and want to get rid of them as long as they receive at least a small compensation for their lendings. This requires the following additional dimensions compared to Lagos and Wright (2005).

6.1.1 Central bank

The central bank is able to control the money supply. In a steady state, the money supply grows (decreases) by a constant factor γ, i.e. $M_{t+1} = \gamma M_t = (1+\tau) M_t$. In contrast to Lagos and Wright (2005), the central bank distributes part of the lump-sum transfers at the beginning of the DM as well as at the beginning of the CM. $\tau_1 M_t$ $(\tau_2 M_t)$ is the amount distributed in the DM (CM). The sum of the transfers equals τ. Hence, every agent, irrespective of its type, holds some money balances at the beginning of the DM if the inflation rate is strictly positive.

6.1.2 Banks

Banks act as financial intermediaries between sellers and buyers. If banks pay a positive interest rate i_d on deposits, they are able to attract money from sellers. The amount of money that banks collect at the beginning of the DM is denoted by d. At the beginning of the next CM banks pay sellers back $(1+i_d)\, d$. The accumulated deposits are lent to buyers such that they are able to relax their liquidity constraint in the DM. Buyers are now able to consume more of the DM good than they could with the accumulated cash balances from the previous CM. At the beginning of the next CM, buyers have to pay back $(1+i)\, l$ to the banks, where l denotes the lendings and i the interest rate banks claim in compensation for issuing loans. There is perfect competition between financial intermediaries such that banks can enter into the market without costs. In equilibrium banks earn zero profits because $i_d = i$. Finally, it is assumed that banks have a costless record keeping technology which allows them to observe agents' financial histories but not their trading histories.

6.1.3 Credit and enforcement

Banks issuance of loans to buyers and their repayment is lagged by one subperiod. Hence, contracts between buyers and banks are on the basis of credit. Credit contracts are restricted to one period. The existence of credit in the context of a search model let two questions arise: (I) Is money necessary in an environment where credit contracts can be implemented? (II) Can banks ensure that buyers repay their debt in the CM? Concerning the first question, one can argue that money is still important because DM trades between buyers and sellers are anonymous such that agents cannot identify their trading partners. Sellers therefore call for immediate compensation for their accrued production costs and buyers have to pay with money. Concerning the second question, Berentsen et al (2007) assume (i) that banks are able to enforce repayment at zero costs or (ii) that they cannot enforce repayment, but are able to exclude buyers from the financial credit market in all future periods if they miss to repay their debt. Irrespective if one considers (i) or (ii), the costs of inflation are calibrated in the same way. Therefore, without loss of generality, I restrict attention to case (i).

6.1.4 Competitive pricing

Due to a preference shock, the probability to be a buyer is $(1-n)$, the probability to be a seller is n. In contrast to the pricing mechanisms presented in section 4.2, terms of trade are not defined through bilateral meetings of buyers and sellers. DM trades take place in big anonymous markets using Walrasian pricing. Sellers produce the quantity that buyers consume, i.e. $(1-n)\,q_b = nq_s$. Buyers exactly compensate sellers accrued production costs. $z(q)$ is defined as follows

$$z(q) = c(q). \tag{30}$$

Competitive pricing differs from Nash-, proportional bargaining and constant markup in the way search frictions arise. In case of competitive pricing, agents get a preference shock at the beginning of the DM what restricts them to be either a buyer or a seller. Buyers always like the goods that are produced by sellers. This stands in contrast to the other pricing mechanisms, where all agents are able to produce and consume in the DM, but not each agent likes the good his bilateral trading partner produces. The introduction of preference shocks and the absence of trading frictions have the following implications on the definition of the society's lifetime welfare

$$(1-\beta)\,W\left(q_b, q_s, X\right) = (1-n)\,u\left(q_b\right) - nc(q_s) + U(X) - X. \tag{31}$$

There are two differences between (31) and (12), the society's lifetime welfare stated in Lagos and Wright (2005). First, search frictions σ are replaced by agents specific preference shocks. Second, since trades are not bilateral, the quantity produced by a seller q_s is not necessarily equal to the quantity consumed by a buyer q_b.

6.1.5 Equilibrium allocation

Berentsen et al (2007) show that a monetary equilibrium with competitive pricing and financial intermediation, where banks can enforce buyers to repay their debt, satisfies

$$\frac{\gamma-\beta}{\gamma} = (1-n)\left[\frac{u'(q_b)}{c'\left(\frac{1-n}{n}q_b\right)} - 1\right] + ni_d. \tag{32}$$

The first term on the right hand side of (32) reflects the buyers' marginal surplus of bringing an additional unit of money into the DM multiplied with the proportion of buyers. The second term reflects the sellers' marginal return depositing an additional unit of money at the banks multiplied with the proportion of sellers. Thus, the right hand side is the surplus that an additional unit of money generates in the DM. Buyers are able to lend money from banks with marginal cost i. Buyers lend money from banks till its marginal product equals its marginal cost. Hence (32) can be reformulated to

$$\frac{\gamma-\beta}{\gamma} = (1-n)\,i + ni_d \overset{i_d=i}{=} i. \tag{33}$$

If there is perfect competition between financial intermediaries, banks make zero profits in equilibrium such that the second equality of (33) holds. (33) links the nominal interest rate i, the real interest rate and the inflation rate. Rearranging yields that (33) is equivalent to the Fisher equation stated in (17). Berentsen et al (2007) prove that the nominal interest rate and the quantity consumed by buyers in the DM are related as follows

$$\frac{u'(q_b)}{c'\left(\frac{1-n}{n}q_b\right)} = 1 + i. \tag{34}$$

In comparison, the same relationship in the model without financial intermediation is given by

$$\frac{u'(\hat{q}_b)}{c'\left(\frac{1-n}{n}\hat{q}_b\right)} = .1 + \frac{i}{(1-n)} \tag{35}$$

The DM consumption level with financial intermediation q_b is larger than the DM consumption level without financial intermediation $\hat{q}_b$ for any fixed i. Hence, the introduction of financial intermediaries ceteris paribus improves welfare.

Berentsen et al (2007) additionally consider the Nash bargaining pricing mechanism where it is equally probable to be a buyer or a seller, i.e. $n = (1-n) = 1/2$. Sellers and buyers bargain bilaterally and the former produce the quantity that the latter want to consume. The society's lifetime welfare function stated in (31) coincides with the one of Lagos and Wright (2005) stated in (12) if the search friction parameter takes the value $\sigma = 1/2$. This is the essential assumption that makes Craig and Rocheteau (2008) and Berentsen et al (2007) comparable. Under Nash bargaining with financial intermediation, DM consumption and the nominal interest rate are related as follows

$$\frac{\gamma - \beta}{\gamma} = \frac{u'(q)}{z'(q)} - 1 = i. \tag{36}$$

Comparing (36) and (16) reveal that the quantity consumed in the DM with financial intermediation is larger than the one without financial intermediation, given any nominal interest rate larger than zero. In these cases, the introduction of financial intermediaries improve the allocation. However, at the Friedman rule financial intermediation do not improve welfare since accumulating cash balances is costless such that agents have no benefit from depositing or lending cash balances. This can also be seen formally since for competitive pricing (Nash bargaining) (34) and (35) ((16) and (36)) coincide and therefore lead to the same DM consumption level.

6.2 Money demand with financial intermediation

The money demand is defined as in (24). In case of a positive nominal interest rate, accumulating money balances or lending money from banks is associated with costs. Hence, buyers spend all of their accumulated cash from the CM plus the lendings from the banks to buy the search good in the DM. Market clearing in the credit sector requires that the lendings of the buyers equal the deposits of the sellers. The nominal GDP in the DM is $(1-n)\,M$, i.e. the proportion of buyers times the aggregated nominal money balances they spend. The nominal GDP in the CM is $PX^* = PA$. M/P correspond to the aggregated real cash balances of the buyers. Since they spend all their money holdings in the DM $M/P = z(q)$ for Nash-, proportional bargaining and constant markup while $M/P = c\left(\frac{1-n}{n}q_b\right)$ in the case of competitive pricing. Using these, the theoretical money demand function for Nash-, proportional bargaining and constant markup can be expressed as follows

$$L(i) = \frac{z(q)}{(1-n)\,z(q) + A}. \tag{37}$$

In case of competitive pricing, the theoretical money demand can be expressed as

$$L(i) = \frac{c\left(\frac{1-n}{n}q_b\right)}{(1-n)\,c\left(\frac{1-n}{n}q_b\right) + A}. \tag{38}$$

7 Quantitative Analysis

In this section, I analyze the models of sections 5 and 6 quantitatively by calibrating the sensitive parameters of the theoretical money demand function for the models with and without financial intermediation. The sensitive parameters are η and A. η is a coefficient of the buyers' utility function that determines their relative risk aversion in consuming the DM good while A is the optimal quantity consumed by each agent in the CM. I use nonlinear regression to fit these parameters to the US data that are reported in Craig and Rocheteau (2008). Then, the calibrated parameter values are used to compute the theoretical money demand function. Finally, the calibrated parameters enable to measure the costs of inflation. I consider all pricing mechanisms presented in this thesis, i.e. (i) Nash bargaining, (ii) proportional bargaining, (iii) constant markup and (iv) competitive pricing.

Moreover, I analyze two versions of each pricing mechanism, one with and one without financial intermediation and compare the results. In particular, I focus on the following questions: Does financial intermediation lower the welfare costs of inflation? If yes, through which channels? In the theoretical model of Berentsen et al (2007), the introduction of financial intermediaries improves the welfare by raising the quantity consumed in the DM. However, the coefficients η and A are unaffected by financial intermediation. This is not true if one calibrates the theoretical model to the data. In this case, the calibrated parameter values vary and therefore influence buyers' relative risk aversion and agents optimal consumption level in the CM.

