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Masterthesis

Division for Economic Theory

Title: *An Overlapping Generations Model of the Search Market for Ideas*

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Contents

Abstract

In this master thesis, the authors' ambition was to construct a microeconomic, monetary model that explains long-time cooperations between heterogenous agents in a three-period overlapping generation framework. Or more precisely, the *overlapping generation's (OLG) model of the search market for ideas'* research question is: How does know-how transfer in the innovation sector influence an entrepreneurs' production possibilities, and what role does money play in trade for expertise and innovation goods?

With the kind support of supervising tutor Prof. Dr. Aleksander Berentsen, the authors' aim was to embed a methodology that orients oneself on Ed Nosal's and Guillaume Rochetetau's [2011] analysis on symmetric and proportional bargaining solutions into Paul Samuelson's [1958] OLG model.

Quickly summarized *the OLG model of the search market for ideas* is constructed in the following way: Agents live for three periods. Each period consists of two stages. In the first stage, the market is decentralized with random matching and proportional bargaining. Entrepreneurs and investors bilaterally meet and trade implementation expertise for shares in future monetary payouts. In the second stage, the market is centralized. Retirees and entrepreneurs meet and trade money for innovative consumption goods.

In *the OLG model of the search market for ideas* the probability to achieve efficient levels of allocations depends on the wedges in the model: The buyer tends to underinvest in real balances the higher the search friction and the lower his bargaining power.

1 Introduction

The capacity for innovation of one country's companies significantly affects the prosperity of a national economy. Besides the innovative performance of established corporations, especially the foundation and funding of new, innovative firms is of great importance for the ability of a country to compete in a rapidly changing global market environment. With the increased internationalization of markets and the Southeast Asian emerging economies growing export potential - in the meanwhile also into superior market segments -, the preservation of international competitiveness becomes a major goal for a high labor-cost country like Switzerland.[1] An intact competitive position requires the innovation sector to perform at its peak capacity.[2]

In the beginning of the process of formation, considerations related to those mentioned above inspired the author to write a master thesis about the influence of the prosperity in the innovation sector on one country's economic performance. As this thesis was compiled in the context of the lecture on 'Monetary Theory' held by Prof. Dr. Berentsen, considerations related to the role of money played a major role for *the overlapping generation's (OLG) model of the search market for ideas*. The research question of this master thesis is therefore: How does know-how transfer in the innovation sector influence an entrepreneurs' production possibilities, and what role does money play in trade for expertise and innovation goods?

The following paragraphs aim to shortly introduce and justify the relevance of previous research work and thoughts for the methodology of this master thesis' *OLG model of the search market for ideas*. The last paragraph in the introduction gives an overview over the organization of the whole master thesis.

[1]For further information see *"Strukturberichtserstattung Nr. 29"* on www.seco.admin.ch

[2]For further information see *"the story behind swiss innovation"* on www.economiesuiesse.ch

Basically there are four directions of thoughts to be mentioned when explaining the composition of the *OLG model of the search market for ideas'* methodology: Firstly [Silveira and Wright, 2010]'s conclusion that the transfer of ideas between agents can be efficiency improving. Secondly [Berentsen et al., 2012]'s statement that the availability of liquidity to the innovation sector is a pre-condition for economic performance. Thirdly [Lagos and Wright, 2005]'s work on an alternating decentralized-centralized market framework that allows [Nosal and Rocheteau, 2011] to investigate the role of agents' money holdings on bargaining and pricing mechanisms. And last but not least [Samuelson, 1958]'s OLG methodology with goods and money that allows the author to embed the search market for ideas' approach, the liquidity and innovation consideration, and the alternating markets approach into a three-period overlapping generation setting. By doing so, the author aims to describe how inter-generational liquidity flows enable the innovation sector to become more productive, preconditioned that the agents manage to overcome search frictions in the first instance.

[Silveira and Wright, 2010]'s paper on the search market for ideas provides the first basic input for the framework of this master thesis. The economists see the development and implementation of new ideas as a major factor for economic performance. Because, if agents are heterogeneous in the ability to come up with ideas and the ability to implement them, some should specialize in innovation and others in implementation [Silveira and Wright, 2010]. The authors investigate if the direct transfer of ideas from innovators to entrepreneurs can lead to a more efficient use of resources than a conjoint implementation. [Silveira and Wright, 2010]'s supposition is approved by many other authors such as [Katz and Shapiro, 1986] who confirm that some inventor-founded start-ups are often second-best, as innovators do not have the entrepreneurial skills to commercialize new ideas or products. Instead of analyzing if innovators should try to implement their ideas by themselves or if they should sell them to already established firms who may be better in implementing them, this master thesis distances oneself from [Silveira and Wright, 2010] in the sense that it lies the focus only on innovators who become entrepreneurs and work together in a partnership with investors who support them with know-how to implement their ideas. *The OLG model of the search market for ideas* aims to model the interaction between agents in a similar way as [Silveira and Wright, 2006] and [Gompers and Lerner, 2004] did: An entrepreneur who aims to build up a start-up searches for a partner to interact with and to form a long-term-partnership with. Once matched, innovators and investors both take active roles in implementation.

An other influencing work for this thesis was the search-theoretic approach to the venture capitalism industry by [Silveira and Amit, 2006] who credit the venture capital industry as one of the major drivers for innovation and productivity improvements in an economy. Now it should have become obvious that the entrepreneurs' search for an investor is the friction that hinders the idea market in *the OLG model of the search market for ideas.* What this means will be described in detail in the theoretical chapter 2.1 concerning monetary models and market frictions.

[Berentsen et al., 2012] provide the second basic framework for *the OLG model of the search market for ideas* as they investigate the role of financial intermediation for the decentralized innovation sector. The authors integrate a model of money and finance into a model of endogenous growth and deal with unrestricted money holdings and divisible money by applying [Shi, 1997]'s representative household approach.

Instead of [Shi, 1997]'s representative households - composed of a continuum of members who pool their money holdings-, this master thesis utilizes the a third basic methodology: The alternating market model with centralized and decentralized markets, introduced by [Lagos and Wright, 2005]. Here, competitive markets allow agents to adjust their money holdings. The [Lagos and Wright, 2005] methodology allows the author to investigate a bargaining and pricing mechanism. Nevertheless, [Berentsen et al., 2012]'s considerations on liquidity and innovation was a major point of origin for the development of *the OLG model of the search market for ideas*.

The last theoretical fundament of this master thesis to mention here, is the OLG model originally from [Samuelson, 1958]. The introduction of money into this model allows the agents to engage in inter-temporal trades [Nosal and Rocheteau, 2011]. The environment of *the OLG model of the search market for ideas* is strongly correlated with the OLG methodology: The old generation of retirees consumes the innovative good produced by the young generation. The young generation is good in coming up with new ideas, but the innovators can only become entrepreneurs if they find middle-generation's investors who provide them with the expertise needed to realize the idea. The investors are willing to provide expertise because they get back shares of monetary payouts from innovation output sales at the end of the period. With this money, an investor can finance its consumption in the next period, when he belongs to the old, retired generation.

Concerning the organization of the paper, section 2 provides an insight into relevant methodologies that build the basis for the development of this master thesis' *OLG model of the search market for ideas*. These are basically models with trading frictions that make money essential, and OLG models. Section 3 presents the environment of the *OLG model of the search market for ideas* in detail. The value functions for the heterogenous agents who play a role in the bargaining processes are explained and the proportional as well as the symmetric bargaining solution is described. The conclusion completes this master thesis in section 4.

