Master's Thesis

Negative Interest Rates as Monetary Policy Tool

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Abstract

With the recent implementation of negative monetary policy rates in Sweden, Denmark, Japan, the European Union and Switzerland, the discussion about the effects of negative interest rates becomes highly relevant. In the first part, this thesis outlines the implementation of negative interest rates as monetary policy tool and discusses some transmission channels targeted by central banks. In the second part, the thesis discusses the effects of negative deposit rates on consumption by revisiting the model of Berentsen and Monnet (2008). Negative interest rates are similar to the inflation tax, since they are taxing the money holdings of agents. Decreasing deposit rates affect consumption and welfare negatively and increases trade in the money market as agents try to avoid the negative deposit rates.

Contents

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1 Introduction

1.1 Motivation

In the aftermath of the financial crisis 2007/08, central banks have adopted unconventional monetary policy instruments to counteract the adverse effects of the crisis. As a reaction, central banks supplied large amounts of liquidity and decreased policy rates (Brunnermeier (2009)). The Taylor rule incorporates levels of unemployment and output to determine the optimal level of interest rates in an economy (Woodford (2001)). As Rudebusch (2009) points out, the Taylor rule suggests optimal interest rates around -5% since the financial crisis. Similarly, Mankiew (18.4.2009) discusses the option to set negative interest rates for the Federal Reserves. Such discussions raised the question of implementing negative interest rates as a monetary policy tool. Recently, central banks have decreased policy rates below zero. While negative short-term interest rates have been considered infeasible, current observations show the opposite (Bernanke and Reinhart (2004)).

The effects of decreasing interest rates as an expansionary monetary policy measure has been frequently analyzed. Several monetary policy channels transmit the decreasing interest rates to the economy and to the yield curve. This affects exchange rates, investments and spending (Goodfriend (2000)). The question remains whether a decrease of interest rates below zero works in a similar way as a decrease above zero. Can we expect that spending and investments increase, when interest rates are lowered into the negative range? With the implementation of negative interest rates as a monetary policy tool, this question becomes highly relevant. However, since the decrease of interest rates below zero is relatively new, there is currently not much literature about the effects on consumption or welfare. In addition, all implementation of negative interest rates have been accompanied by further monetary policy measures, hence the effect of such a decrease below zero is very difficult to measure.

This thesis discusses the implementation of transmission effects of negative interest rates as a monetary policy tool and analyses the effects on consumption and welfare by revisiting the model of Berentsen and Monnet (2008). It is structured as follows. After a brief literature review, an overview of the implementation of negative interest rates as a monetary policy tool follows in section (2). This section focuses on the goals and intentions of central banks. Section (3) revisits the model of Berentsen and Monnet (2008) and discusses negative deposit rates within a channel system. Section (4) discusses the effects of negative interest rates on consumption and welfare in the model of Berentsen and Monnet (2008). The following literature review focuses on previous research related to the questions at hand. More precisely, it outlines the results of theoretical frameworks that are relevant for the discussion in the following sections.

1.2 Literature Review

Short-term interest rates have been low in several countries even before the financial crisis. This has raised the question of implementing monetary policy at very low interest rate levels (Bernanke and Reinhart (2004)). Bernanke and Reinhart (2004) discuss a few proposals to conduct monetary policy with low interest rates without suggesting negative interest rates. Specifically, they discuss steering the expectations of agents, quantitative easing and changing the composition of the central bank balance sheet. Ilgmann and Menner (2011) discuss historical and current policy proposals to implement negative interest rates and outline, that taxing money holdings is not a completely new idea. The feasibility of negative interest rates has been questioned, since agents can always avoid the negative interest rate payments by holding cash instead (Mankiew (18.4.2009)). However, as Goodfriend (2000) states, holding money is not costless, and the current observations show clearly, that banks still choose to hold excess liquidity even when they are subject to negative interest rates.

Central banks used to steer interest rates, which would transmit to the economy over various channels. Boivin et al. (2010) provide an extensive overview of transmission channels and their mechanism. Steering the interest rates effects cost of credit and investment, wealth or the exchange rate. As Goodfriend (2002) points out, paying an interest on reserve holdings allows the central bank to steer the reserve holdings directly beyond the minimum reserve requirements. With a negative interest rate on reserves, banks should have an incentive to hold no more than they are required to. All excess liquidity should be invested into bonds that yield a higher return. As the demand for bonds increases, the price of bonds increases, which will decrease the interest rate paid on bonds. With this mechanism the negative interest rate is transmitted to the yield curve (Goodfriend (2000)).

Altunbas et al. (2010) analyze the effects of monetary policy on the risk-taking behavior of banks with low interest rates. Their results show that if the interest rates are below the suggested interest rate of the Taylor rule, the risk-taking behavior of banks increases significantly. The measure of risk-taking behavior is given by the expected default frequency. To account for a below equilibrium level of interest rates, the authors measure the gap between the Taylor rule interest rate and the actual interest rate. While controlling for additional factors, Altunbas et al. (2010) find that low interest rates increase the risk of financial instability measured by the expected default frequency. These concerns become highly relevant considering the level of interest rates proposed by the Taylor rule compared the current interest rates as indicated by Rudebusch (2009).

Furthermore, in the Lagos and Wright (2005) framework setting, interest rates above zero drive a wedge between marginal utility of consumption and marginal cost of production. To reach the first-best allocation, marginal utility of consumption should equal marginal cost of production. In this framework, the cost of holding money is a function of the time preference and inflation. Since agents face a cost by holding money, consumption decreases and thus they can no longer reach the first-best allocation (Lagos and Wright (2005), (Nosal and Rocheteau, 2011, pp. 131/132.).

Berentsen et al. (2007) developed a model with credit within a Lagos-Wright (Lagos and Wright (2005)) framework. In this setting buyers can lend money to achieve their first-best consumption and sellers can deposit excess cash and earn an interest on their holdings. The authors show that the presence of financial intermediation allows gains in welfare, because sellers can earn an interest on their money holdings and not because the buyer can lend money to reach the first best consumption level. In other words, the lifetime utility of buyers remains the same with financial intermediation while the lifetime utility of sellers increases. Their results suggests that a decrease in the interest rate paid on deposits will reduce the utility of the sellers and thus the gains in welfare. These results arguably raise the question of welfare effects when interest rates are negative.

Berentsen et al. (2005) analyze the effects of unanticipated monetary injections when money holdings are not equal across agents. They find that an unanticipated increase in the money stock increases consumption in a low inflation economy when agents hold different money balances. The monetary injections increase welfare. Agents with less money holdings can increase their consumption, because the loss of value of their money holdings is more than offset by the monetary injections. Agents with high money holdings are subject to a higher inflation tax and the monetary injections cannot offset their loss in consumption. Thus, the increase in the money stock redistributes consumption from rich agents to poor agents (Berentsen et al. (2005)).

Furthermore, Berentsen et al. (2014) compare a floor system to a channel system and find that the floor system is only optimal, if the target rate satisfies the Friedman rule and the central bank is allowed to run a deficit. If these conditions do not hold, a channel system with a positive spread is optimal. Moreover, they show that the interest rate paid on deposits at the central bank is welfare improving. Bernhardsen and Kloster (2010) discuss the differences of a floor and a channel system as well and analyze theoretical aspects based on the model discussed Whitesell (2006) and Keister et al. (2008). Bernhardsen and Kloster (2010) show that, if the central banks provide liquidity in times of financial distress, the key interest rates will decrease in a channel system. However, in a floor system, where the key interest rate is equal to the deposit rate, liquidity providing measures will not reduce the interest rate. They point out, that central banks need to engage in fine-tuning operations to steer the money market rate in a channel system. Similarly, Armenter and Lester (2015) analyze monetary policy normalization for the United States. While the Federal Reserve has not implemented negative interest rates, they have set up two standing facilities for excess liquidity. Since only few financial institutions have access to both standing facilities, the deposit rates on these facilities work similar to a channel system and allow the Federal Reserve to steer the money market rate (Armenter and Lester (2015)).

Arguably, negative interest rates should have some effect on welfare. This monetary policy tool changes the cost of holding money for banks, which is similar to the cost of holding money with inflation. Ennis (2009) extends the Lagos and Wright (2005) framework and shows that agents, who do not have access to the centralized market in every period, will choose to spend more of the money holdings for consumption under inflation than under the Friedman rule. Instead of holding money, which is subject to devaluation, they increase their consumption to avoid the inflation tax. While the classical Lagos and Wright (2005) framework captures the Hotpotato effect of inflation, it fails to explain the consumption front-loading effect. The extension can explain the increased consumption that is observable with inflation (Ennis (2009)).

Lagos and Rocheteau (2005) also analyze the welfare cost of inflation in a search-theoretic framework with Nash bargaining and competitive pricing. They find that increasing inflation reduces the amount of trades as well as the search intensity of the buyer with Nash bargaining. Increasing inflation to moderate levels above the Friedman rule in competitive pricing can increase consumption. However, increasing inflation largely away from the Friedman rule also has negative effects on consumption even with competitive pricing (Lagos and Rocheteau (2005)).

Craig and Rocheteau (2008) compare the measured welfare costs of inflation with two different approaches, one taken by Lagos and Wright (2005) and the other one taken by Bailey (1956). Bailey (1956) and also Lucas (2000) measure the welfare cost of inflation by measuring the area under the inverse money demand curve whereas Lagos and Wright (2005) measure the welfare cost as the percentage amount of consumption that agents would be willing to give up to have an inflation rate of zero rather than 10%. Craig and Rocheteau (2008) find that the area below the inverse money demand curve may underestimate the welfare cost of inflation depending on the terms of trade. Based on the rate of inflation the private return of bringing one additional unit of money into the decentralized market for the buyer is smaller than the social return. Furthermore, the cost of welfare function is subject to the bargaining protocol in the decentralized market. More precisely, Craig and Rocheteau (2008) find that the welfare cost increases if the buyer does not receive all the surplus. Even though the paper focuses on welfare cost of inflation, some tendencies can be drawn for negative interest rates. Firstly, the marginal value of holding money decreases, since agents have to pay negative interest on their money holdings in the settlement market in the following period. Therefore the implementation of negative interest rates may fail to reach the first-best allocation. In addition, the bargaining protocol may affect the cost of negative interest rates in the same way as in Craig and Rocheteau (2008). The more surplus the seller receives, the higher the interest payments will be. Arguably, the cost of negative interest rates may be less if the buyer receives all the surplus.

2 Implementation and Transmission of Negative Interest Rates

2.1 Implementation

While negative interest rats have been observable in financial markets before, central banks have only recently begun to set their policy rates negative. The following section focuses on the implementation of negative policy rates and the targeted channels, in which negative interest rates would transmit to the economy.

Swiss National Bank

In reaction to the low inflation rates and the appreciation of the Swiss Franc against the Euro, the Swiss National Bank introduced a floor for the exchange rate in 2011. The risk of deflation and the decrease in exports, mainly in services has led the SNB to introduce the peg against the Euro. The target range for the 3M-Libor was at 0 at this point in time (Schweizerische Nationalbank (2011)). In December 2014, the Swiss National Bank set a negative target range for the 3M-Libor at 0 to -25 basis points while simultaneously continuing the peg against the Euro. According to the SNB, the risk of deflation had increased until 2014 and the Swiss Franc remained highly overvalued against the Euro, which still put pressure on the export industry (Schweizerische Nationalbank (18.12.2014, 2014)). In January 2016, roughly a month after the central bank set a negative target for the 3M-Libor, the peg was discontinued. During of the implementation of the floor on the exchange rate, the SNB largely increased their foreign reserves. With the dissolving of the peg, the central bank stated to influence the exchange rate continuously when necessary. Simultaneously, the central bank decreased the target range of the 3M-Libor to -25 to -125 basis points and set the interest rate on reserves at -75 basis points (Schweizerische Nationalbank (1.15.2015, 2015a)). As the SNB stated in the press release, the negative interest rates are implemented with a threshold. All reserves held that are below this threshold are subject to a interest rate of 0, while all reserves above that threshold are subject to a interest rate of -75 basis points. There are two methods to calculate this threshold. All banks that are subject to minimum reserve requirements have a threshold of the twenty-fold of the reserve requirements plus or minus changes in cash holdings compared to a reference period. Banks that are not subject to this reserve requirement face a fixed threshold. Beginning of 2016 the central bank has decided not to change the interest rates and thus remain at the same level as at the beginning of 2015 (Schweizerische Nationalbank (2015b, 2016)).

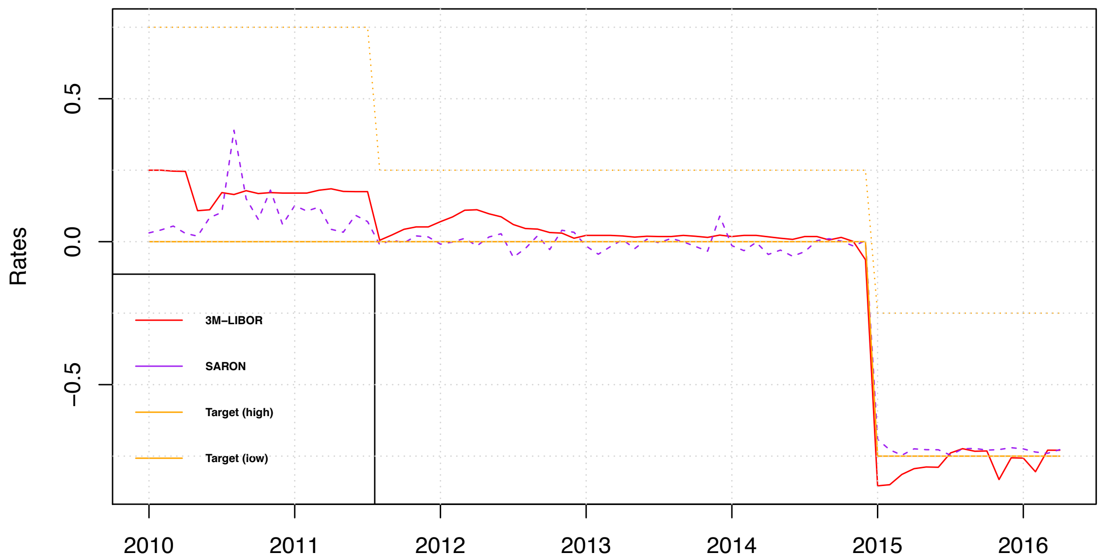

Figure 1: Development of Interest Rates in Switzerland (Data: Schweizerische Nationalbank (2010-2016b); Schweizerische Nationalbank Volkswirtschaftliche Daten (2010-2016), Schweizerische Nationalbank (2010-2016a))

The prime reason for setting negative interest rates was the strong appreciation of the Swiss Franc against the Euro. With the negative interest rates, the SNB intends to reduce the incentive to hold Swiss Francs and thus reduce the capital inflow. This should alleviate the upwards pressure on the Swiss Franc (Schweizerische Nationalbank (18.12.2014, 2014)).