7.1 Nash bargaining

I consider the following parameter values for the buyers bargaining power: $\theta = 1, 0.6, 0.5, 0.3$. These values correspond to those used by Craig and Rocheteau (2008). This allows me to compare the results of the existing literature with the new results that include financial intermediation. A fifth value for θ is constructed such that the aggregated and weighted DM and CM markups yield an average markup of μ. The idea originates from Lagos and Wright (2005) where an average markup of 10% at a benchmark inflation rate of 4% is presumed. The calibration results are presented in *Tables 1* and *2* for the models without and with financial intermediation.

TABLE 1. Nash bargaining without financial intermediation

θ	A	η	$q_{0.1}$	q_0	q_F	$\Delta_{0.1}$	Δ_F
1	1.825	0.144	0.201	0.668	1	1.47%	1.58%
0.6	1.796	0.227	0.160	0.557	0.879	2.66%	3.08%
0.5	1.772	0.265	0.141	0.501	0.814	3.28%	3.94%
0.3	1.620	0.379	0.082	0.306	0.555	5.57%	7.70%
0.325	1.620	0.361	0.091	0.338	0.601	5.19%	6.99%

TABLE 2. Nash bargaining with financial intermediation

θ	A	η	$q_{0.1}$	q_0	q_F	$\Delta_{0.1}$	Δ_F
1	1.839	0.075	0.196	0.674	1	0.76%	0.80%
0.6	1.826	0.122	0.175	0.619	0.944	1.32%	1.46%
0.5	1.818	0.144	0.164	0.591	0.914	1.61%	1.82%
0.3	1.768	0.225	0.126	0.478	0.785	2.85%	3.46%
0.2	1.666	0.307	0.087	0.347	0.616	4.37%	5.83%

The first column in *Table 1* and *2* reports the bargaining power of the buyers. The parameters A and η are calibrated such that the theoretical money demand gives the best possible fit to the US data. Columns 4 and 5 present the quantities buyers consume in the DM at a given inflation rate. q_π is the quantity traded at the inflation rate π. As in the previous sections a nominal interest rate of 3% is consistent with zero inflation. q_F denotes the quantity consumed at the Friedman rule. Column 7 indicates the fraction of income people are willing to give up to reduce the inflation rate from 10% to 0%. Column 8 indicates the respective measure if the inflation rate is reduced from 10% to -3%. There are similarities and differences in the calibration results reported in *Table 1* and *2*:

i. Similarities: In both models, a higher θ yields lower costs of inflation and higher consumption levels in the DM. This is due to the *hold-up problem* detected in Lagos and Wright (2005). Sellers are able to extract part of the aggregated surplus in the DM if they have some bargaining power. Because of this, agents do not get the hole surplus of their spendings which decrease their incentive to accumulate money balances since this is costly. Hence, agents marginal benefit of accumulating an additional unit of money do not correspond to the society's benefit. The efficient outcome is determined through $q^* = 1$. Lagos and Wright (2005) have proven, if $\theta < 1$ the efficient consumption level cannot be attained. Additionally, accumulating real cash balances is associated with opportunity costs if the nominal interest rate is larger than zero. This second inefficiency is discussed extensively in the previous sections. Thus, the welfare maximizing consumption level in the DM can only be attained if buyers have full bargaining power $\theta = 1$ and the nominal interest rate equals zero. Because there is a hold-up problem if $\theta < 1$, the area under the inverse money demand function does not correspond to the welfare costs of inflation. This can easily be seen by comparing the money demand functions which result with and without financial intermediation. *Figure 9* plots these theoretical money demand curves against the US data. The red (black) curve results from the calibration with (without) financial intermediation. Obviously, the two approaches yield nearly identical theoretical money demand functions while the costs of inflation differ substantially. Second, both calibrations indicate that a decrease in θ leads to a decrease in A and an increase in η.

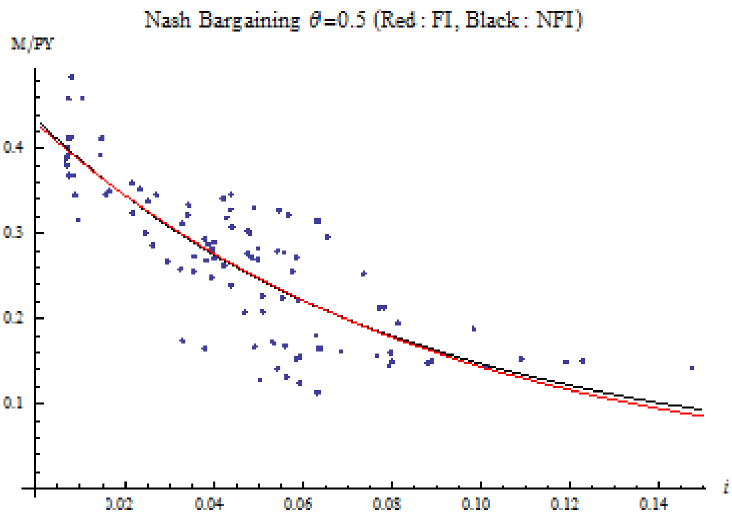

Figure 9: Data and money demand.

ii. Differences: The calibration results reported in *Table 1* differ from *Table 2* in the parameters A and η as well as in the quantity consumed in the DM. First, given θ, A is larger in the model with financial intermediation. For lower values of θ, the discrepancies in A between the two calibrations get larger. Because of higher calibrated values of A, financial intermediation improves welfare in case of Nash bargaining. Second, given any θ, the calibrated values of η are much lower when financial intermediaries are included. For $\theta = 1$ e.g., η with financial intermediation is approximately half the size of the value without. For lower values of θ, the differences get larger in absolute but smaller in relative terms. In any case, financial intermediation lowers the relative risk aversion of the buyers. Third, the quantity consumed in the DM generally increases with financial intermediation. However, there is an exception if the inflation rate equals 10% and $\theta = 1$. Forth, the parameter $\theta_{1+\mu}$ - which is the buyers bargaining power that leads to an average markup of $\mu = 10\%$ - is lower with ($\theta_{1+\mu} = 0.2$) than without ($\theta_{1+\mu} = 0.325$) financial intermediation. For the special case $\theta_{1+\mu}$, A does not differ much between the two models, but η is substantially lower in the model with financial intermediation. Moreover, the costs of inflation are lower with financial intermediation while the impact on q for different rates of inflation is ambiguous. Fifth, introducing fi-

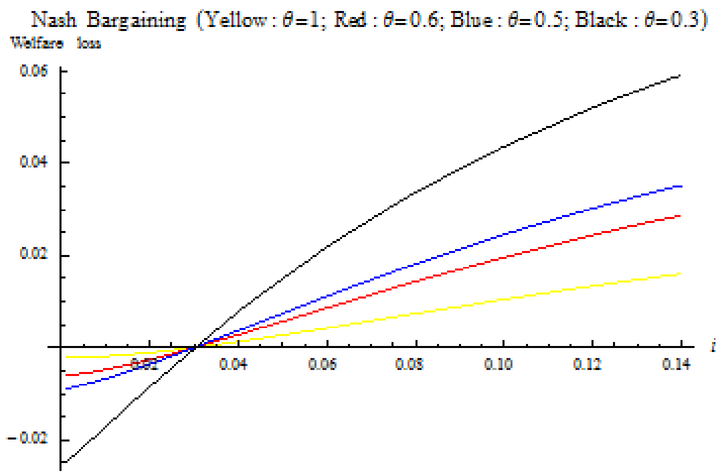

Figure 10: Nash bargaining without financial intermediation.

nancial intermediaries lower the welfare costs of inflation substantially. Given θ, the welfare cost estimates with- are about half of the size of the estimates without financial intermediation. *Figure 10* and *Figure 11* provide the relationship between the nominal interest rate and the costs of inflation for $\theta = 0.3, 0.5, 0.6, 1$ concerning the models without and with financial intermediation, respectively.

Comparing *Figures 10* and *11* reveal that for any nominal interest rate $i \in [0, 14]$, the costs of inflation with financial intermediation are approximately half of the size of the costs without. Hence, the introduction of financial intermediaries considerably lower the negative consequences of perfectly anticipated inflation.

7.2 Proportional bargaining

Again, I consider the parameter values $\theta = 1, 0.6, 0.5, 0.3$ in case of proportional bargaining. *Tables 3* and *4* show the results for proportional bargaining concerning the models without and with financial intermediation, respectively.

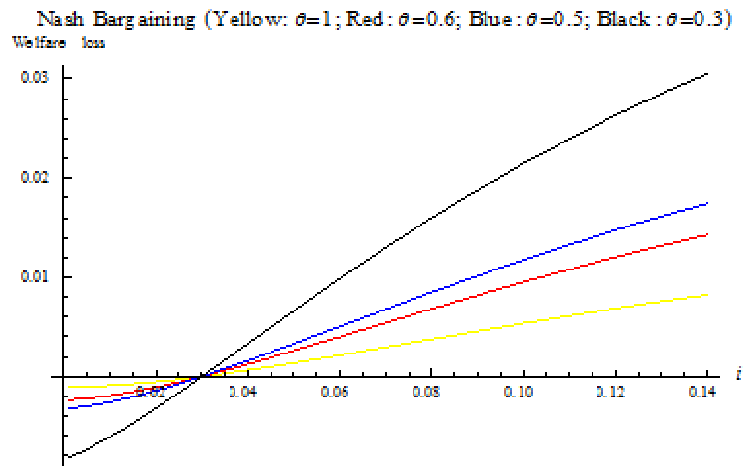

Figure 11: Nash bargaining with financial intermediation.