2 Theory

Chapter 2 gives an insight into the relevant models of which *the OLG model of the search market for ideas* is an extension. These are firstly models that include environments with frictions that make money essential, secondly models that use bilateral meetings as a mechanism of exchange because it explains the agent's behavior in a monetary economy more adequately than in a competitive environment, thirdly search market models with divisible money that determine the exchange value of money, and fourthly search market models with special focus on the innovation sector. In chapter 2.2, inter-temporal market frictions of an OLG model are explained in order to be able to analyze how liquidity flows between generations enable the innovation sector to grow later on in chapter 3. Chapter 2 concludes with a classification of *the OLG model of the search market for ideas* into the current state of research.

2.1 Monetary Models and Market Frictions

To understand why the environment of *the OLG model of the search market for ideas* has trading frictions, it might be helpful to start with the opposite: In one benchmark model for competitive equilibrium in economics, [Arrow and Debreu, 1954] define the environment as frictionless. In the very informative introduction of their book, [Nosal and Rocheteau, 2011] put the importance of market frictions for monetary models in a nutshell by explaining that money cannot play an essential role in an economy with Pareto optimality.[3] In the standard Arrow-Debreu environment with complete markets where people do commit to all future actions, the only constraint

[3] An allocation of goods is Pareto efficient if there is no other allocation possible that makes one agent better off without making at least one agent worse off [Sandler and Smith, 1976].

that agents face is their budget constraint, because if they can fully commit to future actions, individuals demand and supply goods as promised [ibidem]. This equilibrium is already Pareto optimal and money cannot ease trade, meaning cannot play an essential role [Arrow and Debreu, 1954], [Nosal and Rocheteau, 2011].

In order to create a theory that gives value to fiat money [Wallace, 1978] includes frictions into the Walrasian model that make it impossible for individuals to directly acquire the desired goods without money. Furthermore, [Wallace, 1978] describes two features that are essential characteristics of fiat money in a monetary economy: The inconvertibility and the intrinsic uselessness of fiat money. With inconvertibility [Wallace, 1978] means that money cannot be converted into gold for example. Although [Wallace, 1978] admits historical occurrence of convertibility such as in Great Britain during the Napoleonic wars and during World War I, to him, current restoration of convertibility seems unlikely. [Wallace, 1978] also explains that in its intrinsic uselessness, money demand differs from the demand for goods in an economy: A good has value in equilibrium because of its limited supply, and the goods' utility increases in its amount consumed. In contrast to barter trade, individuals do not give up goods for fiat money in order to consume it, but because they believe that someone else will subsequently be willing to give up goods for fiat money too [ibidem]. Intrinsically useless fiat money has value because it facilitates exchange. This is the reason why many monetary economists abandon the frictionless and costless multilateral market clearing in the general equilibrium model where exchange works perfectly and money is not needed to facilitate exchange [Champ et al., 2011], [Wallace, 1978]. Besides his extension of the general equilibrium model, [Wallace, 1978] also contributes to the use of OLG models in monetary theory. We will come back to his reasoning in detail in chapter 2.2.

An other reason that gives money value describe [Nosal and Rocheteau, 2011]: If the endowments and preferences of the trading partners are consistent with each other, there is a coincidence of wants between the trading partners and direct exchange of goods is possible without the need of money. But if the consensus over endowments and goods is not present, the agents have a double-coincidence of wants problem and a medium of exchange needs to emerge in order to avoid autarky [ibidem]. A competitive environment does not cover the double-coincidence of wants problem [ibidem]. This is the reason why, in the following section, we have a look on methodologies with decentralized markets. Outside the competitive market, where agents with inconsistent preferences and endowments meet randomly and bilaterally at different time and place, where the outcome of the trades remain private information, and where agents do not commit to future trades, the double-coincidence of wants problem requests money as a medium of exchange that overcomes those market frictions [Nosal and Rocheteau, 2011].

2.1.1 Models with Bilateral Trade Meetings

The feature of money as a medium of exchange - that individuals are willing to give up goods for money because they rationally think that others will accept the money too - assigns a strategic aspect to agents' money holdings in the process of exchange, how agents meet and trade [Wallace, 1978]. [Jones, 1976] explains the emergence of media of exchange through un-concerted random market behavior, meaning that agents do not meet in a centralized market, but rather do meet bilaterally. In [Jones, 1976]'s model, agents try to minimize the expected time searching for complementary trading partners. Depending on the agents' trading restrictions either direct barter or the use of a common good as medium of exchange arises [ibidem].

After [Jones, 1976], [Diamond, 1982] models the search equilibrium model which is fundamental for research on money in search equilibria. [Diamond, 1982] drops the Walrasian auctioneer and introduces trade frictions that make trade difficult to coordinate in a many-person economy. But whilst [Diamond, 1982] focuses on the explanation of reasons for fluctuations in unemployment by using market frictions, Kiyotaki and Wright integrate [Jones, 1976]'s double-coincidence of wants problem into [Diamond, 1982]'s search equilibrium [Kiyotaki and Wright, 1991], [Kiyotaki and Wright, 1989] and [Kiyotaki and Wright, 1993].

In his work on money, inflation and capital formation, [Von Thadden, 1999] summarizes the Kiyotaki-Wright-scenario as the following: "Agents are specialized in production and consumption in the sense that they never consume the type of good which they themselves are capable of producing [...] Agents bilaterally meet over time according to a random scheme and follow strategic trading rules. These rules specify whether agents should immediately execute a trade or wait for a better match on some future occasion" [Von Thadden, 1999, p. 39]. In the [Kiyotaki and Wright, 1989] setting, the use of money involves strategic elements: For fiat money to be valued, comparatively poor intrinsic properties of money need to be compensated by a sufficient degree of trust in its general acceptability. This degree is endogenously determined: An increase in the acceptability of money by the trading partners makes one agent's money holdings more valuable and increases in turn his own willingness to accept money [Von Thadden, 1999]. As a result, there is scope for multiple equilibria and the exact degree of money-based transactions in equilibrium depends on the trading partners acceptance of money [ibidem].

One very strong feature of the [Kiyotaki and Wright, 1989] setting is that all objects are indivisible and traded one-for-one in all meetings. The rather strong assumption that individuals can hold at most one unit of indivisible money that trades for one unit of output fails to answer the question how the exchange value of money is determined [Nosal and Rocheteau, 2011]. This is the reason why we have a closer look to models that explain the exchange value of money by making it divisible in chapter 2.1.2.

2.1.2 Search Models with Divisible Money

[Trejos and Wright, 1995] endogenise the price level in their search theoretic model of fiat money and bilateral bargaining. In his research work, Zhu [2003, 2005] relaxes the restriction that agents can hold at most one unit of money and examines the question how money supply changes affect inflation and output, both for indivisible [Zhu, 2003] and divisible money [Zhu, 2005]. [Shi, 1997] assumes that households are composed of a continuum of members who pool their money holdings. In such an approach where the economic environment allows degenerated, unrestricted money holdings and divisible money, in equilibrium, the money holdings of all agents of the same type are identical just before the agents are bilaterally matched [Nosal and Rocheteau, 2011].

Shi [1995, 1997] extends the [Kiyotaki and Wright, 1989] model and creates a search model of fiat money by allowing for divisible money and goods. By making money divisible, [Shi, 1995] focuses on the time-consuming trading process that arises from the lack of double coincidence of wants in barter and explains that the trading opportunity, or the number of matches that generate trade, depends on the frequency in which agents choose to shop with money relative to barter. By incorporating search intensity into the model and examining the link between money supply and the number of money holders, [Shi, 1997] shows two effects: On the one hand the effect of money growth on trading opportunities - a non Walrasian feature. And on the other hand the effect of money growth on the real money balance - a Walrasian feature [ibidem].

With the allowance for divisible money, [Shi, 1997] models changes in the fraction of agents holding money relative to the fraction of agents who are sellers. In this way, the probability with which agents have a successful match varies with money growth: After money growth, the search intensity for a match between money holders and producers reduces and the trading opportunity increases [ibidem]. Therefore, after an unstable trading frequency in equilibrium with money holders having low bargaining weight in bilateral exchanges, money growth can increase output and welfare by increasing agent's trading opportunities [ibidem].