Figures (1) and (2) show the development of the key interest rates (in percent) in Switzerland as well as certain balance sheet positions of the central bank. The 3M-Libor rate is currently at the same level as the interest paid on reserves. The reason is the large increase of the balance sheet of the SNB. The large amount of liquidity decreases the money market rate to the floor, which in this case is the interest rate paid on reserves (Berentsen et al. (2014); Bernhardsen and Kloster (2010)). The foreign reserves were also increased largely to influence the exchange rate and even though the peg was discontinued in early 2015, the foreign reserves are still increasing.

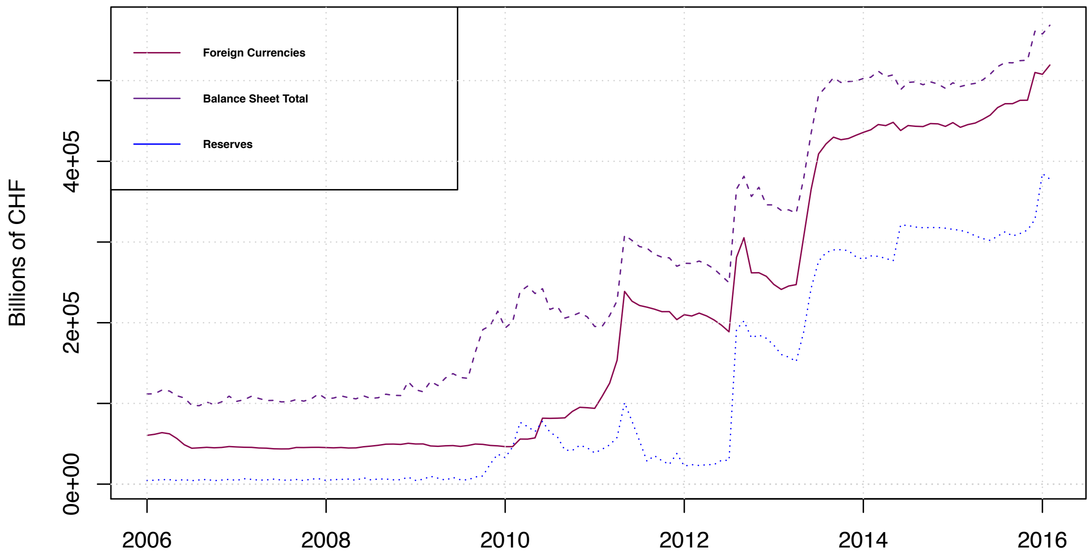

Figure 2: Development of Foreign Reserves, Reserves and Balance Sheet Total (Data: Schweizerische Nationalbank Volkswirtschaftliche Daten (2006-2016))

European Central Bank

In light of low inflation rates in the EU zone, the European Central Bank has set a negative deposit rate of -10 basis points first in June 2014. Together with targeted longer-term refinancing operations, preparatory work regarding ABS outright purchases and the conducting of MRO's at a fixed tender with full allotment, these measures should increase bank lending in the Euro zone (Bech and Malkhozov (2016); European Central Bank (2014)).

The ECB steers money market rates by offering a lending and a deposit facility for banks. The rate on the deposit facility has been set to -10 basis points in June 2014 and since has been continuously lowered to the current level of -40 basis points (European Central Bank (10.03.2016, 2016a)). Furthermore, this deposit rate also applies to all excess reserves held by banks that exceed the minimum reserve requirements. Figure (3) shows the development of the key interest rates (in percent) in the Euro area. As the graph indicates, the EONIA rate has been close to the deposit rate set by the European Central bank, with a few exceptions.

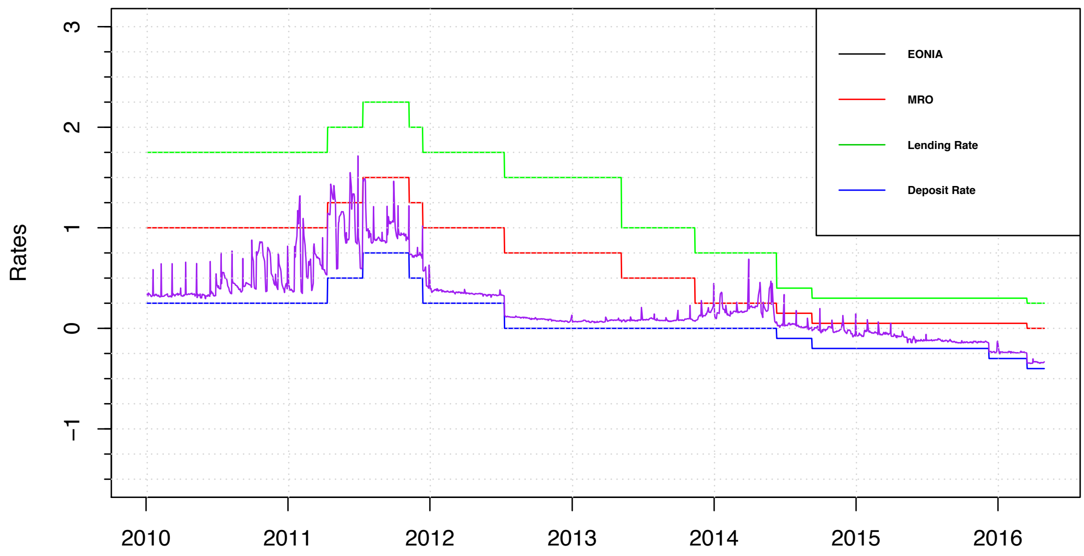

Figure 3: Development of Interest Rates in the EU (Data: European Central Bank (2009-2016, 2010-2016))

The central bank is concerned with increasing bank lending in the Euro zone to increase consumption and investments and anchor inflation expectations at the target of 2% (Bech and Malkhozov (2016); European Central Bank (2014, 2016a)). In the latest issue of their economic bulletin, the ECB reports an increase in bank lending. The low interest rates have been passed through and banks are improving the borrowing conditions for firms and households. The yield curve in the Euro zone has decreased and spot rates up to five years maturity are currently negative. (European Central Bank (19.04.2016, 2016b)).

Inflation rates remain low in the Euro zone and are expected to become negative within the first six months of 2016. Furthermore, they report that long-term inflation expectations for five years are at very low levels.

Danmarks Nationalbank

The central bank of Denmark first implemented negative interest rates in 2012 by setting the rate on one-week deposits to −20 basis points. The monetary policy of Denmark targets a nearly fixed exchange rate to the Euro (Bech and Malkhozov (2016); Danmarks Nationalbank (05.07.2012, 2012)). To alleviate the upwards pressure of the Danish krone, negative interest rates along with foreign currency reserves purchases were implemented. By keeping a fixed exchange rate to the Euro, the central bank of Denmark creates a low inflation environment (Danmarks Nationalbank (2012)). Due to the upwards pressure of the Danish Krone, the central bank has been setting the key interest rates at lower levels than the European Central Bank (Moselund Jensen and Spange (2015)).

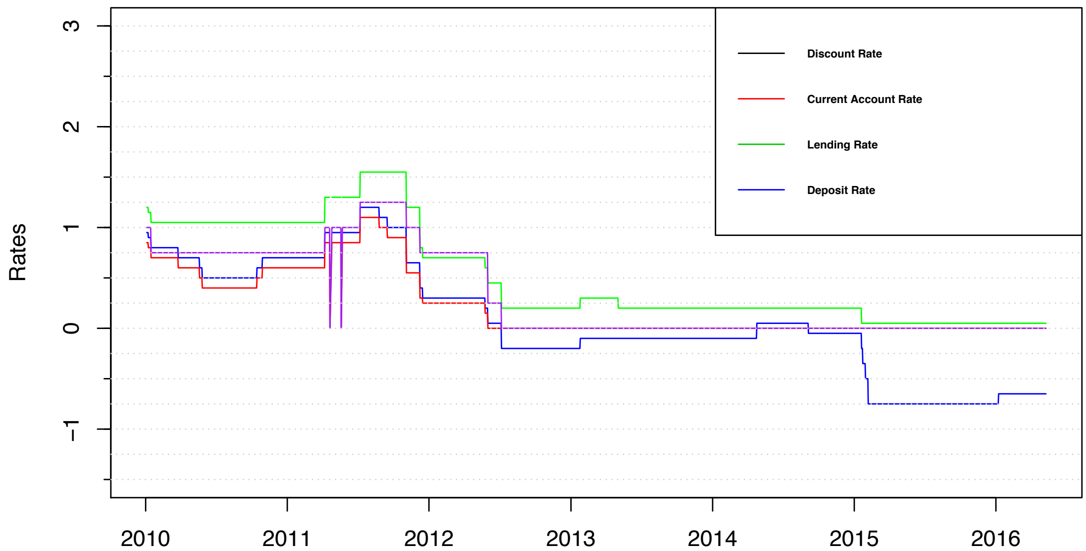

Figure 4: Development of Interest Rates in Denmark (Data: Danmarks Nationalbank (2010-2016))

The development of the key interest rates (in percent) in figure (4) in Denmark shows that the central bank has adjusted the interest rates to keep the target fixed. This also included frequent increases in the deposit rate. The negative interest rate was applied to the one-week deposit rate. In Denmark, banks can hold reserves at the current account up to a certain limit. If the reserves are larger than the current account limit, banks have to make deposits at the central bank. While usually, the interest rate on the deposits is higher than the current account rate, this has changed with the implementation of negative interest rates. Currently, the deposit rate is negative, while the current account rate remains at zero (Danmarks Nationalbank (2012)). In addition, the central bank increased the limit of the current account. This means that less of the reserves are subject to negative interest rates (Danmarks Nationalbank (05.07.2012, 2012)). This method is similar to the threshold method of the Swiss National Bank, since they also decided that negative interest rates should only apply to a part of the excess liquidity. With the implementation of negative interest rates, we would expect agents to increase their cash holdings to avoid the interest rates. However, in Denmark there is no observable sharp increase in the demand for cash, which can be attributed to the fact that holding money is not costless. Furthermore, the negative interest rates have not been passed through to the retail customers of banks (Moselund Jensen and Spange (2015)). As Fremmich Andresen et al. (2015) state, the turnover in the unsecured overnight loan market has decreased since the implementation of negative interest rates, due to the increased limits on the current account. Thus, banks can now manage their daily liquidity demand without necessarily having to borrow in the overnight market. The money market rates have been driven primarily by the reshuffling of liquidity by banks to avoid the negative interest rates (Fremmich Andresen et al. (2015)).

Sveriges Riksbank

The Swedisch Riksbank implemented negative interest rates in February, 2015 by cutting the repo rate to -10 basis points. The bank decided to cut interest rates below zero to ensure the inflation expectation of 2%. Besides the decrease of interest rates, the bank adopted additional measures to safeguard inflation expectations (Sveriges Riksbank (2015), Sveriges Riksbank (12.2.2015), Bech and Malkhozov (2016)). Currently, the repo rate is even lower at -50 basis points. The goal of the central bank remains to anchor inflation expectations at the monetary policy target (Sveriges Riksbank (11.2.2016), Bech and Malkhozov (2016)). The central bank attempts to remove excess liquidity from the banking system by charging negative interest rates. Deposits within the fine-tuning operations are currently at -65 basis points and any liquidity that remains is charged with 125 basis points (Bech and Malkhozov (2016)).

Figure (5) shows the key interest rates in Sweden (in percent). The Swedish central bank implemented a negative deposit rate prior to February 2015. As Bernhardsen and Kloster (2010) outline, even though a negative deposit rate was implemented in 2009, this negative rate had only little effect as the Swedish Riksbank offered fine-tuning operations at a positive rate. Only banks that failed to deposit excess liquidity with these operations had to incur the negative interest payment. This has changed substantially since mid 2014, when the repo rate and the 3 month Stibor rate became negative as well (Bernhardsen and Kloster (2010)).

Sweden implements monetary policy by steering the interest rate through a channel system. Currently, the deposit rate is at -125 basis points, as indicated in the graph below. What remains interesting, is the fact that the market rate is higher than the floor and remains in the middle of the corridor. Most central banks flooded the system with liquidity during the financial crisis. If there is excess liquidity, the money market rate is expected to be at the floor of the channel system (Berentsen et al. (2014); Bernhardsen and Kloster (2010)). Clearly, this is not the case. If a central bank adopts a channel system and provides liquidity to financial institutions, they also have to engage in fine-tuning operations to keep the interest rate on the interbank market in the target range (Bernhardsen and Kloster (2010)). To keep the interest rate close to the repo rate, the Swedish central bank conducts fine-tuning operations (Danmarks Nationalbank (2012)).

Furthermore, the Swedish central bank has introduced much less liquidity than Switzerland, for example. The balance sheet total of the Swedish central bank in 2015 had a value of approximately 3.35 times the value of 2008. Compared to Switzerland, which has a six-fold balance total of the value in 2006, balance sheet expansion in Sweden is not that high. However, the position of fine-tuning operations on the liabilities side have increased substantially (Schweizerische Nationalbank Volkswirtschaftliche Daten (2006-2016); Sveriges Riksbank (2008-2016)). These observations may explain, why the market rate is not at the floor set by the central bank.

Bank of Japan

The Bank of Japan has decided to implement negative interest rates in the beginning of 2016. As the central bank stated, their goal is to achieve price stability around

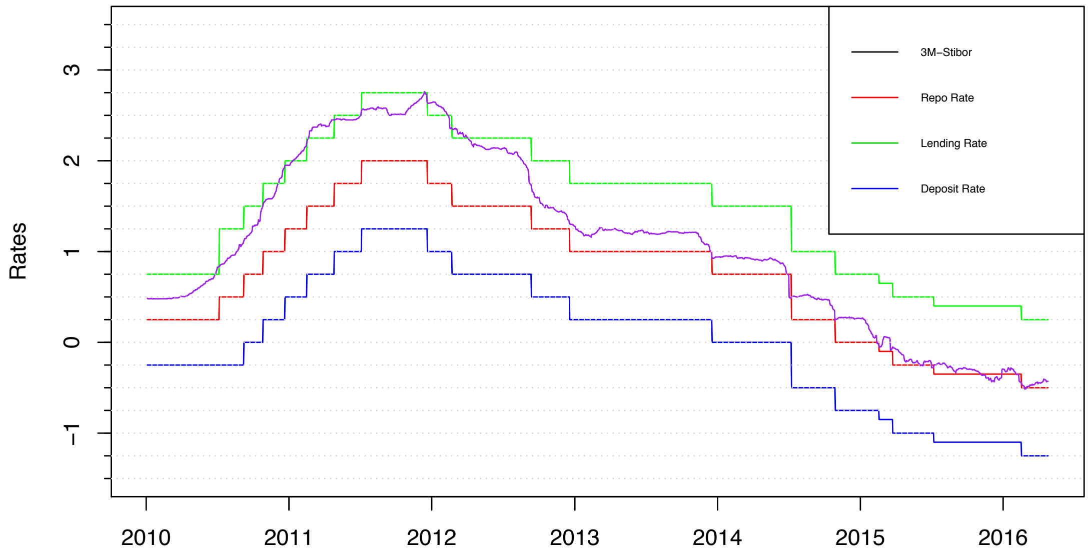

Figure 5: Development of Interest Rates in Sweden (Data: Sveriges Riksbank (2010-2016),STIBOR Data from Sveriges Riksbank (2010-2016) from Thompson Reuters))

the target as soon as possible. They have implemented the so called "Qualitative and Quantitative Easing with Negative Interest Rates" and have clearly stated to pursue this policy as long as necessary (Bank of Japan (29.1.2016), Bank of Japan (2016a)). The negative interest rates are implemented on a three-tiered basis. The balances of the current account of financial institutions are subject to a positive interest rate with currently 10 basis points. In the second tier, with a current interest rate of 0 basis points, there are additional outstanding amounts, which are specified by the central bank. The third tier captures all balances that exceed the amount of tier 1 and 2 and are subject to a negative interest rate of -10 basis points (Bank of Japan (29.1.2016)). Roughly three months after the first implementation of negative interest rates, the short- and medium-term expected inflation remains low. Although with this monetary policy instrument, the Bank of Japan states, that long-term inflation expectations seem to be increasing towards the target (Bank of Japan (2016b)).