TABLE 3. Proportional bargaining without financial intermediation							
θ	A	η	$q_{0.1}$	q_0	q_F	$\Delta_{0.1}$	Δ_F
1	1.825	0.144	0.201	0.668	1	1.47%	1.58%
0.6	2.059	0.226	0.155	0.645	1	2.59%	2.77%
0.5	2.188	0.264	0.134	0.635	1	3.19%	3.42%
0.3	2.811	0.403	0.056	0.596	1	6.14%	6.52%

TABLE 4. Proportional bargaining with financial intermediation							
θ	A	η	$q_{0.1}$	q_0	q_F	$\Delta_{0.1}$	Δ_F
1	1.839	0.075	0.196	0.674	1	0.76%	0.80%
0.6	1.951	0.122	0.174	0.664	1	1.30%	1.38%
0.5	2.010	0.144	0.163	0.659	1	1.58%	1.67%
0.3	2.266	0.229	0.121	0.640	1	2.78%	2.95%

The columns in *Table 3* and *Table 4* contain the same set of parameters as in the previous section. The results of the two calibrations reveal some similarities and some differences:

i. Similarities: In both calibrations, the estimated values of A and η increase if θ decrease. Comparing $q_{0.1}$ and q_0 exhibit that 10% inflation reduces DM consumption considerably. At the Friedman rule the efficient consumption level $q^* = 1$ is attained in both models. This is not the case under Nash Bargaining. Craig and Rocheteau (2008) explain this finding by means of two inefficiencies that arise if $\theta \neq 1$. First, the *rent-sharing externality* reflects the difference in the marginal benefit of money balances from a buyer's compared to the society's point of view. This externality is present under Nash and proportional bargaining. Second, the non-monotonicity inefficiency only arises under Nash bargaining because the buyers share of the trade surplus does not increase with its bargaining power. The non-monotonicity inefficiency is not present under proportional bargaining, because if θ increases the buyers part of the match surplus increases too. Craig and Rocheteau (2008) show that this second inefficiency is responsible for the inefficiently low consumption level at the Friedman rule in case of Nash bargaining. Because there is no such inefficiency in case of proportional bargaining, the consumption levels in the models with and without financial intermediation are optimal at the Friedman rule.

ii. Differences: The calibrated values of A are usually lower if financial intermediation is introduced. If $\theta = 0.3$, the relative difference between the two models is about 19.4 percent. Moreover, the calibrated values of η with financial intermediation are much lower than without. The relative difference is approximately 48% for $\theta = 1$ and about 43% for $\theta = 0.3$. Thus, lower θ lead to lower differences in η. Comparing the steady state consumption levels in the DM at the 10% inflation rate demonstrates that consumption is higher in the model with financial intermediation. Again, there is an exception at $\theta = 1$. In this case, the solutions under proportional- coincide with the solutions under Nash bargaining. In both cases, buyers have full bargaining power and make take it or leave it offers to sellers which enables the former to extract all of the match surplus. If $\theta = 0.3$, consumption $q_{0.1}$ with- is more than twice as large as without financial intermediation. The results are similar for q_0. The lower θ, the larger are the differences in q_0. If $\theta = 0.3$ and financial intermediaries are introduced, the buyers consume about 6.9% more of the search good than without. Finally, a reduction of the inflation rate from 10% to 0% yields much lower welfare gains in the case where financial intermediaries are included. Roughly speaking, financial intermediation lowers the costs of inflation by half. Considering $\theta = 1$, the costs of inflation with financial intermediation are about 50.6% of the level they reach without financial intermediation. As θ decreases, these relative differences are getting smaller. Nevertheless,

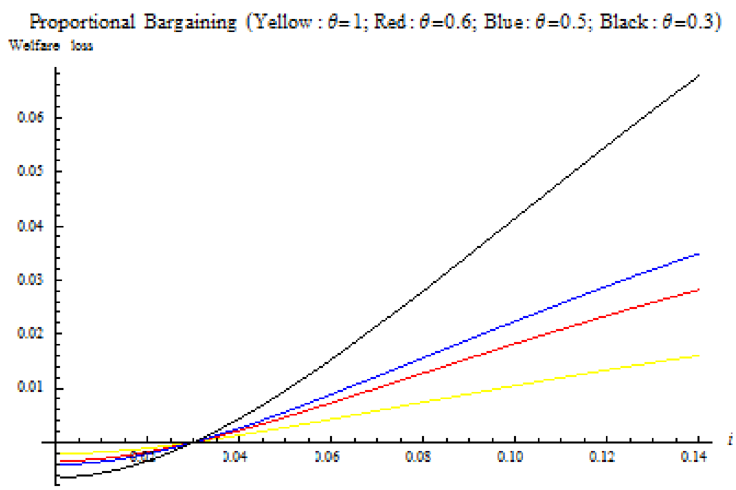

Figure 12: Proportional bargaining without financial intermediation.

the costs of inflation remain considerably large with financial intermediation, then they still range between 0.8% and 2.95%. If $\theta < 1$, the costs of inflation with financial intermediation are even larger than in Lucas (2000). *Figures (12)* and *(13)* exhibit the relationship between the nominal interest rate and the welfare costs of inflation for the two models.

The welfare cost functions are constructed using the nominal interest rates $i \in [0, 0.14]$. A comparison of *Figures (12)* and *(13)* confirm the view that the costs of inflation with financial intermediation are approximately half of the size of the costs without. Moreover, the welfare cost functions in case of proportional bargaining have a more convex shape while the ones in case of Nash bargaining are concave for nominal interest rates larger than zero. The welfare gains if the inflation rate is reduced from 10% to 0% are of similar size for Nash- and proportional bargaining, given the models with respectively without financial intermediation. Nash bargaining generally yields slightly higher estimates, except for $\theta = 0.3$ in the model without financial intermediation.

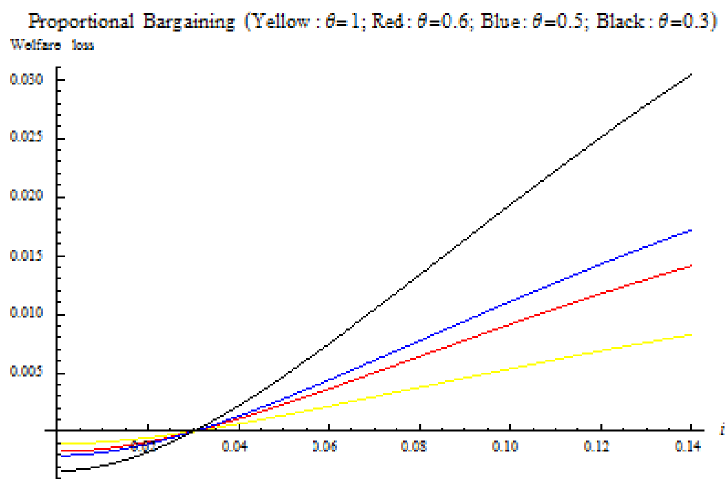

Figure 13: Proportional bargaining with financial intermediation.

7.3 Constant markup

Under constant markup, there is no bargaining power parameter θ. Nevertheless, sellers are able to extract some of the match surplus which is captured by the parameter μ. This parameter depicts the markup that sellers claim in addition to their accrued production costs. The markup can be interpreted as the sellers market power. I consider two models: One that includes financial intermediation and one that does not. To facilitate the comparison with Craig and Rocheteau (2008), I use the same values for the markup parameter. *Tables 5* and *6* show the calibration results for the models without respectively with financial intermediation.

TABLE 5. Constant markup without financial intermediation

μ	A	η	$q_{0.1}$	q_0	q_F	$\Delta_{0.}$	Δ_F
0	1.825	0.144	0.201	0.668	1	1.47%	1.47%
0.1	1.037	0.144	0.104	0.345	0.516	2.45%	2.45%
0.2	0.619	0.144	0.057	0.189	0.282	3.26%	3.26%

TABLE 6. Constant markup with financial intermediation

μ	A	η	$q_{0.1}$	q_0	q_F	$\Delta_{0.1}$	Δ_F
0	1.839	0.075	0.196	0.674	1	0.76%	0.80%
0.1	0.567	0.075	0.055	0.189	0.281	1.75%	2.37%
0.2	0.194	0.075	0.017	0.059	0.088	2.58%	3.67%

Columns two to eight include the same set of parameters as in the previous sections. Note, if $\mu = 0$ the calibration results are equal to the results under Nash- and proportional bargaining if $\theta = 1$. Again, I find similarities and differences in the calibration results of the two models.

i. Similarities: In both models, higher values of μ yield lower values of A. In contrast, μ does not seem to have an influence on η, since the calibrated value of η does not vary with μ. At the Friedman rule the efficient quantity is not traded since there is still a positive markup. Moreover, higher markups are associated with much lower consumption levels in the DM, irrespective if $q_{0.1}, q_0$ or q_F is considered. In both models, the costs of inflation increase for higher values of μ.

ii. Differences: If $\mu > 0$, calibrations yield lower values for A with financial intermediation. The differences are substantial. If $\mu = 0.1$, the optimal consumption level in the CM with financial intermediation is approximately half of the size of the calibrated value without. The difference increases if $\mu = 0.2$ and even reaches with financial intermediation less than one third than without. In the model with financial intermediation the calibrated values of η obtain approximately 52% of the values without financial intermediation. Considering constant markup, the introduction of financial intermediaries lowers the calibrated values of A and η remarkably. The quantity consumed in the DM is generally considerably lower in the model with- than in the model without financial intermediation. Higher μ lead to larger differences in the DM consumption levels among the two models. If $\mu = 0.1$, the values for $q_{0.1}, q_0, q_F$ in the model with financial intermediation are about half of the size of the model without financial intermediation. If $\mu = 0.2$, the values reach approximately 30%. There is an exception if $i = 0$ and $\mu = 0$. Then, the efficient quantity is consumed in both models since accumulating cash balances is costless and sellers are not able to withdraw part of the surplus. It seems surprising, that DM consumption decreases if financial intermediaries are introduced, since the theoretical model of Berentsen et al (2007) predicts the opposite. Nevertheless, costs of inflation $\Delta_{0.1}$ are lower when financial intermediaries are introduced. More precisely, for any μ, the costs $\Delta_{0.1}$ are about 0.7% lower. Because the costs of inflation increase with higher values of μ, the relative gain from introducing financial intermediaries is getting smaller.