[Berentsen et al., 2012] extend [Shi, 1997]'s search model to allow for intermediation in the financial sector and to allow for a balanced growth path. In their model, Berentsen and colleagues [2012] make use of [Shi, 1997]'s representative household approach based on [Lucas, 1990]: A representative household consists of members who are allocated to four activities; producing consumption goods, innovating, working in the financial sector, and leisure [Berentsen et al., 2012]. Money is needed as a medium of exchange to trade innovation time. Trading shocks generate heterogeneous demand for liquid funds among innovators [ibidem]. Financial intermediaries reallocate liquid funds among innovators in the decentralized innovation sector [ibidem]. The feature that this model explains growth in the innovation sector with exogenous changes in money supply, makes it very influencial for *the OLG model of the search market for ideas*. This will be explained in detail in chapter 2.1.3. But first we need to have a look on an alternative to [Shi, 1997]'s approach for search models with money: Alternating markets.

An other approach that captures the unification of money shortly before bilateral trades, as well as the impact of changing money supply on the economy, is the

[Lagos and Wright, 2005] framework for monetary theory and policy analysis. Different to earlier search models, where money holdings are endogenously distributed, [Lagos and Wright, 2005] assume quasi-linear preferences and give agents periodically access to either centralized, competitive markets (CM), or to decentralized search-markets (DM) with bilateral trades. With quasi-linearity, there are no wealth effects in the demand for money, so that all agents make the same choice of money in the CM, except for their choice of the 'quasi-linear good' [Nosal and Rocheteau, 2011]. Hence, the distribution of money is degenerated across agents in the DM [ibidem]. The alternating CM-DM-formulation allows the reintroduction of competitive markets with general equilibria (CM), as well as the valuation of money and the analysis of different pricing mechanisms such as bargaining or price posting in the DM [Lagos and Wright, 2005], [Nosal and Rocheteau, 2011].

Both versions - [Shi, 1997]'s model with large families or households, and the [Lagos and Wright, 2005] model with either centralized or decentralized markets and quasilinear preferences - are tractable for research in monetary theory because they deliver a degenerated distribution of money holdings across agents in equilibrium [Nosal et al., 2007].

Last but not least, the work of [Rocheteau and Wright, 2005] needs to be mentioned. They compare the three dominant market structures in this research field for monetary economies: bargaining (search equilibrium); price taking (competitive equilibrium); and price posting (competitive search equilibrium). The authors introduce a general matching technology and study how the different market structures influence equilibrium and the effects of policies. Rocheteau and Wright [2005, p.175] summarize that: "Under bargaining, trade and entry are both inefficient, and inflation implies first-order welfare losses. Under price taking, the Friedman rule solves the first inefficiency but not the second, and inflation may actually improve welfare. Under posting, the Friedman rule yields the first best, and inflation implies second-order welfare losses".

2.1.3 Search Models with Innovation

Based on the [Lagos and Wright, 2005] alternating CM-DM-formulation, Silveira and Wright [2010] developed a paper on the search market for ideas. They are the first economists to model innovation in a search and matching framework. The authors think that: "This alternating market formulation, borrowed from Lagos and Wright [2005] is convenient because [...] we can allow frictions in the exchange of ideas, but still have centralized trade in assets and other markets" [Silveira and Wright, 2010, p.1553].

The authors see specialization as a key concept for prosperity and define the direct technology transfer to be an optimal strategy in an economy with heterogenous agents: "If agents are heterogeneous in the ability to come up with ideas and extract their returns, some should specialize in innovation (innovators) and others in implementation (entrepreneurs) [...] After a direct technology transfer, the innovators can be sent back to the drawing board to come up with new ideas" [Silveira and Wright, 2010, p. 1551]. As direct technology transfer allows for specialization, direct trade of ideas from innovators to entrepreneurs is better than a conjoint development of the ideas in long-term (start-ups), and or financial (venture capitalism) partnerships: "Involving innovators in implementation, either alone or with partners, is a waste or their time and expertise" [Silveira and Wright, 2010, p. 1551]. The reasoning is that specialization after direct technology transfer makes the use of resources more efficient, makes all parties better off and increases incentives for investment in research [ibidem]. Therefore, the authors focus on a situation where the innovator wants a buyer, not a partner. This requires a market for the exchange of ideas.

[Silveira and Wright, 2010] bypass the idea markets' information constraints of adverse selection and moral hazard by making strong assumptions: Concerning adverse selection, entrepreneurs randomly either have the expertise to recognize the value of an innovator's idea and consequently buy it, or they do not have the expertise and always reject the deal because they are feared of buying a lemon [ibidem].[4] In terms of moral hazard the information and incentive problems are so severe that there are no partnerships or joint implementations: Either the idea is sold and the entrepreneur develops it, or the deal falls through and the innovator tries to implement his idea on his own [ibidem]. The latter assumption goes in line with [Silveira and Wright, 2010]'s notion that joint ventures are second best-, and direct technology transfers are first best solutions. The two main frictions in [Silveira and Wright, 2010]'s market of ideas are bargaining- and liquidity constraints. The mentioning of these two frictions concludes the listing of [Silveira and Wright, 2010]'s influence on the framework of *the OLG model of the search market for ideas*:

Firstly, [Silveira and Wright, 2010]'s bargaining friction lies in the strategic behavior of the innovators who can hinder direct technology transfer. With bargaining, the best deals are the most likely to fall through. The reason is that if the value of the innovator's idea is very high for the entrepreneur, the innovator has a big incentive to behave strategically and to - instead of settle it - put the deal on hold, letting the entrepreneur try to raise additional funds, facing the risk that consequently there is

[4][Akerlof, 1970] describes adverse selection as a problem that arises in markets with asymmetric information: If consumers cannot know the quality of a product and are only willing to pay an average price for it, then this price is more attractive for bad-quality-product-sellers than for good-quality-product-sellers. Consequently, more bad products (lemons) are offered than good products.

no trade. Secondly, [Silveira and Wright, 2010]'s liquidity friction lies in the circumstance that the entrepreneur can only pay up to some amount for the innovator's idea. By ex ante holding more liquid assets, the entrepreneur reduces the probability that the deal falls through. Meaning that the entrepreneur faces a trade-off between the benefit of liquidity, which is a better chance of closing deals, and the cost in terms of rates of return. The liquidity constraint that hinders the market for ideas also rises from the assumption that there are no credits feasible because ideas are difficult to collateralize. What makes liquidity constraints interesting is that the entrepreneur has the choice between different investment opportunities, one being liquid, the other not [Silveira and Wright, 2010].

The innovation sector consideration in *the OLG model of the search market for ideas* were not only influenced by [Silveira and Wright, 2010]'s search model on the market for ideas, but also by Berentsen and colleagues' [2012] model on liquidity, innovation and growth. In this model, liquid funds are initially given by lump-sum transfers from the government to the representative households. This money is afterwards distributed among the household members and used for financial intermediation to face heterogeneous demand for liquid funds among innovators [ibidem]. *The OLG model of the search market for ideas* of this master thesis, on the one hand acknowledges the importance of the innovation sector for economic growth and adopts [Berentsen et al., 2012]'s considerations. On the other hand, instead of using Shi's [2007] representative household approach, this master thesis applies the Lagos-Wright [2005] framework to model bargaining mechanisms in decentralized markets with frictions.

2.2 Overlapping Generations Models

Chapter 2.2 and 2.2.1 aims to describe the reason why OLG models are often used in monetary theory. Or more precisely, to show how it is possible to value money in an environment where agents live for several periods and are willing to hold money from one period to the next [Champ et al., 2011]. After that, subsection 2.2.2 gives an overview over the current state of research in the field of OLG models with search, where [Zhu, 2008]'s OLG model with search plays a major role. The last part of this chapter aims to contextualize this master thesis' *OLG model of the search market for ideas* into the current state of research to accent points of intersection with as well as disparities to the theories described above.