Figure (6) shows the basic loan and discount rate (in percent) which remains positive at 30 basis points. However, due to the decrease of interest rates on excess liquidity

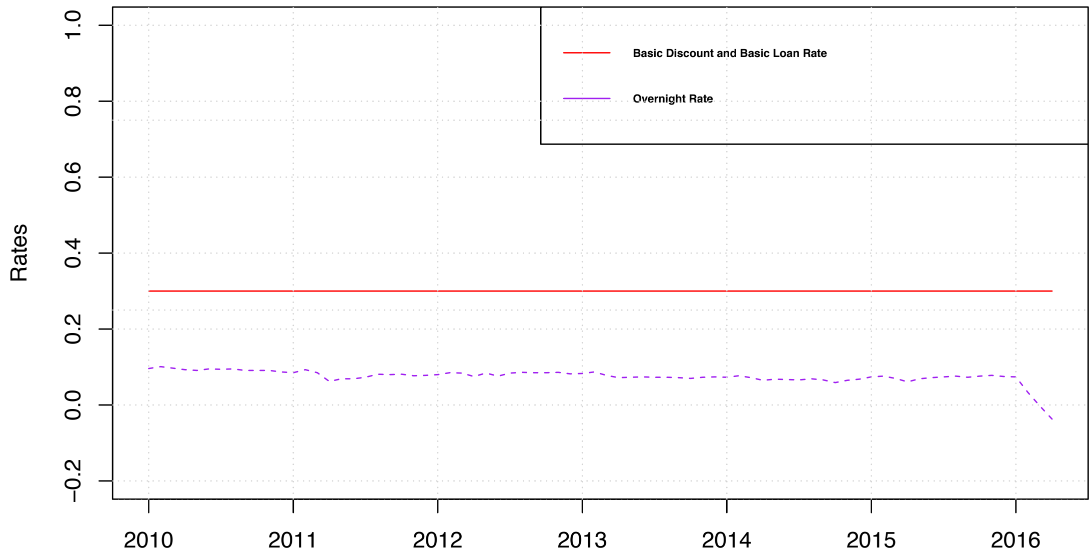

Figure 6: Basic Loan and Discount Rate and Overnight Rate (uncollateralized) (Data: Bank of Japan (2010-2016))

to −10 basis points, the uncollateralized overnight lending rate has decreased and is negative as well since the beginning of 2016.

2.2 Transmission Channels

Negative interest rates were implemented to reach various goals of central banks. In the following subsection a few transmission channels are discussed. There exists a broad literature on the various channels of monetary policy transmission. The following paragraph will focus on a few of them: the management of inflation expectations, interest rate channel, the bank lending channel and the exchange rate channel. These channels were chosen because the five central banks, that have currently implemented negative interest rates, have focused on them consistently in their press releases and economic reports.

Boivin et al. (2010) offer a detailed survey on each of the known transmission channels and analyze the change of transmission channels with an extended version of the New-Keynesian framework of Smets and Wouters (2007). They find that the neoclassical transmission channels, namely, the interest rate channel, , the wealth channel, the inter-temporal substitution effect and the exchange rate channel remain important factors of transmitting monetary policy to the economy.

The direct interest rate channel simply captures the idea that the user cost of capital decreases with lower interest rates and thus, it becomes less expensive for firms to acquire credit. This should lead to an increase in investments (Boivin et al. (2010)). Bernanke and Gertler (1995) discuss the credit channel, which refers to the change in the finance premium, which is the difference between the cost of external funds and internal funds. The authors identify two different transmission effects within the credit channel, the bank lending channel and the balance sheet channel.

The balance sheet channel is based on the decrease in interest rate costs of firms and banks when the central bank decreases interest rates. This causes a strengthening of the balance sheets of households and firms, because the interest rate payments of credit decrease. Thus, the cost of acquiring capital decreases, which should again lead to higher spending. Additionally, a decreasing interest rate can lead to an increase in asset prices and thus improve the health of balance sheet of banks. Similarly, the quality of balance sheets of firms and even households will improve. This makes it easier for firms and households to acquire capital and thus again, investment and consumption should increase ((Peek and Rosengreen, 2010, pp. 257-277) Bernanke and Gertler (1995); Boivin et al. (2010); European Central Bank (2008)).

The bank credit channel refers to the change in reserves of banks with a change in monetary policy and is based on frictions in the credit market, which are overcome by banks. If the central bank increases reserves, the total amount of credit supply available increases. If many borrowers rely on bank loans for investments, this should lead to an increase in credit and thus consumption and investment (Bernanke and Gertler (1995); Boivin et al. (2010); European Central Bank (2008), (Peek and Rosengreen, 2010, pp. 257-277)).

If there is excess liquidity, the central bank can no longer steer the demand for reserves as they used to ((Peek and Rosengreen, 2010, pp. 257-277)). With negative deposit rates, banks should have an incentive to take the excess liquidity and either lend it to firms or households or invest it in other assets. As Goodfriend (2000) points out the increased demand for other assets would lead to an increase in prices and thus the low interest rates would be transmitted across the yield curve. The ECB has tried to improve the conditions for investments and credit since the financial crisis. While consumption has largely driven the economic recovery in the European Union, recently investments have been increasing slowly. The ECB attributes this increase in investments to the monetary policy measures, which included quantitative easing programs and TLTRO among negative interest rates and other policy measures (European Central Bank (2016a)).

Boivin et al. (2010) also discuss several developments that have affected the transmission of monetary policy, such as structural changes and changes in the formation of expectations and point out, that managing inflation expectations has been rising in importance for the conduct of monetary policy. Also Mishkin (2007) points out the importance of managing inflation expectations. The Federal Reserve and many other central banks have started to target inflation more closely and thus were able to anchor inflation expectations. This has lead to the fact that the inflation trend has become more stabilized (Mishkin (2007)).

Galati et al. (2011) use two different approaches, one survey-based, the other one based on financial market data from inflation swaps to analyze how the anchoring of inflation expectations has changed since the financial crisis. They chose this approach, since survey based information does not necessarily reflect the actions of agents while the behavior in financial markets does. Using data from the United Kingdom, the European Union and the USA, they estimate the changes in inflation expectations. Although the survey based analysis yields stable results for inflation expectations, the financial market data based approach shows an increased volatility since 2007 (Galati et al. (2011)).

As agents base their consumption decisions on the expected path of interest rates, their expectations will influence consumption and investment decisions. For example, if agents expect to have low interest rates for prolonged period of time, they will adjust their consumption differently than if the low interest rates are expected to be temporary (Boivin et al. (2010)). This highlights an interesting point. If agents do not believe that negative interest rates will prevail, they will not adjust their spending behavior accordingly. This means that the effect of negative interest rates or any effect of an implemented policy will not reach the same effect when they are not considered to be long-lasting.

Furthermore, targeting inflation expectations is important since monetary policy affects the economy with a lag. Therefore, it is reasonable that a central bank does not only react to current observations but also expected inflation. When inflation expectations become less anchored from the target of the central bank, economic stabilization will be much harder (Orphanides and Williams (2005)).

Switzerland and Denmark are currently both concerned with the effect of monetary policy on the exchange rate which is captured in the exchange rate channel. This transmission channel simply states the relationship between monetary policy and the level of imports and exports. If the central bank reduces the interest rate, the return on domestic assets falls relative to foreign assets, which causes a depreciation of the domestic currency. This leads to a decrease in prices of domestic goods relative to foreign goods and thus to an increase in exports (Boivin et al. (2010)). With negative interest rates, the capital inflow into the domestic country should be reduced, because agents can receive a higher yield on investments abroad. This should alleviate any upwards pressure of the domestic currency. Smets and Wouters (2002) extend a New-Keynesian model to incorporate the effects of monetary policy on the exchange rate. If both import prices and domestic prices are sticky, the central bank faces a trade-off between stabilizing the domestic price level and stabilizing the exchange rate. They show that a central bank, that targets price stabilization cannot react solely on the change of the price level in the domestic country, but has to consider the effects on the exchange rate and thus the price level of imported goods as well. Furthermore, Smets (1995) analyzes the different effects of monetary policy transmission channels in various central bank models. While each central bank adopts a different model, which makes the comparison of the results difficult, it also allows each country to use the model, that suits its economic conditions best. Nevertheless, his results indicate that the exchange rate channel and the cost of capital channel are the most important channels. Also Boivin et al. (2010) point out, there are some studies that highlight the importance of the exchange rate channel for small open economies. Such results support the decisions of central banks to focus their monetary policy on this specific channel.

The results in Smets and Wouters (2002), as well those shown by Mishkin (2007) Smets (1995), and Boivin et al. (2010) suggest that these transmission channels remain highly important for monetary policy and it's effectiveness to reach the goals of the central banks. There are however, some critical aspects that were highlighted in a report of Bech and Malkhozov (2016). For example, in Switzerland an increase in mortgage rates are currently observable and this increase is attributable to the implementation of negative interest rates. This development is due to the fact, that banks insure themselves against the risk of varying short-term interest rates by conducting swaps with pension funds and insurance companies. A bank is subject to a short-term interest rate on it's liabilities side and long-term interest rates on it's asset side. Therefore a bank can circumvent this exposure by agreeing to pay a counter-party the long-term interest rate in return for a short-term interest rate. In an environment with negative short-term rates banks face a negative interest rate from the swap, which they cannot pass onto their customers. This causes their costs to rise and has led to an increase in mortgage rates (NZZ (2.10.2015)). Furthermore Bech and Malkhozov (2016) question the effect on financial stability since negative deposit rates could impair profits of financial institutions and thus have an adverse effect on financial stability.

3 Negative Interest Rates in a Channel System

In this section, I will revisit the theoretical framework of a channel system of Berentsen and Monnet (2008) with a negative deposit facility. Channel systems are adopted by the ECB and the Swedisch central bank. Both central banks have implemented negative interest rates within a channel system by lowering the deposit rate into the negative range.

The theoretical model of this entire section is from Berentsen and Monnet (2008). I will summarize the framework, to make the theoretical considerations more tractable and point out the changes that apply with a negative deposit rate.

Channel systems include an overnight lending facility and a deposit facility. Agents with excess cash holdings can deposit their liquidity at the central bank and earn an interest rate i_d. Agents, that are in need of liquidity overnight, can borrow from the central bank at the interest rate i_l. Since agents, who need liquidity, can always choose to lend from the central bank, they are unwilling to pay a higher interest rate than i_l in the interbank market. Conversely, any agent with excess liquidity is unwilling to lend cash at anything lower than i_d. Therefore the interest rate in the money market, denoted i_m has to lie within this corridor (Berentsen and Monnet (2008)).

Intuitively, a negative deposit rate should induce banks to remove any excess liquidity held at the central bank. If banks were able to store their excess liquidity in cash, we would expect that agents do not access the deposit facility and store their money holdings in cash. The rate of return on cash holdings would be based on the value of the currency. Therefore, if this option would be allowed to agents, the negative deposit rate would no longer function as a floor, but the value of fiat money would act as the lower bound on the money market rate. For this discussion, I will assume that agents cannot avoid the negative deposit rate. Since there are some costs associated with holding cash, agents would not necessarily prefer cash holdings to deposits, even if the rate is negative. For the following discussion, we simply assume that agents cannot avoid the negative deposit rates or in other words, excess liquidity holdings are always subject to the deposit rate. This is observable in reality as well, since most accounts with banks are subject to a fee, and yet people are still willing to deposit their money. Some central banks have implemented thresholds for the application of these negative interest rates (Bech and Malkhozov (2016); Danmarks Nationalbank (05.07.2012); Schweizerische Nationalbank (18.12.2014)). This means that not all reserves held at the central bank are subject to those negative interest payments. Agents with excess liquidity above the threshold only have the incentive to reduce their money holdings down to the threshold. Consequently, the effects outlined below are somewhat less severe, if the negative deposit rate only applies to some part of money holdings.

3.1 The Environment

The environment of the model of Berentsen and Monnet (2008) is as follows. There are three markets that open sequentially. First, there is a settlement market, in which overnight loans are settled and agents can both consume and produce. The second market is the money market. At the beginning of the market agents receive with a signal some probability, about the probability of being a seller or a buyer. Based on this information, they can adjust their money holdings in the money market at rate i_m. In the third market or the so-called goods market, agents can access the central banks facilities. Because agents face uncertainty about their types, they might need to lend liquidity or deposit excess liquidity at the central bank. Furthermore, buyers want to consume in the third market but cannot produce whereas sellers can produce, but do not want to consume.

In the settlement market, agents can produce and consume the general good. If they work to produce the general good, they incur a dis-utility that is linear to the hours worked. Namely, if they work h hours, the utility is $-h$ and consuming h gives a utility of h. They will repay their loans, withdraw their deposits and adjust their money holdings. The goods produced in the first market can be used as collateral for borrowings in the second and third market. Agents can store the general goods with a storing technology, that yields a return on $R > 1$ in the first market of the next period. Furthermore, Berentsen and Monnet (2008) assume that $\beta R \leq 1$. In equilibrium, $\beta R < 1$ will hold. That means the discounted return of collateral is below 1 and thus it is costly for agents to hold them. Agents hold bonds, because there is a liquidity premium on bonds.

In the second market, agents receive a signal ϵ about the probability of being a seller in the third market. Depending on that signal, the agents will choose to trade with each other in the money market. In particular, they will lend or borrow liquidity depending on their expectations about their type. The probability of being a seller is n. Since agents are normalized to one, n is also the fraction of sellers in the economy and consequently the amount of liquidity. At the beginning of the money market agents receive some information with probability σ^k about the probability n^k of being a seller. The authors assume that there is no aggregate uncertainty, or $n = \sum \sigma^k n^k$. The difference of a high probability and a low probability of being a seller is the signal, denoted $\epsilon = n^H - n^L$. If $\epsilon = 0$, the signal contains no information and there is no trade in the money market. If $\epsilon = 1$ there is no uncertainty about the liquidity shock and agents do not access the facilities. Only if $\epsilon \in (0, 1)$, agents receive some imperfect information and trade in the money market and access the facilities at the central bank (Berentsen and Monnet (2008)).