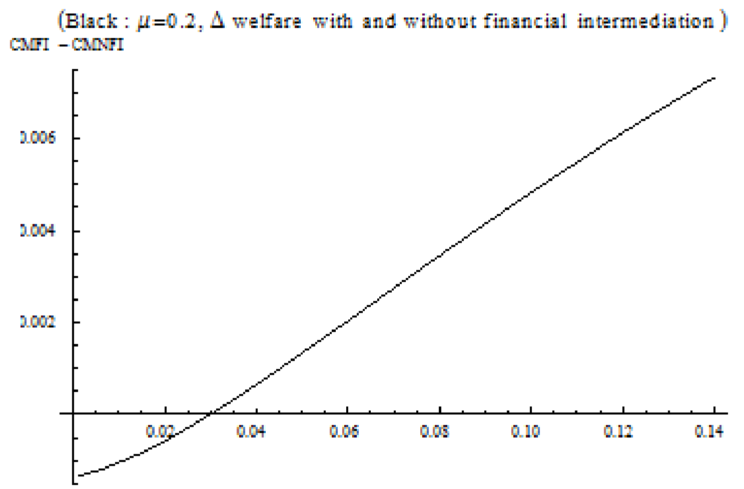

Figure 14: Differences in the costs of inflation.

If $\mu = 0$, the relative gain equals 48.3% while it equals 20.9% if $\mu = 0.2$. Surprisingly, the model with financial intermediation does not always report lower costs of inflation. Exceptions can be found in *Figure 14*. The horizontal axis indicates the nominal interest rate while the vertical axis measures the difference between the welfare cost function with (CMFI) and the welfare cost function without financial intermediation (CMNFI). Hence, a positive (negative) value indicates that the costs of inflation are lower (higher) in the model with financial intermediation than in the model without. The curve is computed for $\mu = 0.2$.

In *Figure 14*, the difference between CMFI and CMNFI is about 0.7% at $i = 0.13$. The difference gets lower if the nominal interest rate gets smaller. A threshold value is reached at $i = 0.03$, the nominal interest rate consistent with zero inflation. If $i > 0.03$ ($i < 0.03$), the introduction of financial intermediaries decrease (increase) the costs of inflation. Hence, financial intermediation does not always lower the costs of inflation. Since for low values of i, the model with financial intermediation reports far smaller values of q, I conclude that the sharp decrease in q is responsible for the higher reported costs of inflation when financial intermediaries are introduced.

7.4 Competitive pricing

In this section, I consider two approaches how competitive pricing can be implemented. First, I deviate from the assumption that the sellers' production costs in the DM are linear in the quantity produced. Instead, sellers' DM cost function takes the form $c(q) = \frac{q^{1+\alpha}}{1+\alpha}$, which equals the specification in Boel and Camera (2011). In a second approach, the production costs are again linear. However, I assume that the proportion of buyers in the population varies.

7.4.1 Competitive pricing with nonlinear cost functions

I consider the parameter values $\alpha = 0.01, 0.02, 0.03$. The proportion of buyers in the society is fixed at 0.6. Taking the first and the second derivative of the nonlinear costs function reveals that $c(q)$ is increasing and convex if α and q are larger than zero. This guarantees that the theoretical results of Lagos and Wright (2005) and Berentsen et al (2007) still hold which allows me to calibrate this specification easily. The results of the calibrations are presented in *Tables 7* and *8*.

TABLE 7. Competitive pricing without financial intermediation (nonlinear cost fct)

α	A	η	$q_{0.1}$	q_0	q_F	$\Delta_{0.1}$	Δ_F
0.01	1.661	0.109	0.186	0.641	0.966	1.38%	1.45%
0.02	1.590	0.100	0.182	0.623	0.935	1.25%	1.31%
0.03	1.521	0.091	0.180	0.605	0.905	1.11%	1.15%

TABLE 8. Competitive pricing with financial intermediation (nonlinear cost fct)

α	A	η	$q_{0.1}$	q_0	q_F	$\Delta_{0.1}$	Δ_F
0.01	1.635	0.063	0.179	0.632	0.946	0.79%	0.83%
0.02	1.531	0.054	0.172	0.601	0.896	0.65%	0.66%
0.03	1.434	0.045	0.166	0.572	0.850	0.49%	0.48%

Columns two to eight remain unaltered compared to the previous sections. Comparing the results in *Tables 7* and *8* again reveals similarities and differences.

i. Similarities: In the calibrated models with and without financial intermediation higher values of α have the following three consequences. First, it yields to lower calibrated values of A and η. Second, the higher α the lower is the quantity consumed in the DM. This result holds in both models for any considered inflation rate. Third, an increase in the parameter α yields lower costs of inflation $\Delta_{0.1}$ despite the fact that this also leads to lower consumption in the DM. Thus, the reduced costs of inflation seems to result from a lower calibrated value of η, which increases the utility from consuming the search good.

ii. Differences: Given a value of α, the calibrated values for A are always lower in the model with financial intermediation. However, the differences are small. The calibrated parameter values of η are lower in the model with financial intermediation. Surprisingly, the absolute differences in η between the two models are constant at 0.046 for any considered α. However, the relative differences between the two models increase since the costs of inflation decrease at higher values of α while the absolute differences stay constant. Irrespective of the steady state inflation rate, the quantity consumed in the DM is lower with financial intermediation for all considered values of α. But, these differences are small and range between 1.5% and 8%. For any inflation rate, a higher value of α leads to bigger differences in the quantity consumed in the DM. In both models there are inefficiencies since the optimal DM consumption level is not attained at the Friedman rule. It can be seen in *Figure 15* that in the model with financial intermediation, the costs of inflation are much lower.

Figure 15 shows the welfare cost functions without (solid curves) and with financial intermediation (dashed curves) for different values of α. Obviously, the introduction of financial intermediaries lower the welfare costs substantially. For example, if one compares $\Delta_{0.1}$, the costs of inflation with financial intermediation are 42.8% lower if $\alpha = 0.01$ and even 55.9% if $\alpha = 0.03$. Because η decreases in both models with higher values of α, this indicates that the reduction in the costs of inflation is due to the change in the parameter η.

7.4.2 Competitive pricing with varying proportion of buyers

The proportion of buyers, sellers in the society is denoted by $(1-n)$ respectively n. I consider the following parameter values: $(1-n) = 0.8, 0.6, 0.4, 0.2$.

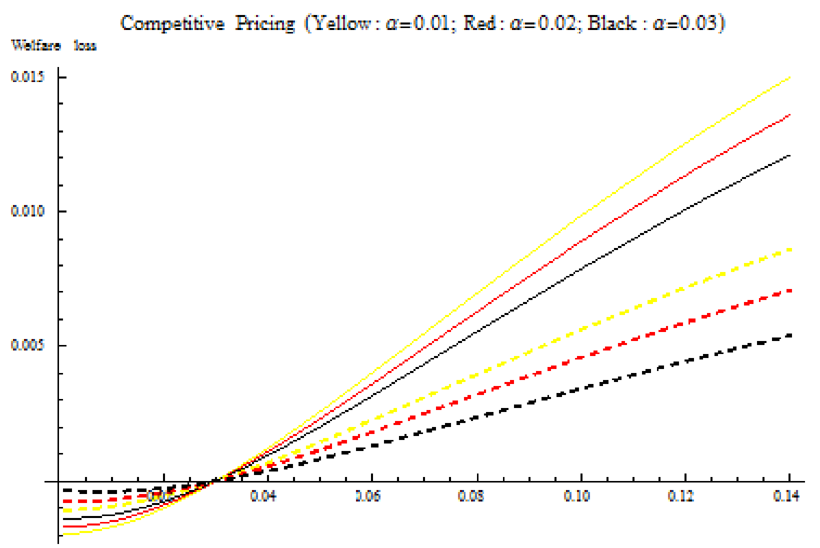

Figure 15: Competitive Pricing with nonlinear cost functions.

Competitive pricing allows to vary the proportion of buyers. This is not possible under Nash-, proportional bargaining and constant markup, as the model with financial intermediation can only be compared to the model without financial intermediation if $(1-n) = n = \sigma = 1/2$. Otherwise, the welfare criterions (12) and (31) do not coincide which makes the two models incomparable. *Tables 9* and *10* exhibit the calibration result for the models without and with financial intermediation.