From the first part of this chapter, we know that [Wallace, 1978] included frictions into the Walrasian model, that value money by making it impossible for agents to directly satisfy their preferences for consumption goods. In his random-matching model with money, [Wallace, 1997] showes short-run, real effects and long-run, nominal effects of changes in money supply. In [Wallace, 1978]'s opinion, the friction in [Samuelson, 1958]'s consumption loan model with overlapping generations gives rise to the best available model of fiat money.

One helpful feature of OLG models with money is the possibility to examine inflationary effects, or in other words, the consequences of an increasing stock of fiat money on economies [Champ et al., 2011]. OLG models with money consider effects of population growth on the supply of fiat money [ibidem]. And, in the framework of an OLG model, it is possible to select a feasible allocation that maximizes the utility of future generations, also called the "golden rule" that identifies the preferences of future generations among feasible stationary allocations [ibidem].

2.2.1 Traditional OLG Models

The OLG Model introduced by [Samuelson, 1958] has been applied to the study of a large number of topics in monetary-, microeconomic- and macroeconomic theory. When [Samuelson, 1958] modeled his famous OLG model in the 1950ies, he had the idea of social security in mind where the generations are hard working and rich when young, and unemployable and poor when old [Nosal and Rocheteau, 2011]. Individuals are born at different dates and live a finite life, but the economy continues forever [ibidem]: Everybody, when young, is incredibly well off. The young are productive, working, and have higher endowments. The old generation is retired, feeble and has not very much to offer to the young. Such a situation creates a problem of no double coincidence of wants and therefore no trade can take place [ibidem]. In this case, the introduction of fiat money reflects a Pareto improvement because if the old generation offers money in exchange for the good produced by the young generation, autarky is avoided and endowments start to circulate in inter-generational, as well as in inter-temporal trades [ibidem].

Furthermore, the use of money as medium of exchange avoids autarky because money is the only nonperishable good in the OLG economy and it can be taken along by the agents into the next period [Nosal and Rocheteau, 2011]. This is the reason why the young generation is willing to exchange goods for intrinsically useless fiat money: They think that in the next period, when they become the old generation, the next young generation will accept money as medium of exchange too and will be willing to trade it for their goods too [Weil, 2008]. Therefore, the use of money enables trade and solves the overlapping generations' economic problem where future generations only have access to the non-storable good when young, but want to consume it when old [Champ et al., 2011].

2.2.2 OLG Models with Search Frictions

In his OLG model with search [Zhu, 2008] embeds search frictions in the OLG model. In his model, people live for two periods and three stages. They are young in stage 1 and 2 in time t and old in stage 3 in time $t + 1$. [Zhu, 2008] models the search friction in the sense that in each period, the young and the old participate in a centralized market. Then, the old die and the young are matched in pairs in the decentralized market. There exists one produced and perishable good per stage and a money stock M_0 held by the initial old. A person born at time t can produce but cannot consume at his first stage. At his second stage, the individual has an equal chance to be a buyer - who can consume but not produce, or a seller - who can produce but not consume. At his third stage, at time $t + 1$, the individual is a buyer who can consume but can not produce. At time t the individuals utility from consuming the consumption good q is $u_t(q)$ and his disutility form producing is $c_t(q)$. An individual maximizes its expected lifetime utility, which is the sum of his stage utilities [Zhu, 2008].

Zhu [2008, p.319] shows that "if the buyers bargaining power in pairwise trade is close to unity and if the old are risk averse, then the golden-rule of money transfer is positive". [Zhu, 2008] incorporates the transaction role of money by using pairwise meetings and anonymity. [Zhu, 2008] finds new implications for the optimal rate of lump-sum money creation compared to the traditional OLG model with money in the utility function like for example in [Abel, 1983]. Once the second stage - where agents have the change to either be a producer or a consumer - is removed, [Zhu, 2008]'s model is equivalent to the two-period-lived OLG model.

Besides [Zhu, 2008], there needs also the work of [Maeda, 1991] and [Russell, 2004] to be mentioned. Both authors embedded search frictions in the form of anonymous bilateral trade meetings into the OLG model, but none of them used the alternating CM-DM environment such as [Zhu, 2008] did. [Russell, 2004] shows that a monetary steady state can deliver lower lifetime expected utility because "[...] intertemporal transactions involving money are inefficient intergenerational transfers" [Russel, 2004, pp. 11-12], and because "[...] some households that are matched with households with money, might prefer to be matched with households with goods" [Russel, 2004, p.1]. In his OLG model with random pairwise trading patterns [Maeda, 1991, p.1] shows a Pareto optimal "[...] stationary monetary equilibrium in which claims bear a higher return than fiat money".

2.2.3 Classification into the Current State of Research

This last part of chapter 2 aims to classify *the OLG model of the search market for ideas* into the current state of research, especially with respect to the monetary models explained above. In chapter 2.1.2 and 2.2.2 two pathbreaking approaches for *the OLG model of the search market for ideas* are described: The Lagos-Wright framework with alternating markets and the OLG model with search.

In contrast to the traditional social security OLG model framework, analyzed by [Wallace, 1978] and [Samuelson, 1958], in *the OLG model of the search market for ideas*, the young generation owns only little money after having invested a lot into education and standing at the beginning of working life and the old generation is able to consume goods with their money payouts from investments in earlier life stages. In this sense, it is the other way round than described by [Nosal and Rocheteau, 2011]: The young generation is poor, and the old generation is rich.

The OLG model of the search market for ideas differs from Silveira and Wright's [2010] search market for ideas model discussed in chapter 2.1.3 in the sense that it rejects direct technology transfer and rather examines conjoint development of ideas in long-term partnerships between the first generation's entrepreneurs and the second generation's investors: A young entrepreneur might have an excellent business idea, but he neither has the know-how, nor the infrastructure to build a start-up firm and start producing the innovative good on his own in order to generate future income with it.

In line with the research work of [Maeda, 1991], [Russell, 2004] and [Zhu, 2008], *the OLG model of the search market for ideas* examines search frictions in the form of random bilateral trade patterns in an OLG environment. What is new in *the OLG model of the search market for ideas* in comparison to previous work on OLG models with search is its setting: The heterogenous agents live for three periods. Each period consists of two stages with alternating markets (DM/CM). The three periods reflect the first-, second-, and third generation of life: On the first stage DM of one period, the first generations' entrepreneur trades shares in future payouts for implementation expertise with the second generations' investor in a random bilateral trade match. This money enables the second generations' investor to consume innovative goods in the next period, when he is a third generations' retiree. On the second stage CM of one period, the first generations' entrepreneur, who successfully made a contract with an investor in the previous stage, is now able to sell his produced innovation good to the third generations' retirees. After that, the entrepreneur pays back the investor as contracted and the first period ends. In the second period, retirees die, former investors become the new retirees, former entrepreneurs become the new investors, and new entrepreneurs are born. The whole process starts from the beginning. In the next chapter *the OLG model of the search market for ideas* is explained in detail.