In the goods market, the type is revealed. Sellers will produce the good at a cost $c(q_s) = q$ and sell these goods at a price p. Buyers will buy goods and derive some utility from the consumption, denoted $u(q)$. $u(q)$ is continuous and twice differentiable with $u'(q) > 0, u''(q) < 0$. Furthermore agents can adjust their money holdings based on their type by accessing the standing facilities. Berentsen and Monnet (2008) first derive the welfare function. To do this, they consider the hours worked by buyers, denoted h_b and sellers, denoted h_s in the first market.

$$h_b = \phi(m_{+1} + (1 + i_\ell)\ell) - Rb + b - \tau M$$

$$h_s = \phi(m_{+1} - (1 + i_d)(m + pq_s)) - Rb + b - \tau M$$

The value of money is denoted ϕ. A buyer has to work so many hours to get the amount of cash holdings he wants to take into the next period and the borrowings he has to repay plus interest plus the amount of bonds he wants to hold minus the return on the bond holdings minus transfers. Equivalently, a seller works so many hours to receive the amount of cash holdings he wants to take into the next period less interest payment on deposits less return of bonds and transfers and the amount of bonds the agent wants to hold. The hours worked by the seller are decreasing in the deposit rate.

$$\begin{aligned} h &= nh_s + (1 - n)h_b \\ h &= n\left[\phi m_{+1} - (1 + i_d)(m + pq_s) - Rb + b - \tau M\right] \\ &\quad + (1 - n)\left[\phi(m_{+1} + (1 + i_\ell)\ell) - Rb + b - \tau M\right] \end{aligned} \tag{1}$$

Market clearing in the third market requires $nq_s = (1 - n)q_b$. Buyers spend all their money holdings in the third market and thus bring in $\tilde{m} = 0$ money holdings into the next period. Conversely, a seller has $\tilde{m} = m + pq_s$ money holdings.

$$h = -(R - 1)b$$

The effects on the hours worked are linear in the deposit rate. The hours worked by a seller increase with a decreasing deposit rate and this holds regardless of whether the deposit rate is positive or negative. Welfare is thus defined as:

$$W = -b + (1 - n)[u(q) - q] + \sum_{j=1}^{\infty} \beta^j \left((1 - n)[u(q) - q] + (R - 1)b\right) \tag{2}$$

which reduces to

$$(1 - \beta)W = (1 - n)[u(q) - q] + (\beta R - 1)b \tag{3}$$

3.2 Negative Deposit Rates with no Trade in the Money Market

The first case considered by Berentsen and Monnet (2008) is the case where the signal bears no information. Therefore, there will be no trade in the money market. Accordingly, the money stock evolves based on transfers to agents and interest payments to and from the central bank.

$$M_{+1} = M - i_\ell L + i_d D + \tau M \tag{4}$$

where L is the total amount of lending by the central bank and D is the total amount of deposits by agents at the central bank. τM denotes transfers to the agents. With a negative deposit rate the deposits paid to depositors becomes negative, which means that holding everything else constant, the money stock will decrease over time. This is not surprising. Agents have to pay for the deposits and thus, liquidity is drawn out of the system. Berentsen and Monnet (2008) assume that the real stock of money is constant over time, which implies $\phi M = \phi_{+1} M_{+1}$. Furthermore $\frac{M_{+1}}{M} = \frac{P}{P_{+1}} = \frac{\phi_{+1}}{\phi} \equiv \gamma$. Usually, a decreasing supply of money would result in an increase in the value of money. However, as Berentsen et al. (2014) outline, a decreasing deposit rates lowers the demand for money in this framework. Therefore the effect on the value of money is not clear and depends on the extent of the decrease of the demand and the supply. As Berentsen and Monnet (2008) show it is optimal to set $\tau = 0$ in equilibrium. However, if $\tau > -i_\ell L + i_d D$, the money stock is growing. That means agents would not only be taxed on their money holdings due to the negative deposit rate but also due to inflation. Arguably, there would be an additional negative effect on consumption.

Berentsen et al. (2015b) set up the maximization problem in the settlement market as follows. Agents maximize their utility by choosing their money holdings, the bond holdings the amount of lending and the amount of deposits.

$$\begin{aligned} W(m,b,\ell,d) = \max_{h,m_2,b_2} \quad & -h + V(m_2,b_2) \\ \text{s.t.} \quad & \phi m_2 + b_2 = h + \phi m + Rb - \phi(1+i_\ell)\ell + \phi\tau M \end{aligned} \tag{5}$$

The first order conditions imply that the marginal value of bringing one extra unit of cash into the second market is equal to the value of fiat money. Furthermore, the marginal value of bringing one additional unit of bonds into the money market equals one.

$$V_m \geq \phi(= \text{ if } m > 0) \tag{6}$$

$$V_b \geq 1(= \text{ if } b > 0) \tag{7}$$

The envelope conditions are as follows.

$$W_m = \phi, W_b = R, W_\ell = \phi(1 + i_\ell), W_d = \phi(1 + i_d) \tag{8}$$

Note, the marginal value of holding one more unit of a deposit is now smaller than one. If agents bring one additional unit into the third market and deposit it with the central bank, the additional value will be less than one. This is different from a positive deposit rate. If the deposit rate decreases, but stays positive, the return on that additional unit of money will still be positive for a seller. With a negative deposit rate, this changes and the rate of return of holding one additional unit of fiat money becomes negative for a seller. Agents will try to avoid this negative deposit rate by decreasing the amount of liquidity they hold at the end of the third market.

Since a seller will never borrow from the central bank and a buyer will never deposit at the central bank in the third market, the value function of the third market can be written as follows.

$$V(m, b) = (1-n)[u(q)+\beta W(m-pq+\ell, b, \ell, 0)]+n[-q_s+\beta W(m+pq_s-d, b, 0, d)] \tag{9}$$

The seller's maximization is as follows:

$$\begin{aligned} &\max_{q_s, d} -q_s + \beta W(m + pq_s, b, 0, d) \\ &m + pq_s = d \end{aligned} \tag{10}$$

And the first-order conditions are as follows:

$$1 - \beta\phi_{+1}p + \beta\phi_{+1}\lambda_d = 0$$

$$\beta\phi_{+1}(1 + i_d) - \beta\phi_{+1}\lambda_d$$

or equivalently

$$\beta\phi_{+1}p + \beta\phi_{+1}\lambda_d = 1 \tag{11}$$

$$i_d = \lambda_d \tag{12}$$

The $\beta\phi_{+1}\lambda_d$ is the Lagrange multiplier of the deposit constraint. The shadow price of the lending constraint is now negative. This means that if the constraint is loosened by one unit, or equivalently, the seller can deposit one unit more, the return on that unit is negative. Clearly, the agent has no incentive to deposit an additional unit of money. Since we assume, that agents cannot avoid the negative deposit rate, the constraint on deposits holds with equality and this directly affects the money holdings of the seller that consist of what he brought into the third market and his earnings from selling the produced goods. These equations result in the following condition.

$$\beta\phi_{+1}p(1 + i_d) = 1 \tag{13}$$

Note, $(1 + i_d) < 1$, that means $p\beta\phi_{+1} > 1$. Since $\beta < 1$, $\phi_{+1}p > 1$ has to hold.

The buyer's maximization problem remains the same, since a buyer will never deposit any money holdings at the central bank and is therefore omitted[1]. The buyer is constrained in the described way. He cannot spend more than his money holdings and his borrowings combined and the amount of borrowings are limited based on his amount of bond holdings. The first-order conditions are:

$$u'(q) = \beta\phi_{+1}p(1 + i_{\lambda_q}) \tag{14}$$

$$\lambda_q = \lambda_\ell + i_\ell \tag{15}$$

The $\beta\phi_{+1}\lambda_q$ is the Lagrange multiplier of the consumption constraint and $\beta\phi_{+1}\lambda_\ell$ is the Lagrange multiplier of the borrowing constraint. From this follows the following condition:

$$\frac{u'(q)}{c'(q)} = \frac{1 + \lambda_\ell + i_\ell}{1 + i_d} > 1 \tag{16}$$

Berentsen and Monnet (2008) show that trades are not efficient, if the borrowing constraint is binding. With a negative deposit rate, trades are never efficient. Even

[1]The maximization problem of the buyer can be found in the appendix.

if the borrowing constraint of the buyer is not binding ($\lambda_\ell = 0$), trades cannot be efficient in equilibrium with a negative deposit rate. The lending rate would have to be equal to the deposit rate. This would imply a negative interest rate on borrowings from the central bank and agents would borrow infinite amounts. A negative deposit rate means, the central bank is paying agents for borrowing money at the central bank. Berentsen et al. (2014) address this question regarding the financing of a central bank deficit and the political aspects associated with it.

In equilibrium, the growth rate of money is determined and according to this equation.

$$\gamma = (1 + i_d) - (1 - n)(i_\ell - i_d)\frac{z_\ell}{z_m} + \tau m \tag{17}$$

where $z_\ell = \frac{\ell}{p}$ and $z_m = \frac{m}{p}$. That means the growth rate of money is determined by the deposit rate and the the amount of borrowings the lenders use to finance their consumption. z_ℓ is the amount of goods one unit of borrowed money can buy, and the z_m is the amount of goods buyers buy with one unit of fiat money. The amount of consumption q is financed by either fiat money or by borrowing at the lending facility. If $\beta R < 1$ holds, the borrowing constraint is binding, which implies $z_\ell > 0$. More precisely, the fraction of consumption financed by borrowing is as follows [2].

$$z_\ell = \beta R b \frac{1 + i_d}{1 + i_\ell} \tag{18}$$

Interestingly, z_ℓ does not only depend on the lending rate implemented by the central bank but also on the deposit rate or more precisely on the difference of those two policy rates. The amount of goods that can be bought with one unit of borrowed money is increasing in the deposit rate. The reason for that lies in the decreasing marginal value of bringing one additional unit of fiat money into the third market.

$$V_m = (1 - n)\frac{u'(q)}{p} + n\beta\phi_{+1}(1 + i_d) \tag{19}$$

Any agent faces a certain probability of becoming a seller and thus would be subject to the negative deposit rate on his money holdings. Therefore, the value of bringing one additional unit of fiat money into the third market is decreasing in the deposit rate. This means agents will hold less money and thus finance a larger part of their

[2] This equation can be derived from 11 as well as the constraint on borrowing for the seller $\ell \leq \bar{\ell} = \frac{Rb}{\phi_{+1}(1+i_\ell)}$ (Berentsen and Monnet (2008))

consumption with credit. Furthermore, with a decreasing deposit rate, the spread is increasing. That means the cost of borrowing money relative to the benefit of holding money increases.

A policy in which $\Delta = 1$ is no longer feasible with negative deposit rate. Thus, out of the three cases in equilibrium in Berentsen and Monnet (2008) only two apply to the case of negative deposit rates. Therefore, we only consider these two cases from Berentsen and Monnet (2008). If:

1. $\bar{\Delta} < \Delta < \tilde{\Delta}$, then $z_\ell > 0$ and $z_m > 0$
2. $\Delta \geq \tilde{\Delta}$, then $z_\ell = 0$ and $z_m > 0$
   where $\tilde{\Delta} = \frac{1-\beta n + \frac{\tau}{(1+i_d)}}{\frac{1}{R} - n\beta}$

The smallest spread possible with a negative deposit rate is a policy, where the lending rate is set positive but close to zero and the deposit rate is negative and close to zero. This lower bound is denoted $\bar{\Delta}$. If we take the negative deposit rate as given, a lower bound for the spread exists, which is uniquely determined by the value of the negative deposit rate. If the central bank implies a policy, for which Δ is between the two critical values, agents will finance a part of their consumption with fiat money and the rest by borrowing at the lending facility.

The second case seems straightforward, since it simply states, that agents will no longer borrow from the central bank if the lending rate is too high. However an increase in the lending rate is identical to a decrease in the deposit rate by the same amount. The corridor widens and case two states that if the corridor increases above a certain threshold, agents will no longer borrow. This comes from the change in the liquidity premium on bonds. The negative deposit rate does not affect the marginal value of holding an additional unit of bonds V_b. However, it will affect the liquidity premium on the bonds. With a probability $(1-n)$ the agent becomes a buyer and can use the bonds to acquire $\frac{R}{\phi_{+1}(1+i_\ell)}$ units of money. With a probability of n, the agent becomes a seller and the additional unit of bonds allow him to acquire $\frac{\beta R(1+i_d)}{(1+i_\ell)}$ units of goods. With a negative deposit rate, the numerator decreases, which implies that the seller can acquire less goods with his bond holdings. The liquidity premium thus decreases with a decreasing deposit rate or with an increasing lending rate. Moreover, the numerator $\beta R(1+i_d) < 1$ holds for all values of βR. This means that the exchange of a bond into fiat money is always associated with a decrease in utility since the fiat money is subject to a negative deposit rate and the bond will yield a discounted return $\beta R < 1$.

$$1 - \beta R = (1 - n) \left(u'(q) \frac{\beta R(1 + i_d)}{(1 + i_\ell)} - \beta R \right) \tag{20}$$

Because of the decreasing liquidity premium of bonds, agents are less willing to hold bonds. Since they face uncertainty about their type, they expect to become a seller with some probability. If they become a seller, they can exchange their bond holdings for fiat money, which is now subject to a negative deposit rate. Therefore, the fraction of consumption financed by borrowing will decrease, if the spread increases. Depending on the level of the deposit rate and the value of βR, sellers will choose to hold bonds across periods or exchange them for fiat money. With positive rates $\beta R < 1$ and $(1 + i_d) > 1$ held and therefore seller could exchange them for fiat money, deposit them at the central bank and earn the deposit rate. Therefore, agents would always choose to exchange their bonds for fiat money. However, with a negative deposit rate, this changes. If the interest rate on deposits is smaller than βR, they will choose to hold bonds across periods rather than exchange them for fiat money and deposit them with the central bank. Nevertheless, since the liquidity premium decreases regardless of whether the deposit rate is more or less than the discounted yield on bonds, agents will finance less consumption by borrowing. These cases do not change with a negative deposit rate are directly taken from Berentsen and Monnet (2008). Moreover, in the framework of Berentsen and Monnet (2008) the collateral is a real good, with a certain return R, that is exogenous. If bonds were nominal, we would expect the following mechanism to take place. Agents would rather hold the bonds than exchange them for fiat money if they become a seller in the third market. That in turn means less agents would be willing to exchange the bonds for fiat money, or equivalently, the supply of bonds is decreasing and thus, causing the price to rise. An increasing price of bonds would decrease the yield of bonds and this mechanism effectively causes the yield curve to shift downwards (Goodfriend (2000)).