TABLE 9. Competitive pricing without financial intermediation (proportion buyers)							
$(1-n)$	A	η	$q_{0.1}$	q_0	q_F	$\Delta_{0.1}$	Δ_F
0.8	1.560	0.084	0.166	0.645	1	1.56%	1.59%
0.6	1.737	0.118	0.189	0.661	1	1.50%	1.59%
0.4	1.912	0.183	0.214	0.673	1	1.44%	1.56%
0.2	2.070	0.357	0.246	0.676	1	1.33%	1.50%

TABLE 10. Competitive pricing with financial intermediation (proportion buyers)							
$(1-n)$	A	η	$q_{0.1}$	q_0	q_F	$\Delta_{0.1}$	Δ_F
0.8	1.563	0.068	0.165	0.647	1	1.25%	1.28%
0.6	1.746	0.073	0.186	0.665	1	0.92%	0.96%
0.4	1.932	0.077	0.206	0.683	1	0.60%	0.64%
0.2	2.119	0.082	0.226	0.698	1	0.29%	0.32%

The first column involves the parameter $(1-n)$ while the other columns show the same parameters as in the previous sections. First, I present similar results for the two models and thereafter their differences are posted.

i. Similarities: Five results are similar. First, lower values of $(1-n)$ yield higher calibrated values of A. The optimal CM consumption level in case of $(1-n) = 0.2$ is about 35.6% larger than the respective value of $(1-n) = 0.8$. Second, lower values of $(1-n)$ are associated with higher calibrated values of η. This result is particularly relevant if financial intermediation is absent which can be seen by comparing the cases $(1-n) = 0.2$ and $(1-n) = 0.8$. The calibrated value of η in the former case is 4.25 larger than the calibrated value in the latter case. Especially in the model without financial intermediation, the calibrated parameters differ much if one compares the highest and the lowest values of $(1-n)$. This effect has implications on the estimated theoretical money demand functions in the model without financial intermediation. *Figure 16* highlights these differences by plotting the money demand functions and the data pairs. The calculated money demand functions differ especially at low and high nominal interest rates for the different values of $(1-n)$. Third, considering 10% and 0% inflation, the calculated values of q are higher if the proportion of buyers in the society is lower. Forth, due to the linear specification of the sellers' production function, the DM consumption level at the Friedman rule is efficient in both models. Fifth, lower values of $(1-n)$ lead to lower costs of inflation $\Delta_{0.1}$ but higher values of η. This is surprising because larger values of η are associated with (partially massive) larger costs of inflation in all other pricing mechanisms.

ii. Differences: The model with financial intermediation indicates higher values of A given $(1-n)$. However, these differences are rather small. Moreover, the calibrated values of η are lower in the model with financial intermediation. These differences increase considerably for lower

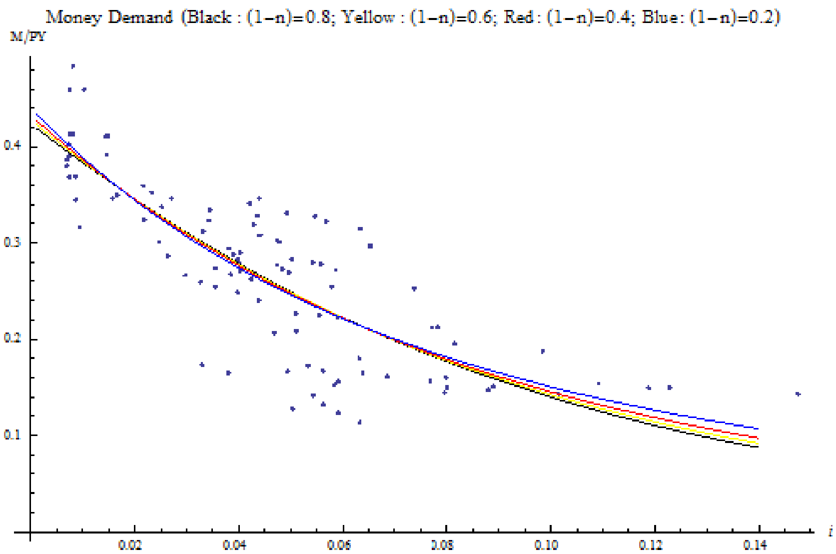

Figure 16: Data and money demand.

proportions of buyers in the society. For example, differences in η among the two models are around 19% in case of $(1-n) = 0.8$ and rise to 77% if $(1-n) = 0.2$. At a steady state inflation rate of 10%, the quantity consumed in the DM is lower in the model with financial intermediation while the opposite is true for an inflation rate of 0%. Considering all calibrations of chapter 7, here, the model with financial intermediation and proportion of buyers $(1-n) = 0.2$ exhibit the lowest costs of inflation. In this case, reducing the inflation from 10% to 0% is just worth 0.29% of total consumption. For all values of $(1-n)$ financial intermediation lowers the costs of inflation. For high values of $(1-n)$ the differences among the two models are not very large, whereas the differences are huge for low values of $(1-n)$. In case of $(1-n) = 0.8$ costs of inflation without financial intermediation are 0.31% larger while for $(1-n) = 0.2$ they even are 1.04% larger. Surprisingly, the relative differences in the costs of inflation are almost identical to the relative differences in the calibrated parameter values of η. This suport the argument that the introduction of financial intermediaries decrease the welfare costs of inflation through lower calibrated values of η. Finally, *Figure 17* plots the welfare cost functions considering the two extreme cases $(1-n) = 0.8, 0.2$ for the model with (solid

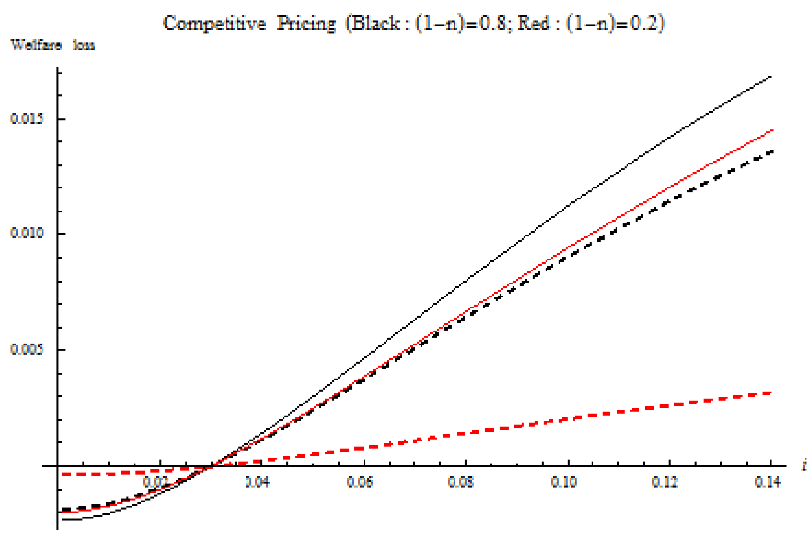

Figure 17: Competitive pricing with varying proportions of buyers.

line) and without (dashed line) financial intermediation.

Figure 17 shows again the results of the previous discussion. Moreover, the welfare cost function for the model with financial intermediation and $(1-n) = 0.8$ gives nearly identical welfare cost estimates as the model without financial intermediation and $(1-n) = 0.2$.

7.5 Summary

The welfare criterions (12) and (31) indicate that four variables determine the costs of inflation. These variables are q_0, q_τ, A, η. The estimates of the different pricing mechanisms give rise to several interesting conclusions. First, the calculated values of q_0 and q_τ do not necessarily increase if financial intermediaries are introduced. Particularly under constant markup and competitive pricing, the quantity consumed in the DM often decrease if financial intermediaries are introduced. This is a difference to the theoretical model of Berentsen et al (2007). In the theoretical model, the introduction of financial intermediaries always improve the allocation by raising the quantity consumed in the DM. This result crucially depends on the assumption that the values of the parameters A and η remain the same for the models with and without financial intermediation. In contrast, the quantitative model calibrates the parameter values of A and η such that they give the best possible fit to the real world data. The calibrated parameters do not coincide in the models with and without financial intermediation. Furthermore, financial intermediation do not always lead to higher consumption in the DM, because the calibrated parameters crucially influence these consumption levels. Second, the calibrated parameter values for A (considerably) change if financial intermediaries are introduced. Nevertheless, the direction of this change in A is not unambiguous. Under Nash bargaining and competitive pricing - variant: varying proportion of buyers - the calibration of A reports higher values in the model with financial intermediation than in the model without. For the other pricing mechanisms A often decrease. Third, the calibrated parameter values for η are always lower in the model with financial intermediation than in the model without. Forth, if $\Delta_{0.1}$ is considered, the model with financial intermediation yields lower costs of inflation. The differences in the calculated costs of inflation are huge. Under Nash-, proportional bargaining and competitive pricing - variant: nonlinear cost function - the calculated costs of inflation in the model with financial intermediation are approximately half as large as in the model without. Considering constant markup, the costs of inflation are lower with financial intermediation but the differences are less than half. Competitive pricing - variant: varying proportion of buyers - is a special case in which the introduction of financial intermediaries reduces the costs of inflation only by a small amount for high values of $(1-n)$, while it reduces the costs more than in all other considered cases for low values of $(1-n)$.

To conclude, the lower calibrated values of η in the model with financial intermediation seem to cause the lower calculated values of the costs of inflation. Two arguments support these findings: *i*) Taking the derivative of the DM utility function with respect to η shows that a decrease in η indeed yields a higher utility for the buyers at any $q \in [0,1]$. Since η represents the relative risk aversion coefficient of the buyers' DM consumption, a decrease in this parameter is equivalent to lower consumption risks which allow buyers to enjoy their consumption of the search good even more. *ii*) The costs of inflation as well as η always decrease if financial intermediaries are introduced. As argued above, all other parameters that could be responsible for the differences in the costs of inflation feature inconsistent outcomes. This supports the hypothesis that η is responsible for the reduction in the costs of inflation.

This should not imply that financial intermediation always lowers costs of inflation. Exceptions are found in case of constant markup. Then, the calculated costs of inflation are higher in the model with financial intermediation if the welfare implications of deflation rates are considered. Nevertheless, these results seem to be special cases, since the calculated values of q, if deflation rates are considered, yield much deeper values under constant markup than they do under the other pricing mechanisms.