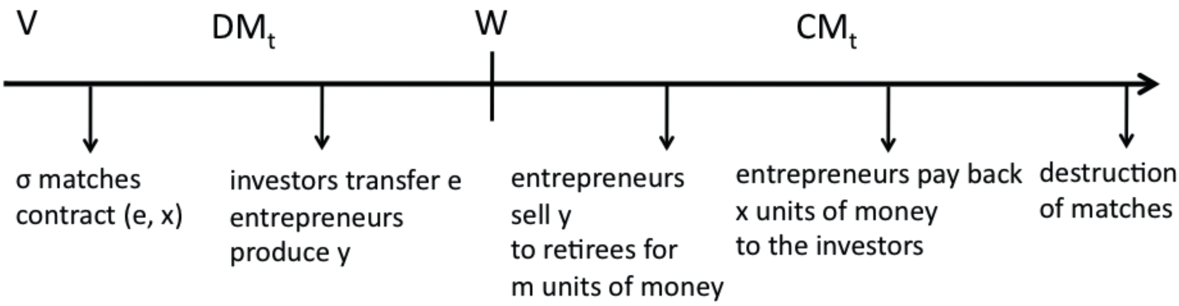
3 The Model

This chapter describes this master thesis' *OLG model of the search market for ideas* in detail and aims to bring the reader closer towards an answer to the research question: How does know-how transfer in the innovation sector influence an entrepreneurs' production possibilities, and what role does money play in trade for expertise and innovation goods? Chapter 3.1. describes the *OLG model of the search market for ideas'* environment, typical sequences of events within one period, and the agents' value functions needed to determine the bargaining solutions in chapter 3.2. Chapter 3.3 discusses the proportional solution with divisible money.

3.1 Environment

In an economy with overlapping generations, time is discrete and continues forever. People live for three periods and there are three types of heterogeneous agents. In the first period, agents are entrepreneurs E, in the second period they become investors I, and in the third period they are retirees R. Afterwards they die. Each period consists of two stages. In the first stage, the market is decentralized with pairwise, random matching and bargaining. Investors and entrepreneurs bilaterally meet and trade implementation expertise e for shares in future monetary payouts x. In the second stage, the market is centralized. Retirees and entrepreneurs meet to trade money m for innovative goods y. It exists one perishable, innovative consumption good y that can only be produced by the entrepreneurs. If an entrepreneur gets expertise from the investor in the DM_t, he produces y and takes it into the second stage of the same-period, the CM_t, to sell it to the retiree. The entrepreneur cannot take y from one period to the next, e.g. from CM_t to DM_{t+1}. The perishability of the consumption good will prevents it from being used as means of payment.

Figure 1: *The two sequences (DM_t, CM_t) in one typical period t:*

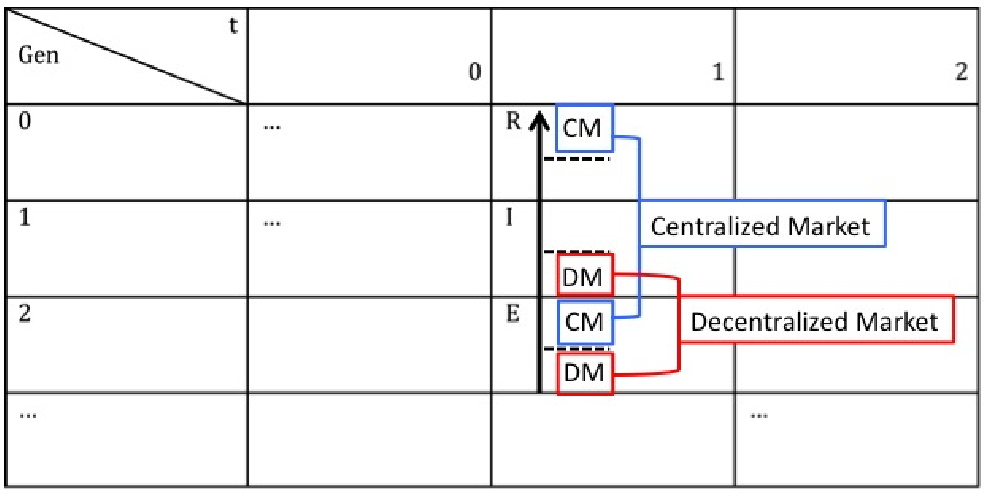


[Illustration by the author]

3.1.1 Sequence of Events Within a Typical Period

In the first period, newborn people become entrepreneurs. They are good in coming up with new, innovative ideas but they do not yet have the expertise to efficiently implement innovative projects. Therefore, an entrepreneur is in need of know-how transfer from an investor. The entrepreneur can produce if and only if he finds an investor in the first period's decentralized innovation market DM_t, willing to provide him with the expertise e needed to realize the innovative idea and to produce y. The matching process between the investor and the entrepreneur is successful with probability σ. This parameter models a search friction in the innovation market and can be interpreted as capturing heterogeneity in terms of the ideas that entrepreneurs have and that investors are willing to support. The higher σ, the more probable each entrepreneur finds a supportive investor. If this is the case, the two types of agents make a contract (e, x) in which the entrepreneur accepts to repay the investor for its provided expertise e, with x units of fiat money, after having produced and sold the innovation projects' output y to the retiree on the centralized consumption good market CM_t at the end of the first period. Afterwards, all matches are destroyed.

Figure 2: *Bilateral trades between the three generations within one period t: First, entrepreneurs and investors meet on the DM_t, afterwards entrepreneurs and retirees meet on the CM_t:*



[Illustration by the author]

As already mentioned, an agent lives for three periods and walks through six sequences - two for each period. In the following, the life of one typical agent and its trading actions are described. He is born in period t and lives for two subsequent periods, $t+1$ and $t+2$, before he dies.

In period t, with probability σ, the knowledge transfer is successful and the entrepreneur starts producing its innovative good in his first period of life DM_t. For the entrepreneur, his output is a function of expertise: $y = f(e)$. The more expertise the entrepreneur gets, the more output can he produce, but the marginal product decreases ($f'(e) > 0, f''(e) < 0$). The output from production is the consumption good y. The entrepreneur sells it to a consumer, the retiree R, at the end of his first period of life on the CM_t. The buyers of the consumption good are the retirees R. A retiree stands in his third and last period of life. Knowing that he will not be able to pay it back in the following period, nobody is willing to provide him with credit. This is the reason why a retiree finances all his consumption by fiat money m. Having received m units of money from the retiree in trade for output y at the end of CM_t, the entrepreneur is now able to settle the accounts with the investor and to pay back x units of money for the investor's provision of implementation expertise. At the end of period t, all matches are destroyed.

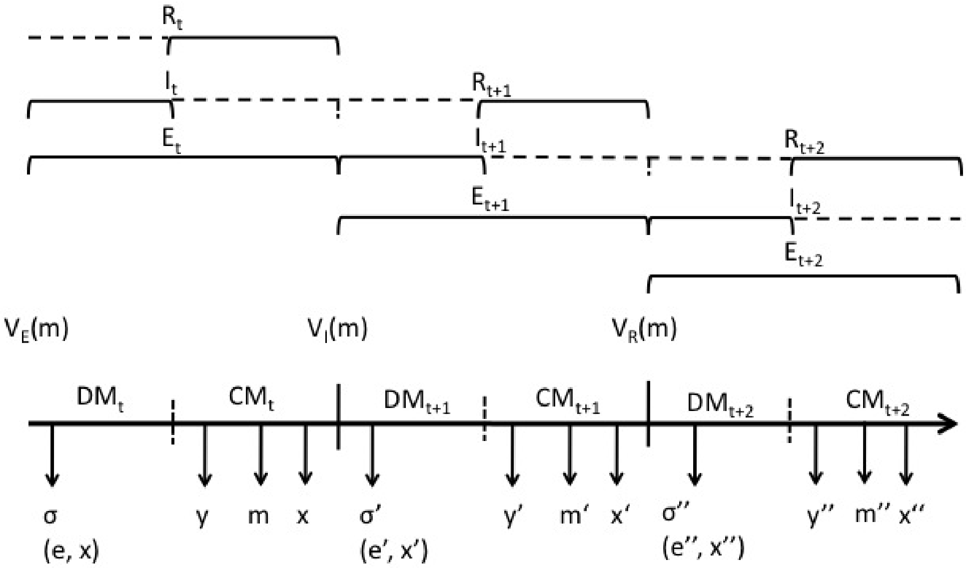
In period $t+1$, with probability σ, a successful entrepreneur is able to enter the second period of life DM_{t+1} with $m - x$ units of money and to become himself an investor I'. The apostrophe $'$ indicates that we are in the agents second period of life $t+1$. Now he is the one providing the next young generation's entrepreneurs with knowledge and expertise to help them realize innovative projects. This is happening with probability σ'. If there is a successful match between I' and E', the new generation entrepreneur produces its innovative product y' and sells it later on in CM_{t+1} to the new retiree R' (former periods' investor). In doing so, the new generation entrepreneur E' is able to pay back the innovator I' with x' units of money at the end of CM_{t+1}.