A central bank's goal is to maximize the lifetime utility by choosing a corresponding level of consumption. There is an upper and lower bound on the consumption level. If it is optimal to set $\Delta > \tilde{\Delta}$, no agent will choose to hold bonds and thus, consumption will be entirely financed by fiat money holdings [3]. Accordingly, the lower bound on consumption can be written as follows. From equation (6), (13) and (19) one can derive $\frac{\gamma-\beta(1+i_d)}{\beta(1+i_d)} = (1-n)[u'(q)-1]$, which can be used to derive equation (21) (Berentsen and Monnet (2008)).

$$\tilde{q}(\tau) = u'^{-1}\left(\frac{1-n\beta-\frac{\tau}{(1+i_d)}}{\beta(1-n)}\right) \tag{21}$$

The lower bound of consumption is increasing in the deposit rate. Moreover, $\tilde{q}(\tau)$ is a concave function of the deposit rate, indicating that the lower bound of consumption decreases more strongly if the deposit decreases severely. Arguably, there is some limit to the extent of negative interest rates in reality. Central banks cannot set the deposit rate too low, because at some point, agents will choose to hold their reserves in cash.

The upper bound of consumption, denoted $\hat{q}$ arises, due to the fact, that the policy $\Delta = \frac{1+i_\ell}{1+i_d}$ cannot be smaller than 1. $\Delta = 1$ corresponds to a policy where the lending rate is equal to the deposit rate. The right-hand side of the equation below is maximized when $\Delta = 1$ is implemented (Berentsen and Monnet (2008)).

$$\frac{1-\beta R}{\beta R} \leq (1-n)\left(\frac{u'(q)}{\Delta}-1\right) \tag{22}$$

If the deposit rate is negative $\Delta = 1$ is not feasible. Therefore, the upper bound decreases with a negative interest rate. Hence, the largest feasible value of delta would be to set the lending rate as close to zero as possible for a given negative deposit rate. In other words the upper bound would be where Δ is as close to one as possible. This value is denoted $\hat{q'}$.

$$\hat{q'} = u'^{-1}\left(\Delta\left[\frac{\frac{1}{\beta R}-n}{(1-n)}\right]\right) \tag{23}$$

The central bank can implement a policy, such that

$$\hat{q'} \geq q' \geq \tilde{q}(\tau) \tag{24}$$

[3] It is optimal to implement this policy, if the real return on bonds is below a certain threshold $\bar{R}$ (Berentsen and Monnet (2008))

The effects of a negative deposit rate on consumption are discussed in section (4).

3.3 Negative Deposit Rates with Trade in the Money Market

In the next step, Berentsen and Monnet (2008) consider the case, that signal contains some, but only little information. This is the case where $\epsilon = n^H - n^L$ is small. The lending and borrowing in the money market are denoted y^k. If an agent borrows liquidity y^k is positive, otherwise it is negative. As a consequence the money stock evolves over time according to a different law of motion.

$$M_{+1} = M[\sigma^H(1-n^H)\ell^H + \sigma^L(1-n)^L\ell^L]i_\ell + [\sigma^H(1-n^H)d^H + \sigma^L(1-n)^L d^L]i_d \quad (25)$$

Thus the growth rate of money is determined by the following equation.

$$\gamma = 1 + i_d - (i_\ell - i_d)\left[\sigma^L(1-n^L)\frac{\ell^k}{M} + \sigma^H(1-n^H)\frac{\ell^H}{M}\right] \quad (26)$$

Again, the money stock is decreasing with a negative deposit rate. In this case $\gamma < 1$, the economy experiences a deflation.

With trade in the money market buyers cannot only borrow at the central bank, but also from potential sellers in the second market. Therefore, the lending and deposit constraint change as follows:

$$\begin{aligned} \ell^k &= \tfrac{Rb}{\phi_{+1}(1+i_\ell)} - \tfrac{y^k(1+i_m)}{(1+i_\ell)}. \\ d^k &= m + y^k + pq_s. \end{aligned}$$

The value function in the first market depends on the hours worked and the expected value of entering the second market with m_2 and b_2 money and bond holdings. The money and bond holdings in the second market are equal to the hours worked and the money holdings from the first market, the return on bonds and the return on deposits plus the deposits less the interest payment on the loans in the second and third market and the repayments.

$$\begin{aligned} W(m,b,\ell,d,y) = \max_{h,m_2,b_2} \quad & -h + Z(m_2,b_2) \\ \text{s.t.} \quad & \phi m_2 + b_2 = h + \phi m + Rb - \phi(1+i_d)d - \phi(1+i_\ell)\ell - \phi(1+i_m)y \end{aligned} \quad (27)$$

The first order conditions and the envelope conditions remain the same. However, there is one additional envelope condition.

$$W_m = -\phi(1 + i_m)$$

This highlights another critical issue. Depending on the implemented policy, the money market rate is not necessarily positive. Moreover, the deposit rate only affected sellers directly, but the money market rate affects both types of agents directly. The level of i_m does not only capture the marginal benefit of lending one additional unit of money for the seller but also the cost of borrowing one additional unit of money for the buyer. This aspect will be discussed below in detail.

The value function in the second market $Z(m, b)$ depends on the money holdings, the borrowing or lending and the bond holdings.

$$Z(m, b) = \sum_{k=H,L} \sigma^k V^k(m + y^k, b, y^k) \tag{28}$$

where y^k solves

$$\begin{aligned} \max_{y^k} \quad & V^k(m + y^k, b, y^k) \\ \text{s.t.} \quad & y^k \geq \frac{Rb}{\phi_{+1}(1 + i_m)} \\ & m + y^k \geq 0 \end{aligned} \tag{29}$$

The first constraint just states that buyers are bounded on the amount they can borrow by the amount of collateral they hold. The second constraint states that sellers cannot lend more liquidity in the second market than their money holdings. The Lagrange multiplier on the first constraint is $\beta\phi_{+1}\lambda_{m\ell}$ and on the latter $\beta\phi_{+1}\lambda_{md}$. If an agent expects to be a seller in the third market, he will not borrow in the money market and thus $\lambda_{m\ell}^H = 0$. And consequently, an agent, who expects to become a buyer will not lend money in the second market and therefore $\lambda_{md}^L = 0$. Therefore the first-order condition simplifies as follows.

$$V_m^H + V_y^H + \beta\phi_{+1}\lambda_{md}^H = 0 \tag{30}$$

$$V_m^L + V_y^L + \beta\phi_{+1}\lambda_{m\ell}^L = 0 \tag{31}$$

As agents receive a signal, they will adjust their money holdings according to their expectations. However, as Berentsen and Monnet (2008) state, since the signal is small, agents are not willing to lend all their money holdings or pledge all their collateral. This means that in the first order conditions (30) and (31) the Lagrange coefficients will be zero, since the lending and the borrowing constraint are not binding.

$$Z_b = \sum_{k=H,L} \sigma^k \left[V_b^k + \sigma^L \beta R \frac{V_m^L + V_b^L}{\beta \phi_{+1}(1+i_m)} \right] \tag{32}$$

The marginal value of bringing one additional unit of bonds in the second market depends naturally on the money market rate and on the marginal value of bringing one additional unit of money and the marginal value of bringing one additional unit of bonds into the third market for the buyer. V_m^L is decreasing with a decreasing deposit rate and so is the money market rate, if all other variables are kept constant. The effect of bringing one additional unit of bonds into the second market with a decreasing interest rates is thus ambiguous.

The marginal value of bringing an additional unit of money into the second market is just the expected value of financing consumption in the third market minus the expected value of lending one additional unit in the money market at the rate i_m.

$$Z_m = \sum_{k=H,L} \sigma^L V_m^L - \sigma^H V_y^H \tag{33}$$

Since the money market rate is decreasing with a decreasing deposit rate, the value of bringing one additional unit of money into the second market is decreasing as well. This is straightforward, since the value of lending this unit in the money market is decreasing with a decreasing money market rate. Moreover, if both the lending and the deposit rate are decreased by the same amount, the result will still be a decrease in the Z_m since the money market rate decreases accordingly [4].

In the third market, agents learn their type and adjust their money holdings accord-

[4]From Berentsen and Monnet (2008) the steps to reach equations (32) and (33) are restated in the appendix

ingly.

$$
\begin{aligned}
V^k(m, y^k, b) = (1 - n^k) & \left(u'(q^k) + \beta W(m - pq^k + \ell^k, b, \ell^k, 0, y\right) \\
& + n^k \left(-q_s^k + \beta W(m + pq_s^k - d^k, b, 0, d^k, y\right) \\
& \text{s.t. } \ell^k \leq \bar{\ell}^l = \frac{Rb}{\phi_{1+i_m}} - \frac{y^k}{\hat{\Delta}} \\
& pq^k \leq m + \ell \\
& d^k \leq m + \ell
\end{aligned} \tag{34}
$$

Here, $\hat{\Delta} = \frac{1+i_\ell}{1+i_m}$. Based on these equations, Berentsen and Monnet (2008) derive the equations that define the equilibrium $\{\gamma, \hat{\Delta}, q^L, q^H, z^L, z^H, z_m, b\}$ and the policy (i_ℓ, i_d). The definition of the equilibrium is replicated here to make the discussion in section (4) more tractable.

$$R\gamma = 1 + i_m \tag{35}$$

$$\frac{\beta Rb}{\Delta} = \sigma^H q^H + \sigma^L q^L - z_m \tag{36}$$

$$z^H = -\frac{\sigma^L}{\sigma^H} z^L \tag{37}$$

$$z_m = \left(\frac{\hat{\Delta}}{\hat{\Delta} - 1}\right) \frac{\{(\hat{\Delta} - 1)[\sigma^L(1 - n^L)q^L + \sigma^H(1 - n^H)q^H] - \epsilon\sigma^L\sigma^H(q^L - q^H)\hat{\Delta}\}R(\Delta - 1)}{R\hat{\Delta} - \Delta + (1 - n)R\hat{\Delta}(\Delta - 1)} \tag{38}$$

$$\hat{\Delta} = \frac{\Delta}{n\beta R(1 - \Delta) + \Delta} \tag{39}$$

$$u'(q^k) = \frac{n^k}{(1 - n^k)} \Delta \frac{1 - n\beta R}{n\beta R}, k = H, L \tag{40}$$

where $b \geq 0$, $z^H > -z^m$. $z^L < \beta Rb\frac{\hat{\Delta}}{\Delta}$. These conditions just mean that the buyer and the seller are not willing to pledge all their collateral or lend all their money holdings in the money market. In the case of a negative deposit rate, these conditions are very restrictive. Berentsen and Monnet (2008) show, that the following two inequalities have to hold in equilibrium. Rewriting both inequalities:

$$\frac{q^L}{q^H} < \frac{\Phi + \frac{\sigma^H}{\sigma^L}(\hat{\Delta} - 1)(1-n) + \sigma^H \epsilon}{\Phi - (\hat{\Delta} - 1)(1-n) + \sigma^H \epsilon}$$

$$\frac{\Phi}{\hat{\Delta}\left(\sigma^H \frac{q^H}{q^L} + \sigma^L\right)} < \Phi - (\hat{\Delta} - 1)(1-n) + \frac{\sigma^L \sigma^H \epsilon \left(1 - \frac{q^H}{q^L}\right)}{\sigma^H \frac{q^H}{q^L} + \sigma^L}$$

$\hat{\Delta}$ increases strongly, with very low negative deposit rates. Therefore, the value of the right-hand side of the second equation becomes small for very low values of i_d. The signal has to contain very little information, for this inequality to hold, since both sides are increasing in ϵ.

Moreover, any implemented policy with a negative deposit rate can never achieve the first-best allocation. The positive spread drives a wedge between the marginal utility of consuming q and the marginal cost of producing q. If the corridor is kept constant, but the policy rates are decreased, consumption decreases as well, as indicated by equation (40). This result holds for positive and negative deposit rates. As agents are essentially taxed on their money holdings, the incentive to hold fiat money decreases. This happens because the marginal value of money is decreasing. Moreover, marginal utility is concave in the deposit rate. That means that for very low values of i_d consumption will decrease more rapidly than for moderate levels of i_d. Nevertheless, such low deposit rates are highly unlikely to be implemented in reality. As the discussion section shows, the effect of decreasing deposit rates appear to be almost linear for moderate decreases in deposit rate.

Since the standing facilities provide a corridor for the interest rate on the market, this rate depends on the policy (i_ℓ, i_d) implemented by the central bank. Berentsen and Monnet (2008) derive the money market rate i_m as follows.

$$i_m = i_\ell - n\beta R(i_\ell - i_d) \tag{41}$$

Furthermore, the authors rewrite the inflation $\pi = \gamma - 1$ as follows.

$$\pi = \frac{1 + i_m}{R} - 1 \tag{42}$$

In Berentsen and Monnet (2008), the money market rate is exactly in the middle of the corridor, if $\beta R n = 0.5$. This result remains valid in the case of negative interest rates, but depending on the level of the corridor, this also implies, that i_m may well be negative.

Central banks usually set a target or a target range for one or more money market rates. In the case of Switzerland for example, this target is negative at the moment. For the rest of this section, the different cases for the value of i_m are discussed. The level of the money market rate depends on the return of bonds, the discount factor as well as the amount of liquidity and the implemented policy. The following three cases discuss the effects of the corridor and the deposit rate on the money market rate assuming the money market rate is targeted above, below or at zero.

Case 1: $i_m = 0$
If we set $i_m = 0$, we can derive the critical value of the corridor and the deposit rate for which the money market rate is zero from equation (41).

$$\delta = \frac{i_\ell}{n\beta R} \tag{43}$$

Or equivalently:

$$i_d = \left(1 - \frac{1}{n\beta R}\right) i_\ell \tag{44}$$

Note, the first term of the right-hand-side of the equation is constant and negative for any value of $n\beta R \leq 1$. If the central bank targets a money market rate of zero, the deposit rate has to be negative. Only if $n\beta R = 1$, the lending rate and the deposit rate can be set to zero. If $1 - \frac{1}{n\beta R}$ is close to -1, the deposit rate has to be the negative value of the lending rate. Consequently, the lower the constant is, the lower the deposit rate has to be in order to keep the money market rate at zero. Moreover, the lower the amount of liquidity n is, the closer the deposit rate has to be to 0.

Since $i_m = 0$, equation (42) simplifies to the following equation, which indicates that with a zero money market rate, the return on bonds describes the money growth rate.

$$\pi_1 = \frac{1}{R} - 1$$

Notably, since $R > 1$, π is always negative, which implies a money stock growth rate of less than one, and therefore, a decreasing money stock. If $i_m = 0$, there is no cost of borrowing in the money market for the buyers. However, if the signal is small enough, they would still be reluctant to pledge all their collateral in the money market, because they have a high uncertainty about their type. Sellers are willing to lend money, since they are faced with a negative deposit rate in the third market. However, the signal ϵ is small in the discussed case. This means, that even though sellers want to lend money, they are reluctant to lend all of their money holdings. The same holds for the case with $i_m < 0$.