8 Conclusion

Extensive economic literature engages with the question how to measure the welfare implications of inflation. This master thesis proceeds on this line of research by empirically investigating the consequences of the introduction of financial intermediaries on the costs of inflation. The model environment is based on the approach of Lagos and Wright (2005). A theoretical model including financial intermediaries into this framework is given in Berentsen et al (2007). They proof analytically that the introduction of financial intermediaries improve the allocation by allowing banks to pay interest to sellers deposits. In this master thesis, their principal ideas are adopted to calibrate the parameter values of A and η such that they match the data for the US money demand from 1900 to 2000. Using these calibrated parameter values, the theoretical money demand function and the costs of inflation can be calculated. Afterwards, the results of the models with and without intermediation are compared for the pricing mechanism Nash-, proportional bargaining, constant markup and competitive pricing. The main result is that financial intermediation considerably reduces the costs of inflation. In this master thesis, the costs of 10% in comparison to 0% inflation ranges between 1.11%, 0.29% and 6.14%, 2.85% for the models without respectively with financial intermediation. In both models, the costs of inflation reach their maximum under Nash- and proportional bargaining at low values of θ and their minimum under competitive pricing. Despite the fact that financial intermediation lowers the costs of inflation considerably, inefficiencies, such as the non-monotonicity inefficiency and the rent-sharing externality, do not disappear. The reduction in the costs of inflation seem to result from lower calibrated values of η, which allows buyers to get even more enjoyment from their DM consumption. Compared to that, the different recalibrations of Lucas' (2000) traditional approach that measures the welfare costs of inflation by calculating the area under the inverse money demand function, reports values between 0.64% and 1.47%. Finally, despite the fact that financial intermediation reduces the costs of inflation substantially, inflation remain a serious problem for the society's welfare, especially since the tax aspect of perfectly anticipated inflation is not the only source of inflation costs.

9 Apendix

9.1 Lucas

In this section, I prove that the log-log and the semi-log welfare cost functions can be written as stated in (10) and (11). To do so, I use the Lucas (2000) methodology stated in (9) to determine the welfare costs of inflation.

Proof. Log-log case: $m(r) = Ar^{-\eta}$,

$$w(r) = \int_0^r m(x)dx - rm(r) = \left[\left(\tfrac{1}{1-\eta}\right) Ax^{1-\eta}\right]_0^r - Ar^{1-\eta}$$

$$= \left(\tfrac{1}{1-\eta}\right) Ar^{1-\eta} - Ar^{1-\eta} = Ar^{1-\eta}\left(\tfrac{1-(1-\eta)}{1-\eta}\right) = A\left(\tfrac{\eta}{1-\eta}\right) r^{1-\eta}. \quad \blacksquare$$

Proof. Semi-log case: $m(r) = Be^{-\xi r}$,

$$w(r) = \int_0^r m(x)dx - rm(r) = \left[-\tfrac{1}{\xi} Be^{-\xi x}\right]_0^r - rBe^{-\xi r}$$

$$= -\tfrac{1}{\xi}Be^{-\xi r} - \left(-\tfrac{1}{\xi}B\right) - rBe^{-\xi r} = \left(\tfrac{B}{\xi}\right)\left(1 - e^{-\xi r}\left(1+\xi r\right)\right). \quad \blacksquare$$

9.2 Calibration without financial intermediation

9.2.1 Constructing the money demand

In Lagos and Wright (2005), the nominal GDP is given by

$$PY = \sigma M + PA,$$

where σM (PA) corresponds to the nominal GDP in the DM (CM). The money demand function is given by

$$L(i) = \frac{M}{PY}.$$

Inserting PY into the money demand function yields

$$L(i) = \frac{M}{PY} = \frac{M}{\sigma M + PA}.$$

This can be rearranged to

$$L(i) = \frac{M/P}{\sigma M/P + A}.$$

Since $z(q) = M/P$, the money demand is

$$L(i) = \frac{z(q)}{\sigma z(q) + A} = \frac{1}{\sigma + \frac{A}{z(q)}}.$$

Two equations are essential to calibrate the welfare costs of inflation for the model without financial intermediation. First, equation (16) shows how the nominal interest rate relates to the consumption level in the DM. Second, (25) indicates the theoretical money demand function. (16) can be solved for q. Thereafter, q can be inserted into $z(q)$ and $z(q)$ into the money demand function. Unless otherwise indicated, the following specifications - the same as in Lagos and Wright (2005) - are used:

$$\begin{array}{lll} u(q) = \frac{q^{1-\eta}}{1-\eta}; & u'(q) = q^{-\eta}; & u''(q) = -\eta q^{-\eta-1} \\ c(q) = q; & c'(q) = 1; & c''(q) = 0 \\ \sigma = \frac{1}{2} & & \end{array}$$

Using these specifications, the money demand depends on the parameters A and η. These parameters are calibrated with nonlinear regression such that they match the U.S. data over the period 1900-2000. In the following, this procedure is described for the pricing mechanisms (I) Nash bargaining (II) proportional bargaining (III) constant markup and (IV) competitive pricing.

9.2.2 Nash bargaining

In case of Nash bargaining $z(q)$ is determined (derivation is provided on request) through

$$z(q) = \frac{\theta c(q)u'(q) + (1-\theta)\, u(q)c'(q)}{\theta u'(q) + (1-\theta)c'(q)}.$$

Taking derivative with respect to q (derivation is provided on request) yields

$$z'(q) = \frac{u'(q)c'(q)\left[\theta u'(q) + (1-\theta)c'(q)\right] + \theta\,(1-\theta)\,(u(q) - c(q))\,(u'(q)c''(q) - c'(q)u''(q))}{\left[\theta u'(q) + (1-\theta)c'(q)\right]^2}.$$

Inserting the specifications into $z'(q)$ lead to

$$\begin{aligned} z'(q) &= \frac{q^{-\eta}\left[\theta q^{-\eta}+(1-\theta)\right]+\theta\left(1-\theta\right)\left(\frac{q^{1-\eta}}{1-\eta}-q\right)\left(\eta q^{-\eta-1}\right)}{\left[\theta q^{-\eta}+(1-\theta)\right]^{2}} \\ &= q^{-\eta}\left(\frac{\left[\theta q^{-\eta}+(1-\theta)\right]+\theta\left(1-\theta\right)\left(\frac{q^{-\eta}}{1-\eta}-1\right)\eta}{\left[\theta q^{-\eta}+(1-\theta)\right]^{2}}\right). \end{aligned}$$

Using $z'(q)$ and $u'(q)$, (16) is given by

$$\begin{aligned} \frac{u'(q)}{z'(q)} &= \frac{q^{-\eta}}{q^{-\eta}\left(\frac{\left[\theta q^{-\eta}+(1-\theta)\right]+\theta(1-\theta)\left(\frac{q^{-\eta}}{1-\eta}-1\right)\eta}{\left[\theta q^{-\eta}+(1-\theta)\right]^{2}}\right)} \\ &= \frac{\left[\theta q^{-\eta}+(1-\theta)\right]^{2}}{\left[\theta q^{-\eta}+(1-\theta)\right]+\theta\left(1-\theta\right)\left(\frac{q^{-\eta}}{1-\eta}-1\right)\eta}=1+\frac{i}{\sigma}. \end{aligned}$$

Substitute $X=q^{-\eta}$ into (16) and then solve it for X with Mathematica. Convert back, there results two values of q. Only one of them can be used to construct a reliable $z(q)$ and $L(i)$. This value is complex and therefore not stated here, but the author provid the necessecary information if it is requested. Instead, I present the special case $\theta=1$, which can be identified explicitly.

If $\theta=1$, then $z(q)$ yields $c(q)$ and $z'(q)$ yields $c'(q)$ such that (16) can be rewritten to

$$\frac{u'(q)}{c'(q)}=\frac{q^{-\eta}}{1}=1+\frac{i}{\sigma}.$$

Solving this for q yields

$$q=\left(1+\frac{i}{\sigma}\right)^{-\frac{1}{\eta}}.$$

Insert q into $c(q)$ and $c(q)$ into $L(i)$ yields

$$L(i)=\frac{c(q)}{\sigma c(q)+A}=\frac{1}{\sigma+\frac{A}{c(q)}}=\frac{1}{\sigma+A\left(1+\frac{i}{\sigma}\right)^{\frac{1}{\eta}}}.$$

This money demand function can be calibrated with respect to A and η.

Nash bargaining markup μ The aim is to construct an aggregated markup of $(1 + \mu) = 1.1$. The markup is defined as price over marginal cost. In the CM, there is perfect competition and hence the markup equals 1. In the DM, the real marginal cost equals $c'(q)$ while the nominal marginal cost corresponds to $\frac{c'(q)}{\phi}$. The price in the DM is $\frac{M}{q}$. Hence, the markup in the DM is given by

$$\frac{\frac{M}{q}}{\frac{c'(q)}{\phi}} = \frac{M\phi}{c'(q)q}.$$

Since $z(q) = M\phi = M/P$, this can be rewritten to

$$\frac{z(q)}{c'(q)q} \overset{c'(q)=1}{=} \frac{z(q)}{q}.$$

The nominal GDP in the CM is	$\frac{X^*}{\phi} = \frac{A}{\phi}$.
The nominal GDP in the DM is	σM.
The CM consumption share is	$\frac{A/\phi}{A/\phi+\sigma M} = \frac{A}{A+\sigma M\phi} = \frac{A}{A+\sigma z(q)}$.
The DM consumption share is	$\frac{\sigma M}{A/\phi+\sigma M} = \frac{\sigma M\phi}{A+\sigma M\phi} = \frac{\sigma z(q)}{A+\sigma z(q)}$.