In period $t+2$, with probability σ', a successful investor I' is able to enter the third and last period of life DM_{t+2} with $m-x+x'$ units of money and to become a retiree R''. The two apostrophes $''$ indicate that the agents are in their third period of life $t+2$. The money enables him to enjoy consumption of the innovative good x'' - given that the new generation of entrepreneurs E'' had a match with the last generations entrepreneurs E' who now have become investors I''. With probability $1-\sigma'$ there was no successful match between I' and E' in period $t+1$ and the investor holds only $m-x$ units of money in his last period of life as a retiree. At the end of the third period, the agent dies.

On the following page, figure 3 gives a two-dimensional insight into the life of an agent that walks through three periods, each with two stages. During his lifetime, the agent transforms himself from an entrepreneur in t to an investor in $t+1$ and finally to a retiree in $t+2$.

Also the agents' value functions, dependent on the agents' money holdings $V_E(m)$, $V_I(m)$, and $V_R(m)$ are visible. The value functions are described in detail in the subsequent chapter 3.1.2.

Figure 3: *Life of one agent (E, I', R" = E_t, I_{t+1}, R_{t+2}) and the relevant parameters:*



[Illustration by the author]

3.1.2 Value Functions

Assumed that there is a match between the investor and the entrepreneur with probability σ, both accept the contract (e, x). The entrepreneur produces y units of the innovative good and sells it to the retiree before he recompenses the investor for the knowledge transfer. But with probability $1 - \sigma$ there is no match and the entrepreneur enters both subsequent periods with zero money. In an overlapping generations model with three periods, the utility function of consumption for an agent born at time t is $U_t[c_t, c_{t+1}, c_{t+2}] = u(c_t) + \beta u(c_{t+1}) + \beta^2 u(c_{t+2})$ with $u' > 0$ and $u'' < 0$. In the setting of an OLG model, the consumption of a young agent is c_t. The consumption of the same individual being middle-aged is c_{t+1}, and the consumption of the same individual being old is c_{t+2}. The entrepreneurs lifetime value function at the beginning of the DM_t is therefore:

$$V_E(0) = \sigma\{\beta V_I(m-x) + [\sigma'\{\beta^2 V_R(m-x+x')\} + (1-\sigma')\beta^2 V_R(m-x)]\}$$

$$+(1-\sigma)\{\beta V_I(0) + \beta^2 V_R(0)\}$$

The equation above becomes quite large as the interaction between the entrepreneur - being an investor in its second period of life - with the next generation's new entrepreneurs $(\sigma'(...)+(1-\sigma')(...))$ is included. This interaction determines the amount of money with which the entrepreneur enters his third period of life. The equation can be rewritten and simplified to get a clearer representation of the entrepreneur's lifetime value function in the DM_t:

$$V_E(0) = \sigma\{\beta V_I(m-x) + \beta^2 V_R(.)\} + (1-\sigma)\{\beta V_I(0) + \beta^2 V_R(0)\}$$

In the following, only the entrepreneur's value function respective his next period of life is of interest. In this case, with probability σ, the entrepreneur has a match, produces in DM_t, sells in CM_t and settles his account before he enters the DM_{t+1} with $m - x$ units of money. With probability $1 - \sigma$, the entrepreneur has no match in the CM_t. Then he enters the CM_{t+1} with zero balances:[5]

$$V_E(0) = \sigma V_I(m - x) + (1 - \sigma)V_I(0) \tag{1}$$

Provided that there was a match one period before, the entrepreneur enters the new period as investor with $m - x$ units of money. Due to his activities as an entrepreneur one period before, the investor now has the knowledge e' to help the next generation of entrepreneurs E' to implement their ideas. In order to let the new generation's entrepreneur participate on his expertise, the investor has to sacrifice some consulting time $c(e)$ that occurs like an opportunity cost in the investor's value function. With matching probability σ', an agreement is reached and the new generations' entrepreneur can produce the new innovative good y' and pay back x' units of money to the investor I'. This enables the investor to consume for x' more units of money in the next period CM_{t+2} when he becomes a retiree: $m - x + x'$. With probability $1-\sigma'$ there is no match between an investor and a new generation's entrepreneur. If this is the case, the investor enters the last period with the same amount of money that he entered the second period with: $m - x$. The investor's lifetime value function at the beginning of the DM_{t+1} is therefore:

$$V_I(m - x) = \sigma' V_R(m - x + x') - c(e') + (1 - \sigma')V_R(m - x) \tag{2}$$

Provided that there was a match two periods before (σ), and that there was a match one period before (σ'), or not $(1 - \sigma')$ the investor enters the third period DM_{t+2} with either $m-x+x'$ or with $m-x$ units of money and becomes a retiree. The value

[5] Discount factor β is dismissed from now on because in this OLG setting, agents do not live infinitely long but die at the end of period 3. For further explanation see [Champ et al., 2011]

function of the retiree is a function of its real balances ($z = \phi m$) expressed in terms of the innovation good y'' traded with the second next generation of entrepreneurs E'' in the CM_{t+2}. As the third is his last period of life, the retiree spends all his money holdings. The real value of the retiree's money holdings is also his budget constraint ($y'' = \phi m$). If the investor enters the third period with m units of money and trades money for good y'' at the competitive price ϕ, his utility is

$$V_R(m) = U_R(y'') = \phi m \tag{3}$$

A recapitulatory view indicates that entrepreneurs have direct access to both, DM and CM, whilst investors only have direct access to the DM and retirees only have direct access to the CM. Therefore, the bilateral matching process between investors and entrepreneurs not only plays a role in the decentralized market bargaining: Once an investor and an entrepreneur have found each other in the DM, they negotiate over the quantity of expertise and money that their contract should content. This directly influences the quantity of the investment good y that the retiree is able to acquire ($\phi m = y$) in the following CM [Nosal and Rocheteau, 2011]. This is shown in detail in the following chapter.

3.2 Equilibria

Chapter 3.2 describes the role of money in *the OLG model of the search market for ideas*. It should become obvious that the value of money depends on the fundamentals of the model, such as the search frictions σ and the competitive price ϕ. *The OLG model of the search market for ideas* is strongly influenced by and oriented on the work of [Nosal and Rocheteau, 2011] who provide the methodological framework for its mathematical analysis. The calculations in chapter 3 therefore show some similarities to [Nosal and Rocheteau, 2011]'s alternative bargaining solutions in their book 'Money, Liquidity and Payments'. Chapter 3.2.1 explores money in equilibrium under a trading protocol for the DM with a proportional bargaining solution. Chapter 3.2.2 explores money in equilibrium under a trading protocol for the CM with a symmetric bargaining solution. In chapter 3.3. money is not restricted to be indivisible anymore. By letting the price of money and the quantity of the search good vary, chapter 3.3 shows how money as a medium of exchange is valued in a multi-generation setting with search frictions. Thereby this chapter aims to answer how know-how transfers in the innovation sector influences an entrepreneurs' production possibility, and what role does money play in trade for expertise and innovation goods.