Case 2: $i_m > 0$
For the money market rate to be positive the following condition has to hold:

$$i_d > \left(1 - \frac{1}{n\beta R}\right) i_\ell$$

Again, the constant is negative. This means that if the central bank lowers the lending rate, the deposit rate has to be high enough for the money market rate to be above zero. Even with a negative deposit rate the money market rate is above zero, if the policy rates are high enough. If amount of liquidity increases, the deposit rate has to increase by a larger amount than the right-hand-side of the equation to keep the money market rate above zero. In such a case, this would lead to a decrease in the spread of the policy rates, which is welfare enhancing (Berentsen and Monnet (2008)) . Inflation is determined by the following equation:

$$\pi_2 = \frac{1 + i_m}{R} - 1 \tag{45}$$

With a positive money market rate, inflation can be zero, below or above zero, depending on the money market rate and the rate of return on bonds.

Case 3: $i_m < 0$
The money market rate is negative if the following inequality holds:

$$i_d < \left(1 - \frac{1}{n\beta R}\right) i_\ell$$

If the lending rate is increased, the deposit rate has to be decreased by more than a one-to-one change, which increases the difference between the two policy rates. Moreover, with a high amount of liquidity, the deposit rate can have higher values than with lower liquidity. This is straightforward, an increasing supply of liquidity decreases the money market rate. The money market rate can thus be kept below zero without a decrease in the deposit rate. Inflation is determined as equation (45) states. Since the money market rate is negative the first term of the right-hand side of equation (45) will always be smaller than one, indicating again a negative inflation rate or deflation. Rewriting equation (45), an increase in the spread leads to a decreasing inflation rate (Berentsen and Monnet (2008)).

$$\pi_3 = \frac{1 + i_\ell}{R} - n\beta\delta - 1 \tag{46}$$

If the money market rate is below zero, that means there is a negative cost associated with borrowing and a negative return associated with lending. Just as the negative deposit rate in the third market, the negative money market rate punishes agents, that hold liquidity. While the money market rate stays within the corridor, the sellers still will prefer to lend their money in the money market rather than deposit it with the central bank, but they can no longer avoid the negative interest rates on their money holdings. Clearly, this affects consumption negatively.

Note, even though we are currently observing money markets rates equal to case 1 and 3, these conditions do not pose a steady-state. Only if the amount of general goods produced in the first market can be limited somehow, or the signal is close to zero, borrowing will not be infinite. Agents could be constraint in their bond holdings or not willing to pledge all their collateral. Without these additional assumption, agents in case 1 and 3 would borrow as much as they can with their bonds. The equilibrium would be a situation, where agents borrow infinite amounts of fiat money. As long as the money market rate is above the deposit rate, sellers would still be willing to lend money. Nevertheless, with a money market rate of zero or lower, they still face will have a negative effect on consumption.

Comparison of Cases 1-3 From (45) it is straightforward, that inflation is higher in the second case than in the two others. As indicated by (46), a higher i_m is associated with a higher inflation rate. Therefore, the following equation has to hold.

$$\pi_2 > \pi_1 > \pi_3$$

Moreover, if the amount of sellers becomes large, relatively more money holdings compared to the total money stock will be charged with a negative deposit rate. Therefore, the total amount of interest payments from sellers to the central bank increases. Thus, the money stock should increase by less with a high amount of sellers.

While in cases 1 and 3 the economy experiences a deflation, in case 2 the economy can either have an increasing, constant or decreasing money stock. Nevertheless, the growth rate will always be larger in case 2 than in any other case.

In a next step, we consider the effects of the three cases on consumption by analyzing equation (40). Clearly, as Δ increases, consumption will decrease and therefore we only need to consider the value of Δ for the different cases. Δ increases with decreasing policy rates, which causes the marginal utility of consumption to increase, which implies decreasing consumption. Therefore the following equation has to hold:

$$q_2 > q_1 > q_3$$

This clearly shows the adverse effects of a negative deposit rate. Not only, is the first-best allocation no longer feasible, but if the money market rate is targeted at zero or even negative, consumption will be reduced even further. The money market rate does not affect the consumption directly, but the central bank adopts a policy to reach a money market rate target and that policy will affect consumption. Therefore, the target for the money market rate affects consumption indirectly.

4 Discussion

In the following section, the effects of a negative deposit rate are discussed more closely. I computed a few scenarios the framework of Berentsen and Monnet (2008) to show the effects of negative interest rates on consumption. For the choice of parameters, I followed closely the approach taken in Berentsen et al. (2015b). The deposit and lending rate are set equal to the average deposit and lending facility rate during the years 2014 and 2015. To simplify the interpretation, the transfers τ are set to zero. Berentsen et al. (2015b) calculate the time preference β as follows. Since $\beta = \frac{1}{1+r}$, they calculate the real interest rate as the difference of a one year Swiss government bond yield and the inflation rate. Here, the monthly average of government bond yields over the Euro Area, which are in total 647 observations from 2014 and 2015 of 27 countries of the Euro zone, were taken to calculate the average yearly yield on government bonds. Then, the average of the average yearly yields in 2014 and 2015 were calculated. The inflation rate is taken from the average monthly HICP rates from 2014 and 2015 from the Euro zone and then again the average over the two years was taken. The average of the one year government bonds less the average inflation rate results in the real interest rate r. Since in equilibrium $\beta R < 1$ has to hold, R was set arbitrarily slightly less than r to match this condition. Most research uses data from several years to calculate the parameters used in the simulation (see Berentsen et al. (2015a); Craig and Rocheteau (2008); Lagos and Rocheteau (2005)). Since negative interest rates have only been implemented in the Euro zone since 2014, there are only two years of observations. For some points during the discussion, I assume a fixed lending rate at the average over the past two years in the Euro Zone. Considering a longer time span would increase the average lending rate. This would not provide any additional insights besides the effects of a larger spread between the lending rate and a negative deposit rate. The scenarios and paramters were computed using Matlab and R, the data was taken from European Central Bank (2009-2016, 2014-2016a) and European Central Bank (2014-2016b).

The utility function is $u(q) = log(q)$, as in Berentsen et al. (2015b). Berentsen et al. (2015b) also set a log-normal distribution for the liquidity preference shock and draw random numbers from that distribution for their simulation. With negative deposit rates, the signal ϵ has to be very close to zero, if the equilibrium conditions are still to hold. Namely, if the signal is too large, the lending and borrowing constraint will start to bind and the equation for marginal utility in equilibrium will no longer hold. Therefore, the parameter of the signal will be chosen to fit the equilibrium criteria even for low values of the deposit rate. n is set equal to 0.5 and σ^k is chosen accordingly. All further variables are computed according to the definitions in equilibrium.

No trade in the money market As discussed above, the bounds on the consumption level decrease with a decreasing deposit rate. The graph shows the upper and lower bound of feasible consumption as a function of the deposit rate. The lending rate is computed as a function of the deposit rate and the corridor is set according to the data. The graph is depicted at the interval $[-0.5, 0.5]$ to show the relationship of the bounds on consumption with the deposit rate. However the lending rate is negative for all values of $i_d < 0.0056$. Although the lower bound does not react to

Parameter Values			
Parameter	Target Description	Parameter Values	Target Values
β	Average real interest rate	0.97930	0.02114
i_ℓ	Average marginal lending rate	0.00410	0.00410
i_d	Average deposit rate	-0.00147	-0.00147
n	Amount of sellers	0.5	0.5
n^H	signal type L	0.5000055	0.5000055
n^L	signal type H	0.499995	0.499995
σ^H	Prob. signal type H	0.50	0.50
σ^L	Prob. signal type L	0.50	0.50
R	$R <$ Real Interest Rate	1.0200	1.0200
δ	Difference in rates	0.0056	0.0056

Table 1: Parameter Values

the decrease in the policy rates, since $\pi = 0$, the upper bound decreases. Moreover, as visible in the plot, the upper bound is concave in the deposit rate and thus decreases more strongly as the deposit rate decreases into the negative range. This clearly shows that with the implementation of a negative deposit rate, the central bank lowers the upper bound of consumption. It is important to mention, that the upper bound of consumption only decreases little within the range of current observable levels of negative interest rates.

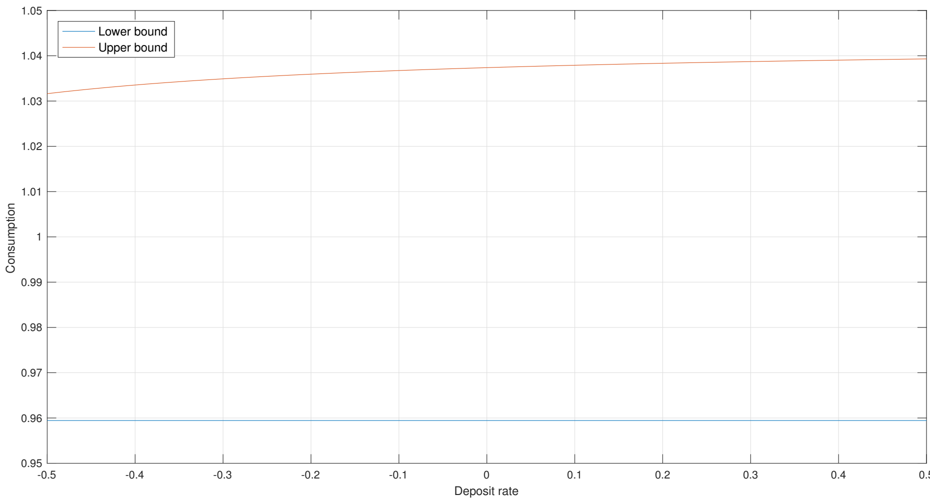

Figure 7: Upper and lower bound of consumption

Figure (8) shows two graphs. The first one depicts the change in consumption

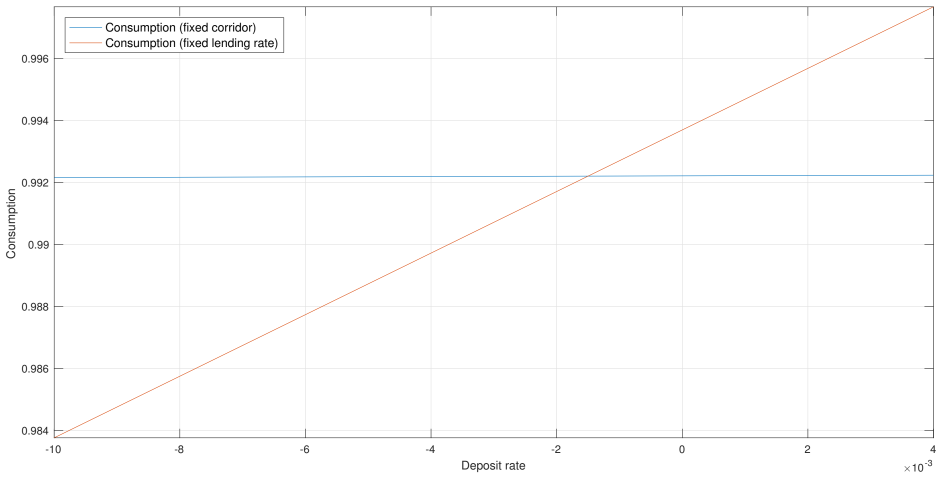

Figure 8: Consumption

with a decreasing deposit rate, when the corridor is kept constant. Therefore also the lending rate decreases accordingly. The second graph shows the effect on consumption if the lending rate is kept constant. If the central bank keeps the spread constant, the decrease in consumption is far less severe, than with an increasing spread. As Berentsen and Monnet (2008) point out, welfare decreases if the central bank increases the corridor and this holds also with negative deposit rates.

Trade in the money market As indicated by the definition of the equilibrium above, consumption is decreasing in the deposit rate. This result holds regardless of whether the deposit rate is positive or negative. However, the function of consumption with respect to the deposit rate is slightly concave and not linear, which is barely observable in the plot due to the very close interval. With severely low deposit rates, consumption will decrease more strongly than with deposit rates around zero. As the graph indicates the effects are very small for deposit rates that range from -10 to 4 basis points. Furthermore, $q^H < q^L$ holds. Agents, that receive a signal that they will become a seller with a high probability will consume less than agents that expect to become a buyer. Since the dynamics are similar for q^L, only q^H is depicted here.

This graph highlights another important implication. The central bank is inevitably forced to increase the spread, if it chooses to decrease the deposit rate. As Berentsen and Monnet (2008) point out, whether a monetary policy is expansionary or restric-

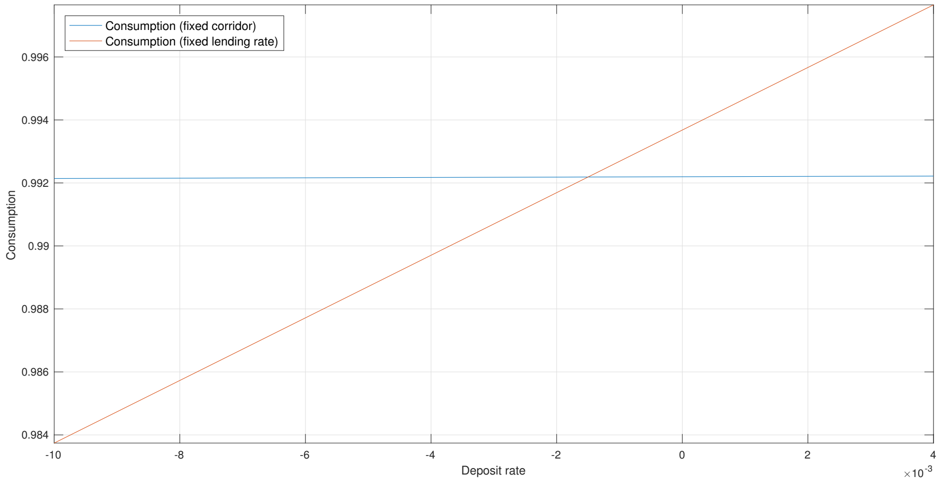

Figure 9: Consumption (q^H) with a fixed corridor and a fixed lending rate

tive does not depend on the level of the money market rate, but rather on the spread between the lending rate and the deposit rate. With decreasing negative deposit rate, the spread will have to increase at some point and therefore monetary policy will become more restrictive, even though the deposit rate is decreasing. Setting negative deposit rates can therefore not necessarily be considered as an expansionary monetary policy.

Starting from a given policy (i_ℓ, i_d) implemented by the central bank, the loss in consumption will be larger, if the central bank simply decreases the deposit rate and thus increases the corridor, than if the central bank would lower both the lending rate and the deposit rate to keep the corridor constant. For example, the ECB has increased the corridor by decreasing the deposit rate more than the lending rate. The graph indicates the welfare loss of an increasing corridor relative to the welfare loss of a constant corridor with decreasing rates. The loss in welfare due to an increase in the corridor is a result of Berentsen and Monnet (2008) and also applies here.