Multypling the markups of each sub market with its respective consumption share and then add them together reveal the aggregated markup

$$(1 + \mu) = \frac{z(q)}{q}\frac{\sigma z(q)}{A + \sigma z(q)} + 1\left(\frac{A}{A + \sigma z(q)}\right).$$

9.2.3 Proportional bargaining

In case of proportional bargaining $z(q)$ is determined through

$$z(q) = \theta c(q) + (1 - \theta)\, u(q).$$

Taking derivative with respect to q yields

$$z'(q) = \theta c'(q) + (1 - \theta)\, u'(q) = \theta + (1 - \theta)\, q^{-\eta}.$$

Using $z'(q)$ and $u'(q)$, (16) can be reformulated to

$$\frac{u'(q)}{z'(q)} = \frac{q^{-\eta}}{\theta + (1 - \theta)\, q^{-\eta}} = 1 + \frac{i}{\sigma}.$$

Solving this for q yields

$$\begin{aligned}
&\Leftrightarrow \quad \frac{1}{\theta q^{\eta}+(1-\theta)}=1+\frac{i}{\sigma} \\
&\Leftrightarrow \quad \theta q^{\eta}+(1-\theta)=\frac{\sigma}{i+\sigma} \\
&\Leftrightarrow \quad \theta q^{\eta}=\frac{\sigma}{i+\sigma}-(1-\theta) \\
&\Leftrightarrow \quad q=\left[\frac{1}{\theta}\left(\frac{\sigma}{i+\sigma}-(1-\theta)\right)\right]^{\frac{1}{\eta}}=\left[\frac{1}{\theta}\left(\frac{\sigma-(1-\theta)(i+\sigma)}{i+\sigma}\right)\right]^{\frac{1}{\eta}}=\left[\frac{\theta\,[i+\sigma]-i}{\theta\,(i+\sigma)}\right]^{\frac{1}{\eta}}.
\end{aligned}$$

Inserting q into $z(q)$ reveals

$$z(q)=\theta\left[\frac{\theta\,[i+\sigma]-i}{\theta\,(i+\sigma)}\right]^{\frac{1}{\eta}}+\frac{(1-\theta)}{(1-\eta)}\left[\frac{\theta\,[i+\sigma]-i}{\theta\,(i+\sigma)}\right]^{\frac{1-\eta}{\eta}}.$$

Plugging in $z(q)$ into $L(i)$ yields

$$L(i)=\frac{1}{\sigma+\frac{A}{\theta\left[\frac{\theta[i+\sigma]-i}{\theta(i+\sigma)}\right]^{\frac{1}{\eta}}+\frac{(1-\theta)}{(1-\eta)}\left[\frac{\theta[i+\sigma]-i}{\theta(i+\sigma)}\right]^{\frac{1-\eta}{\eta}}}}.$$

This money demand function can be calibrated with respect to A and η.

9.2.4 Constant markup

In case of constant markup $z(q)$ satisfies

$$z(q)=(1+\mu)\,c(q).$$

Taking derivative with respect to q yields

$$z'(q)=(1+\mu)\,c'(q).$$

Using $z'(q)$ and $u'(q)$, (16) can be reformulated to

$$\frac{u'(q)}{z'(q)}=\frac{q^{-\eta}}{1+\mu}=1+\frac{i}{\sigma}.$$

Solving this for q yields

$$\Leftrightarrow \quad q^{-\eta} = (1+\mu)\left(1+\frac{i}{\sigma}\right)$$

$$\Leftrightarrow \quad q = \left[(1+\mu)\left(1+\frac{i}{\sigma}\right)\right]^{-\frac{1}{\eta}}.$$

Inserting q into $z(q)$ reveals

$$z(q) = (1+\mu)\left[(1+\mu)\left(1+\frac{i}{\sigma}\right)\right]^{-\frac{1}{\eta}}.$$

Plugging in $z(q)$ into $L(i)$ yields

$$L(i) = \frac{1}{\sigma + \frac{A}{(1+\mu)\left[(1+\mu)\left(1+\frac{i}{\sigma}\right)\right]^{-\frac{1}{\eta}}}}.$$

This money demand function can be calibrated with respect to A and η.

9.2.5 Competitive pricing

In case of competitive pricing, there are the following new specifications:

$q_s \equiv$	quantity that sellers produce
$q_b \equiv$	quantity that buyers consume
$n \equiv$	proportion sellers
$(1-n) \equiv$	proportion buyers

$$\text{Market clearing} \quad nq_s = (1-n)\, q_b \Leftrightarrow q_s = \left(\tfrac{1-n}{n}\right) q_b; \quad q_b \equiv q$$

$$\text{Cost function} \quad c(q_s) = c\left(\tfrac{1-n}{n}q\right) = \frac{\left(\frac{1-n}{n}q\right)^{1+\alpha}}{1+\alpha}; \quad c'(q_s) = \left(\tfrac{1-n}{n}q\right)^{\alpha}; \quad c''(q_s) = \alpha\left(\tfrac{1-n}{n}q\right)^{\alpha-1}$$

Under competitive pricing $z(q)$ and $z'(q)$ are determined through

$$z(q) = c(q_s),$$

$$z'(q) = c'(q_s).$$

q is determind through equation (35). Using $c'(q_s)$ and $u'(q)$, (35) can be reformulated to

$$\frac{u'(q_b)}{c'(q_s)} = \frac{q^{-\eta}}{\left(\frac{1-n}{n}q\right)^{\alpha}} = 1 + \frac{i}{1-n}.$$

Solving this for q yields

$$\begin{aligned} \Leftrightarrow \quad q^{-\eta-\alpha} &= \left(1+\frac{i}{1-n}\right)\left(\frac{1-n}{n}\right)^{\alpha} \\ \Leftrightarrow \quad q &= \left[\left(1+\frac{i}{1-n}\right)\left(\frac{1-n}{n}\right)^{\alpha}\right]^{-\left(\frac{1}{\alpha+\eta}\right)}. \end{aligned}$$

Inserting q into $c(q)$ reveals

$$c(q) = \frac{\left[\left(1+\frac{i}{1-n}\right)\left(\frac{1-n}{n}\right)^{\alpha}\right]^{-\left(\frac{1+\alpha}{\alpha+\eta}\right)}}{1+\alpha}.$$

Plugging in $c(q)$ into $L(i)$ yields

$$L(i) = \frac{1}{(1-n) + \frac{A(1+\alpha)}{\left[\left(1+\frac{i}{1-n}\right)\left(\frac{1-n}{n}\right)^{\alpha}\right]^{-\left(\frac{1+\alpha}{\alpha+\eta}\right)}}}.$$

This money demand function can be calibrated with respect to A and η.

9.3 Calibration with financial intermediation

9.3.1 Constructing the money demand

The nominal GDP is given by

$$PY = (1-n)\,M + PA,$$

where PA $((1-n)\,M)$ corresponds to the nominal GDP in the CM (DM). The money demand function is given by

$$L(i) = \frac{M}{PY}.$$

Inserting PY into the money demand function yields

$$L(i) = \frac{M}{PY} = \frac{M}{(1-n)\,M + PA}.$$

This can be rearranged to

$$L(i) = \frac{M/P}{(1-n)\,M/P + A}.$$

Since $z(q) = M/P$, the money demand is

$$L(i) = \frac{z(q)}{(1-n)\,z(q) + A} = \frac{1}{(1-n) + \frac{A}{z(q)}}.$$

Two equations are essential to calibrate the welfare costs of inflation for the model with financial intermediation. First, equation (36) shows how the nominal interest rate relates to the consumption level in the DM. Second, (37) indicates the theoretical money demand function. (36) can be solved for q. Thereafter, q can be inserted into $z(q)$ and $z(q)$ into the money demand function. Unless otherwise indicated, the following specifications are used:

$$\begin{array}{lll} u(q) = \frac{q^{1-\eta}}{1-\eta}; & u'(q) = q^{-\eta}; & u''(q) = -\eta q^{-\eta-1} \\ c(q) = q; & c'(q) = 1; & c''(q) = 0 \\ (1-n) = n = \frac{1}{2} & & \end{array}$$

Using these specifications, the money demand depends on the parameters A and η. These parameters are calibrated with nonlinear regression such that they match the U.S. data over the period 1900-2000. In the following, this procedure is described for the pricing mechanisms (I) Nash bargaining (II) proportional bargaining (III) constant markup and (IV) competitive pricing.

9.3.2 Nash bargaining

$z(q)$ and $z'(q)$ are defined as in Appendix 9.2.2. Using $z'(q)$ and $u'(q)$, (36) is given by

$$\frac{u'(q)}{z'(q)} = \frac{\left[\theta q^{-\eta} + (1-\theta)\right]^2}{\left[\theta q^{-\eta} + (1-\theta)\right] + \theta\,(1-\theta)\left(\frac{q^{-\eta}}{1-\eta} - 1\right)\eta} = 1 + i.$$

Substitute $X = q^{-\eta}$ into (36) and then solve it for X with Mathematica. Convert back, there result two values for q. Only one of them can be used to construct a reliable $z(q)$ and $L(i)$. This value is complex and therefore not stated here, but the author provid the necessecary information if it is requested. Instead, I present the special case $\theta = 1$, which can be identified explicitly.

If $\theta = 1$, then $z(q)$ yields $c(q)$ and $z'(q)$ yields $c'(q)$ such that (36) can be rewritten to

$$\frac{u'(q)}{c'(q)} = \frac{q^{-\eta}}{1} = 1 + i.$$

Solving this for q yields

$$q = (1+i)^{-\frac{1}{\eta}}.$$

Insert q into $c(q)$ and $c(q)$ into $L(i)$ yields

$$L(i) = \frac{c(q)}{\sigma c(q) + A} = \frac{1}{\sigma + \frac{A}{c(q)}} = \frac{1}{\sigma + A(1+i)^{\frac{1}{\eta}}}.$$

This money demand function can be calibrated with respect to A and η.