3.2.1 Bargaining Set and Proportional Solution in the DM

The proportional bargaining solution requires that agents' surpluses increase as the bargaining set expands and as the buyer's real balances increase [Nosal and Rocheteau, 2011]. This implies that the solution is monotonic and also Pareto efficient and that linearity is convenient when solving the bargaining problem [ibidem]. In the DM-match between entrepreneur and investor, every agent receives a constant share $(\theta, 1-\theta)$ of the total match surplus TS: θTS for the 'buyer' of expertise (entrepreneur) and $(1-\theta)TS$ for the 'seller' of expertise (investor), where $\theta \in (0,1)$ is the buyer's bargaining power. An agreement is a pair (e, x), where the buyer E receives $e \geq 0$ units of the search good expertise provided by the seller I. In exchange for expertise the investor receives $x \in [0, m]$ units of money earned by the entrepreneur by selling its innovative good y to the retiree in the CM. If an agreement is reached, then the entrepreneur's lifetime utility is $V_I(m-x) > 0$. If there is no agreement, the entrepreneur's lifetime utility is simply $V_I(0) = 0$. The use of equation (2) helps us to define the entrepreneur's surplus from trade:

$$S_E = V_I(m-x)$$

$$S_E = \sigma'\{V_R(m-x+x') - c(e')\} + (1-\sigma')V_R(m-x)$$

$$S_E = \sigma'\{\phi(m-x+x') - c(e')\} + (1-\sigma')\phi(m-x) \tag{4}$$

$$S_E = \theta TS$$

If an agreement is reached, the investor's lifetime utility is $V_R(m-x+x') - c(e')$. If there is no agreement, the investor's lifetime utility is still $V_R(m-x)$. The investor's surplus from trade is therefore:

$$S_I = V_R(m-x+x') - c(e') - V_R(m-x)$$

$$S_I = \phi(m-x+x') - c(e') - \phi(m-x)$$

$$S_I = \phi x' - c(e') \tag{5}$$

$$S_I = (1-\theta)TS$$

The total surplus is the sum of the entrepreneur's and investor's surpluses (4) + (5):

$$TS = \theta TS + (1-\theta)TS$$

$$TS = S_E + S_I$$

$$TS = \sigma'\{\phi(m - x + x') - c(e')\} + (1 - \sigma')\phi(m - x) + \phi x' - c(e')$$

$$TS = \sigma'\{\phi(m - x + 2x') - 2c(e')\} + (1 - \sigma')\phi(m - x)$$

$$TS = \sigma'\{\phi 2x' - 2c(e')\} + \phi(m - x) \tag{6}$$

Now we can define amount of money (x') that the entrepreneur has to pay to the investor by substituting the results from equations (4) and (6):

$$S_I = (1 - \theta)TS$$

$$\phi x' - c(e') = (1 - \theta)\Bigg\{\sigma'\{\phi 2x' - 2c(e')\} + \phi(m - x)\Bigg\}$$

It is obvious that an entrepreneur can never repay more money than he has ($x \leq m$):

$$x' = \frac{(1 - \theta)\Bigg\{\phi(m - x) - c(e')(1 + 2\sigma')\Bigg\}}{\phi - (1 + \theta)2\phi\sigma'} \leq m \tag{7}$$

3.2.2 Symmetric Bargaining Solution in the CM

So far investors and entrepreneurs have met bilaterally in the DM. For the trade meetings between the retirees R and the entrepreneurs E, we think in terms of competitive markets (CM). Buyers (R) and sellers (E) of the innovative good y meet in large groups in the CM. Particularly, all retirees want to consume, but not all of them have the required amount of money x' at their disposal. Only a fraction σ of retirees owns x' from the bargain one period before. The retirees solve the following problem:

$$\phi(1 - \sigma')(m - x) + \phi(1 - \sigma')(m - x + x') = \sigma y \tag{8}$$

Assume a situation where there was a match between the entrepreneur and the investor in DM_t. In the next period the entrepreneur becomes an investor. When the investor has no match in DM_{t+1} he keeps the money holdings $(m-x)$ he entered the DM_{t+1} with and enters the DM_{t+2} as a retiree with $(m-x)$. There he has the probability too of having no match and he cannot consume at all. This is expressed in the first term of equation (8). When the investor has a match in DM_{t+1}, enters the next period DM_{t+2} with money holdings $(m-x+x')$, but has no match with an entrepreneur as a retiree in CM_{t+2}, he neither can consume. This is expressed by the second term on the left side of equation (8). The retiree can only consume in CM_{t+2} with probability σ. He spends all his money to consume σ units of innovation good y. This is expressed by the right side of equation (8).

The amount of money x' that the retiree pays to the entrepreneur for the innovation good y is calculated by inserting x' from equation (7) into equation (8), and by assuming symmetry $(x=x', \sigma=\sigma', e=e')$. Solving (8) for ϕm, results in equation (9), which is exactly the retiree's utility function described in equation (3):

$$U_R(y) = \phi m \approx \frac{\sigma y}{2(1-\theta) - \frac{2-2\theta(2-\theta)-2x(1-\theta)}{1-2\sigma(1-\theta)}} \tag{9}$$

Equation (9) shows that $U_R(y)$ increases with a higher σ. In order to consume, the retiree must accumulate ϕm real balances into the CM_{t+2}. So far, this chapter illustrated the set of allocations that can be achieved in an economy with indivisible and durable fiat money. As the goal is to illustrate how money as a medium of exchange helps the agents to achieve preferred outcomes that could otherwise not be achieved, in the next chapter fiat money becomes perfectly divisible.

3.3 Proportional Solution with Divisible Money

Fiat money is now assumed to be perfectly divisible. The terms of trade in the DM are determined by the proportional solution where each player receives a constant share $(\theta, 1-\theta)$ of the match surplus. The search good expertise e and money m are traded in the DM between I and E. The innovative good y and money m are traded between E and R in the CM. In the DM, the real value of money ϕ and the search good expertise e are now allowed to vary over time.

The timing of events in a typical period is as follows: At the beginning of the DM, a measure of σ buyers (E) and sellers (I) are randomly matched, where the buyer can offer a payback of x units of money and the seller offers implementation expertise e. In each match, the buyer (E) and seller (I) make a contract (e, x), where e represents the amount of the search good to be provided by I and x represents the amount of money I receives from E at the end of the period. $E's$ and $R's$ participate at the same-period CM and exchange m units of money for the general innovation good y at price ϕ.

Consider the bargaining set between E and I. An agreement is a pair (e, x), where the buyer (E) receives $e \geq 0$ units of the search good expertise provided by the seller (I), in exchange for $x \in [0, m]$ units of money [Nosal and Rocheteau, 2011]. If an agreement is reached, then the buyer's utility is $u^E = V^I(m - x)$. R will pay ϕm units of money to E for the innovation good y in the CM. Therefore, before he pays back the I with x, $E's$ income at the end of the first period is $y = \phi m$ and E holds $m - x = \frac{y}{\phi} - x$ units of money at the end of the first period. $\frac{y}{\phi}$ can be rewritten as $\frac{f(e)}{\phi}$ because for E the output y is a function of expertise e: $y = f(e)$. The more expertise E gets, the more output can he produce ($f'(e) > 0$), but with decreasing marginal product ($f''(e) < 0$). If there is a match, the buyer's utility depends on the value of his money holdings when he enters the next period's DM and becomes an I:

$$u^E = V^I(\frac{f(e)}{\phi} - x) \tag{10}$$

We can now put (10) into $I's$ lifetime value function (2) from subchapter 3.2.1. and get:

$$u^E = \sigma\{V^R(\frac{f(e)}{\phi} - x + x_{+1}) - c(e_{+1})\} + (1-\sigma)V^R(\frac{f(e)}{\phi} - x) \tag{11}$$

Because V^R is linear in money, we can rewrite equation (11) as

$$u^E = \sigma\{f(e) - \phi x + \phi x_{+1} - c(e_{+1})\} + (1-\sigma)(f(e) - \phi x) + u_0^I \tag{12}$$