Furthermore, the central bank can only achieve the first-best allocation when $\Delta = 1$. This is the optimal policy, if the borrowing constraint of the buyer is not binding. Since this cannot be the case, trades are not efficient (Berentsen and Monnet (2008)). Ignoring the borrowing constraint for the moment, the inefficiency does not only come from the buyer, who faces a cost of holding money and thus does not carry enough money, but also from the seller who faces a negative yield on his money holdings because of the negative deposit rate. This directly affects consumption, because the seller faces a lower yield on his earnings with a negative deposit rate and the buyer faces a lower marginal value of holding money. This negative effect is especially severe since the seller cannot avoid this negative deposit rate. Furthermore, the wedge between marginal utility and marginal cost of producing is increased, if buyers are constrained by the amount of bonds they hold. In this case buyers cannot purchase as many consumption goods as preferred because they cannot borrow more at the standing facility.

Costs of negative deposit rates The welfare cost of inflation is calculated as the fraction of consumption that agents are willing to give up to move from a inflation of 10% to 0% (Lagos et al. (2015)). This method has also been applied in Berentsen et al. (2015a); Craig and Rocheteau (2008); Lagos and Wright (2005). Since the cost of negative interest rates is similar to the cost of inflation in the sense that both are a form of taxation on money holdings, it stands to reason that the cost of negative deposit rates can be calculated according to the same method.

There are however several issues arising when computing the cost of negative interest rates. First, lending rates cannot be to high, because in equilibrium agents would not borrow at the standing facility if the lending rate is too high. Secondly, there are additional welfare effects with a change in the spread. Thirdly, buyers prefer a scenario with low lending rates to a scenario with high lending rates, which also influences the cost of decreasing interest rates (Berentsen and Monnet (2008)). Moreover, with negative interest rates, the money market rate could be negative as well. In the model of Berentsen and Monnet (2008), the money market rate cannot be zero or lower in equilibrium, since agents would borrow infinite amounts in the second market. Based on these factors, the calculations are based on the following three scenarios. The baseline will be a policy $(i_\ell, i_d) = (0.020, 0)$ and there are two scenarios, where the interest rates are decreased to $(0.015, -0.005)$ and to $(0.015, -0.01)$. I follow the approach taken in Lagos and Wright (2005). Namely, the consumption q in the first case is multiplied by a factor ζ. To calculate ζ, the welfare of the baseline is then set equal to the welfare in the second case (Lagos and Wright (2005)). The cost $1-\zeta$ is thus the amount of consumption, agents would be willing to give up to be in the baseline scenario. The subscript 1 refers to the case with no trade, the subscript 2 refers to the case with trade. The variable ζ is the amount the consumption in the baseline scenario has to be multiplied to reach the same welfare as in the other two cases.

$$
\begin{aligned}
(1-\beta)W_{\zeta}(\Delta_{baseline}) &= (1-\beta)W(\Delta_{15,5}) \\
(1-n)(u(q(\Delta_{baseline})\cdot\zeta) - q_{baseline}) + (\beta R)b_{baseline}\cdot\zeta - b_{baseline} & \\
= (1-n)(u(q_{15,5}) - q_{15,5}) + (\beta R - 1)b_{15,5} &
\end{aligned}
$$

Parameter Values			
	Baseline	15,5	15,10
i_ℓ	0.020	0.015	0.015
i_d	0.0	-0.005	-0.015
q_1	0.9782	0.9781	0.9732
q^H	0.9782	0.9781	0.9732
$1-\zeta_1$		$-1.733\cdot10^{-6}$	$-8.402\cdot10^{-5}$
$1-\zeta_2$		$3.669\cdot10^{-4}$	$3.817\cdot10^{-5}$
$1-\zeta_1, b$		$2.152\cdot10^{-6}$	$1.248\cdot10^{-4}$

Table 2: Cost of negative deposit rates

Since the signal can only contain very little information, the value of n^k is very close to n. This means that consumption will be very similar, even if agents trade in the money market and q^H is almost identical to q^L.

Although consumption is the same whether agents can trade or not, the costs differ and most notably, the cost are negative in the case without trade in the money market. This would imply that agents would have to receive a compensation to move from the case $(0.015, -0.005)$ to the baseline scenario. Though welfare and consumption are both higher in the baseline scenario, this value seems to be counter-intuitive. This result stems from the bond holdings in both cases. The utility in the first market is just the discounted return on bonds minus the newly produced collateral. Since $\beta R < 1$ in all scenarios, the utility in the first market is negative and since in the collateral in the baseline scenario is larger than in the other two, the utility in the first market in the baseline scenario is lower than in the other two scenarios. If $\beta R = 1$, agents would be willing to give up a positive part of consumption to move from the scenarios with negative deposit rates to the baseline scenario. Hence, these results are negative, because there is some costs associated with holding bonds. As indicated in section (3), the liquidity premium of holdings bonds is decreasing in Δ. Since Δ in the baseline scenario is smaller than in the other two cases, the liquidity premium on bonds is higher. That means agents can use the bonds to purchase a higher amount of goods in the scenarios with negative deposit rates. This is the reason that agents would have to be compensated to move from a scenario with negative deposit rates to the baseline scenario. The costs denoted with subscript b shows the costs if $\beta R = 1$.

Considering now only the positive costs of negative interest rates, the cost moving from the first scenario $(0.015, -0.005)$ to the baseline scenario is higher in the case without trade while in the second case moving from $(0.015, -0.01)$ to the baseline is higher. When agents cannot trade the costs associated with negative deposit rates are higher if the they move from a negative deposit rate to a positive while keeping the corridor constant. An additional decrease in the deposit rate is also costly, but the costs are less high. In the case with trade, the costs associated with an increasing corridor are higher, which is exactly what we would expect from the theory. Even though these costs are very close to zero, the change in the policy of the three scenarios only affects a few basis points. Therefore it is not surprising that agents are willing to give up only a small part of their consumption.

Similarly, to the results of Ennis (2009), agents will try to avoid the taxation of money holdings. However, in contrast to his framework, negative interest rates act a tax for sellers and not buyers. Sellers experience a taxation on their money holdings, which consists of the money, they brought into the goods market as well as the earnings in the goods market. As they are faced with a higher cost of bringing money to the next period, sellers will try to avoid this cost, by reducing the money holdings at the end of the period. This leads to a decrease in the good q and thus, to a decrease in consumption. This result is also similar to the cost of holding money in the Lagos and Wright (2005) framework. As (Nosal and Rocheteau, 2011, pp.131/132) explain, the cost of holding money comes from inflation and the time preference. This cost drives a wedge between the marginal utility of consumption and marginal cost of production and agents can no longer reach the first-best allocation ((Nosal and Rocheteau, 2011, pp.131/132) from Lagos and Wright (2005)). In an environment with positive inflation, the buyer will try to avoid the taxation on his money holdings by spending it (Ennis (2009)). However, with negative deposit rates, the sellers are taxed, and they cannot avoid the cost of holding money by spending it. The only way to avoid the taxation is to hold less money.

It is reasonable to assume that the amount traded in the money market is also affected. The variable z^k describes the fraction of consumption that is financed with borrowing in the second market. For values of the deposit rate that are currently observable in reality, z^k does not change. However, these functions are not linear if the corridor changes. As long as the corridor is constant, z^k remains a constant fraction of consumption. However, with a fixed lending rate, this is no longer true. With a decreasing deposit rate combined with a fixed lending rate, the buyers borrow more in the money market. Consequently, those who expect to become a seller lend more of their money holdings in the second market with a decreasing deposit rate. Therefore, with a decrease in the deposit rate combined with less than one-to-one decrease in the lending rate, we would expect more turnover in the money market. Note, this result is again not specific to negative deposit rate, but applies in general to decreasing interest rates. Since the central bank directly affects the money market rate with its policy rates, it can steer the behavior in the money market in this framework. There is a vast literature that discusses the beneficial effects of financial systems. If banks are unwilling to lend to other financial institutions the financial network becomes very fragile. With an increase in liquidity in the interbank market the financial network can be stabilized and is less prone to liquidity shocks (see Acemoglu et al. (2015); Allen and Gale (2000); Nier et al. (2007)). Since such effects are not included in this framework, it is not possible to weigh the negative effects on consumption with the possibly beneficial effects of an increased trade in the money market.

5 Conclusion

Negative interest rates have only been recently implemented as monetary policy tool. Currently, there are five central banks that have lowered their interest rates into the negative range, albeit with largely different ways of implementation. The effects of such policy measures remain uncertain, as there is only little literature and empirical evidence of this policy tool. The analysis of the framework of Berentsen and Monnet (2008) shows that negative deposit rates cause a decline in welfare. As the sellers are taxed on their money holdings consumption decreases. While the loss in consumption is far less, if the central bank decreases the deposit and the lending rate simultaneously and by the same amount, an increase in the spread will cause much larger losses in consumption. Since the lending rate is bound from below, the central bank will have to increase the corridor at some point, if they want to lower interest rates further. The large decrease in consumption seems to stem mostly from an increase in the spread rather than from a decline in the deposit rate. Since the corridor of the monetary policy determines how strict or loose the monetary policy is, an increase in the spread is considered as restrictive monetary policy. This means that lowering the deposit rate is not necessarily an expansionary monetary policy. Even though a decrease in the spread of the policy rates causes negative welfare effects, it might improve the amount of borrowing and lending in the money market.

If the central bank does not only target consumption maximization but, also factors like financial stability, inflation expectations or exchange rate effects the total effect of negative interest rate remains uncertain.

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List of Figures

Appendix

Maximization problem of the buyer in the settlement market

$$
\begin{aligned}
&\max_{q,\ell} u(q) + \beta W(m - pq + \ell, b, \ell, 0)\\
&m + \ell \geq pq \qquad (47)\\
&\ell \leq \bar{\ell} = \frac{Rb}{\phi_{+1}(1 + i_\ell)}
\end{aligned}
$$

(Berentsen and Monnet (2008))

Hours worked in the first market with trade in the money market
The hours worked by a buyer and seller are as follows:

$$h_b = \phi_{+1}(m_{+1} + (1 + i_\ell)\ell + (1 + i_m)y^k) - (R - 1)b - \tau M \qquad (48)$$

$$h_s = \phi_{+1}(m_{+1} - (1 + i_d)d + (1 + i_m)y^k) - (R - 1)b - \tau M \qquad (49)$$

Market clearing in the money market requires $(\sigma^H(1-n^H)+\sigma^L(1-n^L))(1+i_m)y^k = (\sigma^H n^H + \sigma^L n^L)(1 + i_m)y^k$. The total amount of hours worked are $h = nh_s + (1 - n)h_b$. Furthermore, since there is no aggregate uncertainty, the following condition has to hold as well. $n = \sum_{k=H,L} \sigma^k n^k$ This means that $n = \sigma^H n^H + \sigma^L n^L$ and $\sigma^H(1 - n^H) + \sigma^L(1 - n^L)$. Using equation (25), hours worked can be written as follows. The hours worked do not change with trade in the second market.

$$
\begin{aligned}
h &= \sigma^H n^H + \sigma^L n^L(\phi_{+1}(m_{+1} + (1 + i_d)d + (1 + i_m)y^k) - (R - 1)b - \tau M)\\
&+ \sigma^H(1 - n^H) + \sigma^L(1 - n^L)(\phi_{+1}(m_{+1} - (1 + i_\ell)\ell + (1 + i_m)y^k) - (R - 1)b - \tau M)\\
&= -(R - 1)b + \phi_{+1}(m_{+1} - (\sigma^H n^H + \sigma^L n^L)(1 + i_d)d + (\sigma^H n^H + \sigma^L n^L)(1 + i_m)y^k\\
&+ (\sigma^H(1 - n^H) + \sigma^L)(1 - n^L)(1 + i_m)y^k - (\sigma^H(1 - n^H) + \sigma^L(1 - n^L))(1 + i_\ell)\ell - \tau M\\
&= -(R - 1)b
\end{aligned}
$$

Thus the welfare function does not change with trade in the money market (Berentsen and Monnet (2008)).

Marginal value of holding an additional unit of bonds or money in the money market

If we take the first derivation of the maximization problem in the second market (29) with regards to bonds, b.

$$Z_b = \sum_{k=H,L} \sigma^k \left[V_b^k + \sigma^k \frac{\lambda_{m\ell}^k}{1+i_m} \right]$$

From (31), the equation simplifies to (32).

Similarly, the first derivation of (29) with regards to money holdings m is as follows.

$$Z_m = \sum_{k=H,L} \sigma^k (V_m^k - +\sigma^k \beta \phi_{+1} \lambda_{md})$$

Again, using equation (30), the marginal utility of money can be written as (25)(Berentsen and Monnet (2008)).