Nash bargaining markup μ Multypling the markups of each sub market with its respective consumption share and then add them together reveal the aggregated markup with financial intermediation

$$(1+\mu) = \frac{z(q)}{q} \frac{(1-n)\, z(q)}{A + (1-n)\, z(q)} + 1 \left(\frac{A}{A + (1-n)\, z(q)} \right).$$

9.3.3 Proportional bargaining

$z(q)$ and $z'(q)$ are defined as in Appendix 9.2.3. Using $z'(q)$ and $u'(q)$, (36) is given by

$$\frac{u'(q)}{z'(q)} = \frac{q^{-\eta}}{\theta + (1-\theta)\, q^{-\eta}} = 1 + i.$$

Solving this for q yields

$$\Leftrightarrow \quad \theta q^{\eta} + (1-\theta) = \frac{1}{1+i}$$

$$\Leftrightarrow \quad \theta q^{\eta} = \frac{1}{1+i} - (1-\theta)$$

$$\Leftrightarrow \quad q = \left[\frac{1}{\theta}\left(\frac{1}{1+i} - (1-\theta)\right)\right]^{\frac{1}{\eta}} = \left[\frac{1}{\theta}\left(\frac{1-(1-\theta)(1+i)}{1+i}\right)\right]^{\frac{1}{\eta}} = \left[\frac{\theta\,[1+i]-i}{\theta\,(1+i)}\right]^{\frac{1}{\eta}}.$$

Inserting q into $z(q)$ reveals

$$z(q) = \theta\left[\frac{\theta\,[1+i]-i}{\theta\,(1+i)}\right]^{\frac{1}{\eta}} + \frac{(1-\theta)}{(1-\eta)}\left[\frac{\theta\,[1+i]-i}{\theta\,(1+i)}\right]^{\frac{1-\eta}{\eta}}.$$

Plugging in $z(q)$ into $L(i)$ yields

$$L(i) = \frac{1}{\sigma + \frac{A}{\theta\left[\frac{\theta[1+i]-i}{\theta(1+i)}\right]^{\frac{1}{\eta}} + \frac{(1-\theta)}{(1-\eta)}\left[\frac{\theta[1+i]-i}{\theta(1+i)}\right]^{\frac{1-\eta}{\eta}}}}.$$

This money demand function can be calibrated with respect to A and η.

9.3.4 Constant markup

$z(q)$ and $z'(q)$ are defined as in Appendix 9.2.4. Using $z'(q)$ and $u'(q)$, (36) is given by

$$\frac{u'(q)}{z'(q)} = \frac{q^{-\eta}}{1+\mu} = 1+i.$$

Solving this for q yields

$$\Leftrightarrow \quad q^{-\eta} = (1+\mu)(1+i)$$

$$\Leftrightarrow \quad q = [(1+\mu)(1+i)]^{-\frac{1}{\eta}}.$$

Inserting q into $z(q)$ reveals

$$z(q) = (1+\mu)\,[(1+\mu)(1+i)]^{-\frac{1}{\eta}}.$$

Plugging in $z(q)$ into $L(i)$ yields

$$L(i) = \frac{1}{\sigma + \frac{A}{(1+\mu)[(1+\mu)(1+i)]^{-\frac{1}{\eta}}}}.$$

This money demand function can be calibrated with respect to A and η.

9.3.5 Competitive pricing

The same specifications for competitive pricing presented in Appendix 9.2.5. are used.

$c(q_s)$ and $c'(q_s)$ are defined as in Appendix 9.2.5. Under competitive pricing, q is determind through equation (34). Using $c'(q_s)$ and $u'(q)$, (34) can be reformulated to

$$\frac{u'(q_b)}{c'(q_s)} = \frac{q^{-\eta}}{\left(\frac{1-n}{n}q\right)^{\alpha}} = 1 + i.$$

Solving this for q yields

$$\Leftrightarrow \quad q^{-\eta-\alpha} = (1+i)\left(\frac{1-n}{n}\right)^{\alpha}$$

$$\Leftrightarrow \quad q = \left[(1+i)\left(\frac{1-n}{n}\right)^{\alpha}\right]^{-\left(\frac{1}{\alpha+\eta}\right)}.$$

Inserting q into $c(q)$ reveals

$$c(q) = \frac{\left[(1+i)\left(\frac{1-n}{n}\right)^{\alpha}\right]^{-\left(\frac{1+\alpha}{\alpha+\eta}\right)}}{1+\alpha}.$$

Plugging in $c(q)$ into $L(i)$ yields

$$L(i) = \frac{1}{(1-n) + \frac{A(1+\alpha)}{\left[(1+i)\left(\frac{1-n}{n}\right)^{\alpha}\right]^{-\left(\frac{1+\alpha}{\alpha+\eta}\right)}}}.$$

This money demand function can be calibrated with respect to A and η.

9.4 Measuring the welfare cost of inflation

9.4.1 Lagos and Wright

Lagos and Wright (2005) define the society's lifetime welfare facing an inflation rate of τ as follows

$$(1-\beta)\, V(\tau) = U(X^*) - X^* + \sigma\left[u(q_\tau) - c(q_\tau)\right].$$

The society's lifetime welfare facing an inflation rate of 0 and reduced consumption possibilities $(1-\Delta) < 1$ in both markets is given by

$$(1-\beta) V_{(1-\Delta)}(0) = U(X^* (1-\Delta)) - X^* + \sigma [u(q_0 (1-\Delta)) - c(q_0)] .$$

The costs of inflation are determined by

$$V_{(1-\Delta)}(0) = V(\tau) .$$

Solving this equation with respect to Δ yields the fraction of income that people are willing to give up to have zero instead of τ inflation. Let's rearrange this equation

$$\begin{aligned} V_{(1-\Delta)}(0) &= \left(\frac{1}{1-\beta}\right) [U(X^* (1-\Delta)) - X^* + \sigma [u(q_0 (1-\Delta)) - c(q_0)]] \\ &= V(\tau) = \left(\frac{1}{1-\beta}\right) [U(X^*) - X^* + \sigma [u(q_\tau) - c(q_\tau)]] . \end{aligned}$$

$$\Leftrightarrow U(X^* (1-\Delta)) - U(X^*) + \sigma [u(q_0 (1-\Delta)) - c(q_0)] = \sigma [u(q_\tau) - c(q_\tau)] .$$

Recall that $U(X^*) = A \log(A)$. Then the equation can be rewritten as

$$A \log(A (1-\Delta)) - A \log(A) + \sigma [u(q_0 (1-\Delta)) - c(q_0)] = \sigma [u(q_\tau) - c(q_\tau)] .$$

This is equivalent to

$$A \log(1-\Delta) + \sigma [u(q_0 (1-\Delta)) - c(q_0)] = \sigma [u(q_\tau) - c(q_\tau)] . \qquad (39)$$

9.4.2 Craig and Rocheteau

Instead of (39), Craig and Rocheteau (2008) use the following measure to determine the costs of inflation

$$-A\Delta + \sigma [u(q_0 (1-\Delta)) - c(q_0)] = \sigma [u(q_\tau) - c(q_\tau)] . \qquad (40)$$

$A \log(1-\Delta) \approx -A\Delta$ for values of Δ converging zero. But, (39) solely coincide with (40) if $A \log(1-\Delta) = -A\Delta$. Since costs of inflation are larger zero if the inflation rate is positive, the two measures are not identical. Because of this, Craig and Rocheteau (2008) call it a compensated meausere. In my thesis, I use the compensated measure to determine the costs of inflation.

9.4.3 Berentsen et al

The welfare criterion is stated in (31). The society's lifetime welfare facing an inflation rate of τ is given by

$$(1-\beta) W(\tau) = U(X^*) - X^* + (1-n) u(q_b(\tau)) - nc(q_s(\tau)).$$

Society's lifetime welfar facing 0 inflation and reduced consumption possibilities in both markets is given by

$$(1-\beta) W_{(1-\Delta)}(0) = U(X^*(1-\Delta)) - X^* + (1-n) u(q_b(0)(1-\Delta)) - nc(q_s(0)).$$

To determine the costs of inflation one needs to solve

$$W_{(1-\Delta)}(0) = W(\tau)$$

with respect to Δ which correspond to the fraction of income that people are willing to give up to have 0 instead of τ inflation. Let's rearrange this equation using the compensated welfare cost measure of Craig and Rocheteau (2008)

$$-A\Delta + (1-n)\left[u(q_b(0)(1-\Delta)) - u(q_b(\tau))\right] = n\left[c(q_s(0)) - c(q_s(\tau))\right].$$

Using the specifications under competitive pricing one gets

$$-A\Delta + (1-n)\left[\frac{(q(0)(1-\Delta))^{1-\eta}}{1-\eta} - \frac{(q(\tau))^{1-\eta}}{1-\eta}\right] = n\left[\frac{\left(\left(\frac{1-n}{n}\right)q(0)\right)^{1+\alpha}}{1+\alpha} - \frac{\left(\left(\frac{1-n}{n}\right)q(\tau)\right)^{1+\alpha}}{1+\alpha}\right].$$

This can be rearranged to

$$\begin{aligned} & -A\Delta + (1-n)\left[\frac{(q(0)(1-\Delta))^{1-\eta}}{1-\eta} - \frac{(q(\tau))^{1-\eta}}{1-\eta}\right] \\ = \; & (1-n)\left(\frac{1-n}{n}\right)^{\alpha}\left[\frac{(q(0))^{1+\alpha}}{1+\alpha} - \frac{(q(\tau))^{1+\alpha}}{1+\alpha}\right]. \qquad (41) \end{aligned}$$

Solving (41) for Δ yields the costs of inflation in the model with financial intermediation under competitive pricing. For Nash-, proportional and constant markup set $(1-n) = n = \frac{1}{2}$ and $\alpha = 0$ to get the costs of inflation in the model with financial intermediation. Under these pricing mechanisms, the costs of inflation in the model with financial intermediation can only be compared to the costs of inflation in the model without financial intermediation if $\sigma = (1 - n) = \frac{1}{2}$.

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