Which reduces to:

$$u^E = f(e) - \phi x + u_0^I \tag{13}$$

If no agreement is reached, E has zero utility because without e he can neither start producing, nor has he something to trade for money with R. Furthermore, E has no match possibilities in the following period DM as he has nothing to offer in the bargain:

$$u_0^E = V^I(0) = 0 \tag{14}$$

If an agreement is reached, in order to let E participate on his expertise, I has to sacrifice some consulting time $c(e)$ that occurs like an opportunity cost in $I's$ value function. If I helps E to implement his idea and to produce y, I gets paid out by E with x, after that E has sold y for m to R. The seller's utility is therefore:

$$u^I = V^R(m_{-1} - x_{-1} + x) - c(e) \tag{15}$$

Because V^R is linear in money, we can rewrite equation (15) as

$$u^I = \phi m_{-1} - \phi x_{-1} + \phi x - c(e) + u_0^I \tag{16}$$

Which reduces to:

$$u^I = \phi x - c(e) + u_0^I \tag{17}$$

If there is no agreement, then the seller's utility is

$$u_0^I = V^R(m_{-1} - x_{-1}) = \phi m_{-1} - \phi x_{-1} \tag{18}$$

because he enters the last period of life with the same amount of money, he entered the second period with. The buyer's (E), respectively the sellers (I) surpluses from an agreement are:

$$S_E = u^E - u_0^E = f(e) - \phi x$$

$$S_I = u^I - u_0^I = \phi x - c(e)$$

Those two equation can be combined into one:

$$u^E - u_0^E = \frac{\theta}{1-\theta}(u^I - u_0^I) \tag{19}$$

where $\theta \in (0, 1)$ is the buyer's bargaining power. The total surplus is the sum of the buyer's and seller's surpluses:

$$TS = S_E + S_I = f(e) - \phi x + \phi x - c(e) = f(e) - c(e)$$

The proportional bargaining solution requires that each agent receives a constant share of the match surplus. $S_E = \theta TS$ and $S_I = (1 - \theta)TS$:

$$S_E = f(e) - \phi x = \theta[f(e) - c(e)]$$

$$S_I = \phi x - c(e) = (1 - \theta)[f(e) - c(e)]$$

The model solves

$$(e, x) = arg\ max_{x \leq m}[f(e) - \phi x] \tag{20}$$

subject to

$$f(e) - \phi x = \frac{\theta}{1-\theta}[\phi x - c(e)] \tag{21}$$

and

$$x \leq \frac{f(e)}{\phi} = m \tag{22}$$

Solving expression (21) for ϕx results in $\phi x = (1 - \theta)f(e) + \theta c(e)$. Substituting ϕx into expression (20) simplifies the problem to:

$$e = arg\ max_e \theta[f(e) - c(e)] \tag{23}$$

subject to

$$\phi x = \theta c(e) + (1 - \theta)f(e) \tag{24}$$

If restriction (22) binds, e is simply the solution to:

$$\phi m = \theta c(e) + (1 - \theta)f(e) \tag{25}$$

Recap that $\theta \in [0, 1]$ represents the buyer's (E's) bargaining power, and $1 - \theta$ represents the seller's (I's) bargaining power, m is the buyer's money holdings, and x is what the buyer spends for e. If the constraint (22) $x \leq m$ does bind, then the solution to (20) is

$$e = e^* \tag{26}$$

and

$$x = m^* \equiv \frac{\theta c(e^*) + (1 - \theta)f(e^*)}{\phi} \tag{27}$$

With proportional bargaining, the buyer's ($E's$) surplus is strictly increasing in his real balances, until the match output e^* is achieved. Hence, if the cost of holding money balances is zero, then the buyer will accumulate sufficient real balances to purchase the efficient level of the search good e^* [Nosal and Rocheteau, 2012].

This is in contrast to the generalized Nash solution in [Nosal and Rocheteau, 2012]. The generalized Nash bargaining solution maximizes a weighted geometric mean of the buyer's and the seller's surpluses from trade, where the weights are given by the agent's bargaining powers and a surplus is the difference between lifetime utility when there is an agreement and lifetime utility when there is disagreement [ibidem]. The difference between the proportional solution and Nash solution is that in the latter, the buyer's share is a function of e, whereas for the proportional bargaining solution it is a constant [ibidem].

If however constraint (22) $x \leq m$ does not bind, then $m < m^*$. In this case, E buys an inefficiently low amount of expertise $e < e^*$ with his money holdings x earned form selling y to R. The DM output is determined by a proportional distribution of the buyers 'utility' $f(e)$ of using e, and the sellers disutility $-c(e)$ of providing e. The buyer will not accumulate more balances in the CM than he would spend in the contract in the next DM.

In the last section the results of *the OLG model of the search market for ideas* will be concluded and the most important points of this work are summarized.

4 Conclusion

The key feature of *the OLG model of the search market for ideas* was to analyze how money can effect more efficient endowment allocations for heterogenous agents in an multi-generation-setting with search frictions. The model makes clear that the result of the bilateral negotiations determines if optimality can be achieved or not. Besides this it is also important to pay attention to the monetary wedge introduced by search frictions since they occur like a cost of holding money to the buyer. The buyer tends to underinvest in real balances the higher the search frictions and the lower his bargaining power.

The proportional bargaining solution with divisible money between E and I determines the traded amount of e between the two types of agents. In turn, the traded amount of e from the contract (x, e) influences E's production possibilities and as $f(e) = y$, E's output too. Considering E's monetary earnings $\frac{f(e)}{\phi}$, as $f(e) = y = \phi m$, E earns $y = \phi m$ units of money in trade with R in the CM. This shows that the monetary earnings $\frac{f(e)}{\phi}$ of E are also dependent on the results (x, e) from the negotiations between E and I.

In the symmetric equilibrium of the economy described above, the money supply equals the money demand. Money that the agents hold in the model equals the money demand m. Consider money that a central bank injects into the system as money supply M. As long as money supply M is exogenously given by an instance such as the central bank, the real value of money, ϕ is also exogenously determined. But thanks to the negotiations between the heterogenous agents, ϕ as well as $\frac{f(e)}{\phi}$ adjust until the equation $\frac{f(e)}{\phi} = m$ is correct. In this case we have money demand equal to money supply.

In *the OLG model of the search market for ideas*, the bargaining set becomes larger when the buyer (E) is able to use more of his money for contract (x, e). This solution is consistent with [Nosal and Rocheteau, 2012]'s exemplifications and illustrates the idea that fiat money allows traders to achieve utility and output levels that otherwise would not be attainable.

With proportional bargaining, the buyer's surplus is strictly increasing in his real balances, until the match output e^* is achieved. But in *the OLG model of the search market for ideas* there is a monetary wedge introduced by search frictions (σ) and bargaining powers $(\theta, (1-\theta))$. A rise in the seller's bargaining power reduces the buyer's bargaining power θ and raises the monetary wedge. The buyer will tend to underinvest in real balances since he incurs the cost from holding money, but only receives a fraction θ of the match surplus. But if the cost of holding real balances is zero, match output approaches e^*. In contrast to the Nash solution, in the proportional bargaining solution the buyer accumulates sufficient real balances to purchase the efficient level of the search good e^* - if costs of holding real balances are not severe [Nosal and Rocheteau, 2012].

It is possible to extend the model in several ways. First, one could allow heterogeneity among entrepreneurs in their ability to develop their idea. This would induce a risk factor of start-up-bankruptcy for the investor and therefore influence his strategic behavior in contracting with the entrepreneur in the DM. Second, as mentioned in the beginning of this thesis, the innovation sector is seen as a major driver of economic performance. Therefore it would be interesting to model growth in the three-generations-setting, that could for example emerge from or at least be associated with the optimal quantity of innovative goods produced and sold by the entrepreneurs.

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