R Code

```
install.packages("gdata")
library("gdata")
install.packages("Hmisc")
library("Hmisc")
install.packages("zoo")
library("zoo")

#Sweden
setwd("/Users/rominaruprecht/Desktop")
sw <- read.csv("interest_rates_sw.csv", sep=";")
View(sw)
sw <- sw[-c(1:3, 1588:1633),]
date <- strptime(as.character(sw$Interest.rates.and.exchange.rates),
"%d/%m/%Y")

lending <- as.numeric(as.character(sw$X.1))
deposit <- as.numeric(as.character(sw$X))
repo <- as.numeric(as.character(sw$X.2))
stibor <- as.numeric(as.character(sw$X.7))

par(mfrow=c(2,1))
plot(date, lending, col="green", type="l", ylab="Rates",
xlab="", ylim=c(-1.50, 3.5))
par(new=TRUE)
plot(date, deposit, col="blue", type="l", ylab="",
xlab="", ylim=c(-1.50, 3.5), axes=FALSE)
par(new=TRUE)
plot(date, repo, col="red", type="l", ylab="",
xlab="", ylim=c(-1.50, 3.5), axes=FALSE)
par(new=TRUE)
plot(date, stibor, col="purple", type="l", ylab="",
xlab="", ylim=c(-1.50, 3.5), axes=FALSE)
```

```
par(new=TRUE)
grid(nx=NA, ny=NULL, col = "lightgray", lty ="dotted")
minor.tick(nx=0, ny=4, tick.ratio=0.5)
par(new=TRUE)
abline(h=0.25, col = "lightgray", lty ="dotted")
abline(h=0.5, col = "lightgray", lty ="dotted")
abline(h=0.75, col = "lightgray", lty ="dotted")
abline(h=1.25, col = "lightgray", lty ="dotted")
abline(h=1.5, col = "lightgray", lty ="dotted")
abline(h=1.75, col = "lightgray", lty ="dotted")
abline(h=2.25, col = "lightgray", lty ="dotted")
abline(h=2.5, col = "lightgray", lty ="dotted")
abline(h=2.75, col = "lightgray", lty ="dotted")
abline(h=3.25, col = "lightgray", lty ="dotted")
abline(h=3.5, col = "lightgray", lty ="dotted")
abline(h=-0.25, col = "lightgray", lty ="dotted")
abline(h=-0.5, col = "lightgray", lty ="dotted")
abline(h=-0.75, col = "lightgray", lty ="dotted")
abline(h=-1.25, col = "lightgray", lty ="dotted")
abline(h=-1.5, col = "lightgray", lty ="dotted")
abline(v=as.Date("2010-01-04"), col = "lightgray", lty ="dotted")
abline(v=as.Date("2011-01-03"), col = "lightgray", lty ="dotted")
abline(v=as.Date("2012-01-02"), col = "lightgray", lty ="dotted")
abline(v=as.Date("2013-01-02"), col = "lightgray", lty ="dotted")
abline(v=as.Date("2014-01-02"), col = "lightgray", lty ="dotted")
abline(v=as.Date("2015-01-02"),col = "lightgray", lty ="dotted")
abline(v=as.Date("2016-01-02"), col = "lightgray", lty ="dotted")
par(new=TRUE)
legend("topright", legend = c("3M-Stibor", "Repo Rate",
"Lending Rate", "Deposit Rate"), col = 1:4, lty = 1, cex=0.5)

#ECB
setwd("/Users/rominaruprecht/Desktop")
eonia <- read.csv("eonia.csv", sep=";")
View(eonia)
```

```
date <- as.Date(eonia$Datum, "%d.%m.%y")
lending <- as.numeric(as.character(eonia$Lending.Rate))
deposit <- as.numeric(as.character(eonia$Deposit.Rate))
mro <- as.numeric(as.character(eonia$MRO))
eonia <- as.numeric(as.character(eonia$Eonia))

par(mfrow=c(2,1))
plot(date, lending, col="green", type="l", ylab="Rates",
xlab="", ylim=c(-1.50, 3))
par(new=TRUE)
plot(date, deposit, col="blue", type="l", ylab="",
xlab="", ylim=c(-1.50, 3), axes=FALSE)
par(new=TRUE)
plot(date, mro, col="red", type="l", ylab="",
xlab="", ylim=c(-1.50, 3), axes=FALSE)
par(new=TRUE)
plot(date, eonia, col="purple", type="l", ylab="",
xlab="", ylim=c(-1.50, 3), axes=FALSE)
par(new=TRUE)
grid(nx=NA, ny=NULL, col = "lightgray", lty ="dotted")
minor.tick(nx=0, ny=4, tick.ratio=0.5)
par(new=TRUE)
abline(h=0.25, col = "lightgray", lty ="dotted")
abline(h=0.5, col = "lightgray", lty ="dotted")
abline(h=0.75, col = "lightgray", lty ="dotted")
abline(h=1.25, col = "lightgray", lty ="dotted")
abline(h=1.5, col = "lightgray", lty ="dotted")
abline(h=1.75, col = "lightgray", lty ="dotted")
abline(h=2.25, col = "lightgray", lty ="dotted")
abline(h=2.5, col = "lightgray", lty ="dotted")
abline(h=2.75, col = "lightgray", lty ="dotted")
abline(h=-0.25, col = "lightgray", lty ="dotted")
abline(h=-0.5, col = "lightgray", lty ="dotted")
abline(h=-0.75, col = "lightgray", lty ="dotted")
```

```
abline(h=-1.25, col = "lightgray", lty ="dotted")
abline(h=-1.5, col = "lightgray", lty ="dotted")
abline(v=as.Date("2010-01-04"), col = "lightgray", lty ="dotted")
abline(v=as.Date("2011-01-03"), col = "lightgray", lty ="dotted")
abline(v=as.Date("2012-01-02"), col = "lightgray", lty ="dotted")
abline(v=as.Date("2013-01-02"), col = "lightgray", lty ="dotted")
abline(v=as.Date("2014-01-02"), col = "lightgray", lty ="dotted")
abline(v=as.Date("2015-01-02"),col = "lightgray", lty ="dotted")
abline(v=as.Date("2016-01-02"), col = "lightgray", lty ="dotted")
par(new=TRUE)
legend("topright", text.font = 2, legend = c("EONIA", "MRO",
"Lending Rate", "Deposit Rate"), col = 1:4, lty = 1, cex=0.5)

#SNB
setwd("/Users/rominaruprecht/Desktop")
snb.bilanz <- read.csv("snb_bilanz.csv", sep=";")
libor <- read.csv("libor.csv", sep=";")
saron <- read.csv("saron.csv", sep=";")
low <- read.csv("untere_grenze.csv", sep=";")
high <- read.csv("obere_grenze.csv", sep=";")

libor$Libor <- as.numeric(as.character(libor$Libor))
saron$Saron <- as.numeric(as.character(saron$Saron))
low$Untere.Grenze <- as.numeric(as.character(low$Untere.Grenze))
high$Obere.Grenze <- as.numeric(as.character(high$X))

LIBOR <- ts(libor$Libor, start=c(2010, 1), end=c(2016, 04),
frequency=12)
SARON <- ts(saron$Saron, start=c(2010, 1), end=c(2016, 04),
frequency=12)
Low <- ts(low$Untere.Grenze, start=c(2010, 1), end=c(2016, 04),
frequency=12)
High <- ts(high$Obere.Grenze, start=c(2010, 1), end=c(2016, 04),
frequency=12)
```

```
par(mfrow=c(2,1))
ts.plot(LIBOR, SARON, High, Low, gpars=list(col=c("red", "purple",
"orange", "orange"),xlab="", ylab="Rates", lty=c(1:3)))
legend("bottomleft",text.font = 2,  legend = c("3M-LIBOR", "SARON",
"Target (high)", "Target (low)"),
col =c("red", "purple", "orange", "orange") ,
lty = 1, cex=0.5)
par(new=TRUE)
minor.tick(nx=0, ny=2, tick.ratio=0.5)
par(new=TRUE)
grid(nx = NULL, col = "lightgray", lty ="dotted")
abline(h=0.25, col="lightgray", lty="dotted")
abline(h=-0.25, col="lightgray", lty="dotted")
abline(h=-0.75, col="lightgray", lty="dotted")
abline(h=0.75, col="lightgray", lty="dotted")

View(snb_bilanz)
snb_bilanz$Datum <- snb_bilanz$Datum
snb_bilanz$Devisen <- snb_bilanz$Aktiven...Devisenanlagen
snb_bilanz$Aktiven <- snb_bilanz$Aktiven...Total
snb_bilanz$Notenumlauf <- snb_bilanz$Passiven...Notenumlauf
snb_bilanz$Giro <- snb_bilanz$Passiven...
Girokonten.inlaendischer.Banken
snb_bilanz$Passiven <- snb_bilanz$Passiven...Total
View(snb_bilanz)
snb <-snb_bilanz
View(snb)
Devisen <- ts(snb$Devisen, start=c(2006, 1), end=c(2016, 02),
frequency=12)
Aktiven <- ts(snb$Aktiven, start=c(2006, 1), end=c(2016, 02),
frequency=12)
Notenumlauf <- ts(snb$Notenumlauf, start=c(2006, 1), end=c(2016, 02),
frequency=12)
Giro <- ts(snb$Giro, start=c(2006, 1), end=c(2016, 02), frequency=12)
Passiven <- ts(snb$Passiven, start=c(2006, 1), end=c(2016, 02),
```

```
frequency=12)

par(mfrow=c(2,1))
ts.plot(Devisen, Aktiven, Giro, gpars=list(col=c("deeppink4",
"darkorchid4", "blue") ,xlab="", ylab="Billions of CHF", lty=c(1:3)))
legend("topleft",text.font = 2,  legend = c("Foreign Currencies",
"Balance Sheet Total", "Reserves"), col=c("deeppink4",
"darkorchid4", "blue") , lty = 1, cex=0.5)
par(new=TRUE)
minor.tick(nx=0, ny=2, tick.ratio=0.5)
par(new=TRUE)
grid(nx = NULL, col = "lightgray", lty ="dotted")
abline(h=0.25, col="lightgray", lty="dotted")
abline(h=-0.25, col="lightgray", lty="dotted")
abline(h=-0.75, col="lightgray", lty="dotted")
abline(h=0.75, col="lightgray", lty="dotted")


#Denmark
setwd("/Users/rominaruprecht/Desktop")
danmark <- read.csv("danmark.csv", sep=";")
View(danmark)
date <- strptime(as.character(danmark$Datum), "%Y/%m/%d")
View(danmark)
discount <- as.numeric(as.character(danmark$Discount.rate))
current <- as.numeric(as.character(danmark$Current.account.deposits))
lending <- as.numeric(as.character(danmark$Lending))
deposit <- as.numeric(as.character(danmark$Certificates.of.deposit))

par(mfrow=c(2,1))
plot(date, lending, col="green", type="l", ylab="Rates",
xlab="", ylim=c(-1.50, 3))
par(new=TRUE)
plot(date, deposit, col="blue", type="l", ylab="",
xlab="", ylim=c(-1.50, 3), axes=FALSE)
```

```
par(new=TRUE)
plot(date, current, col="red", type="l", ylab="",
xlab="", ylim=c(-1.50, 3), axes=FALSE)
par(new=TRUE)
plot(date, discount, col="purple", type="l", ylab="",
xlab="", ylim=c(-1.50, 3), axes=FALSE)
par(new=TRUE)
grid(nx=NA, ny=NULL, col = "lightgray", lty ="dotted")
minor.tick(nx=0, ny=2, tick.ratio=0.5)
par(new=TRUE)
abline(h=0.25, col = "lightgray", lty ="dotted")
abline(h=0.5, col = "lightgray", lty ="dotted")
abline(h=0.75, col = "lightgray", lty ="dotted")
abline(h=1.25, col = "lightgray", lty ="dotted")
abline(h=1.5, col = "lightgray", lty ="dotted")
abline(h=1.75, col = "lightgray", lty ="dotted")
abline(h=2.25, col = "lightgray", lty ="dotted")
abline(h=2.5, col = "lightgray", lty ="dotted")
abline(h=2.75, col = "lightgray", lty ="dotted")
abline(h=-0.25, col = "lightgray", lty ="dotted")
abline(h=-0.5, col = "lightgray", lty ="dotted")
abline(h=-0.75, col = "lightgray", lty ="dotted")
abline(h=-1.25, col = "lightgray", lty ="dotted")
abline(h=-1.5, col = "lightgray", lty ="dotted")
abline(v=as.Date("2010-01-04"), col = "lightgray", lty ="dotted")
abline(v=as.Date("2011-01-03"), col = "lightgray", lty ="dotted")
abline(v=as.Date("2012-01-02"), col = "lightgray", lty ="dotted")
abline(v=as.Date("2013-01-02"), col = "lightgray", lty ="dotted")
abline(v=as.Date("2014-01-02"), col = "lightgray", lty ="dotted")
abline(v=as.Date("2015-01-02"),col = "lightgray", lty ="dotted")
abline(v=as.Date("2016-01-02"), col = "lightgray", lty ="dotted")
par(new=TRUE)
legend("topright",text.font=2, legend = c("Discount Rate",
"Current Account Rate", "Lending Rate", "Deposit Rate"),
col = 1:4, lty = 1, cex=0.5)
```

```
#Japan
setwd("/Users/rominaruprecht/Desktop")
japan <- read.csv("japan.csv", sep=",")
View(japan)
japan <- japan[-c(1, 79:85),]

japan$BJ.MADR1M <- as.numeric(as.character(japan$BJ.MADR1M))
japan$ST.STRACLUCON <- as.numeric(as.character(japan$ST.STRACLUCON))
View(japan)

basic <- ts(japan$BJ.MADR1M, start=c(2010, 1),
end=c(2016, 04), frequency=12)
overnight <- ts(japan$ST.STRACLUCON, start=c(2010, 1),
end=c(2016, 04), frequency=12)

par(mfrow=c(2,1))
ts.plot(basic, overnight, gpars=list(col=c("red", "purple"),
xlab="", ylab="Rates", lty=c(1:3), ylim=c(-0.2, 1)))
legend("topright",text.font = 2,  legend =
c("Basic Discount and Basic Loan Rate", "Overnight Rate"),
col =c("red", "purple") , lty = 1, cex=0.5)
par(new=TRUE)
minor.tick(nx=0, ny=2, tick.ratio=0.5)
par(new=TRUE)
grid(nx = NULL, col = "lightgray", lty ="dotted")
abline(h=0.25, col="lightgray", lty="dotted")
abline(h=-0.25, col="lightgray", lty="dotted")
abline(h=-0.75, col="lightgray", lty="dotted")
abline(h=0.75, col="lightgray", lty="dotted")

#Data ECB
library("gdata")
setwd("/Users/rominaruprecht/Desktop")
eonia <- read.csv("eonia.csv", sep=";")
```

```
eonia$date <- as.Date(eonia$Datum, "%d.%m.%y")
eonia$deposit.rate <- as.numeric(as.character(eonia$Deposit.Rate))
#deposit rate in percent
eonia$lending.rate <- as.numeric(as.character(eonia$Lending.Rate))
#lending rate in percent
eonia$deposit <- (eonia$deposit.rate/100)
eonia$lending <- (eonia$lending.rate/100)
View(eonia)


subset <- eonia[-c(1:83, 595:1630),]
View(subset)
mean(subset$lending) #mean of 2014/2015
il <- 0.00410
mean(subset$deposit) #mean of 2014/2015
id <- -0.00147
mean(im$Eonia)
delta <- (il - id)
yield <- 0.02339 #average yearly yield of government bonds
of 27 countries in the EU (647 observations)
#average yield of 2014/2015
hicp <- 0.00225 #average yearly yield of HICP inflation
in the euro zone -> average of 2014/2015 (24 observations)
r <- yield-hicp
View(r)
beta <- (1/(1+r))
View(beta)
```

Matlab Code

```
%Parameters
beta = 0.9793;
r = (1/beta);
R = 1.020;
```

```
n = 0.5;
nh = 0.5000055;
nl = 0.499995;
epsilon = nh-nl;
iL = 0.0041;
iD = -0.0015;
pi = 0;
c = iL-iD;
sigmah = 0.50;
sigmal = 0.50;
D1 = 0;
L1 = 0.02;
D2 = -0.005;
L2 = 0.015;
D3 = -0.010;
L3 = 0.015;


%No Trade in the Money Market
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Lower  and upper bound of consumption as a function of the policy
syms d
l = (beta*(1-n))/(1 - (beta*n) + (pi/(1+d)))
u = ((1+d)/(1+(d+0.0056)))*((1-n)/((1/R)-n))

fplot(l, [-0.1, 0.1])
hold on
fplot(u, [-0.1, 0.1])
grid on
legend('show')


%Constant corridor
syms d
```

```
delta0 = (1+(d+0.0056))/(1+d)
fd = (1/delta0)*(1+(((1-n)*(delta0-1))
/(1 + (beta*n*(delta0-1)) - (delta0/R))))
q = ((R*beta*(1-n))/((1-(n*beta*R))*delta0))
b = q/(beta*R*fd)
w = (((1-n)*(log(q)-q)) + (((beta*R)-1)*b)/(1-beta))
zl0 = (beta*R*b/delta0)
zm0 = q - zl0

% Fixed lending rate
syms d
delta0l = (1+iL)/(1+d)
fdl = (1/delta0l)*((1+(((1-n)*(delta0l-1))
/(1 + (beta*n*(delta0l-1))- (delta0l/R)))))
ql = ((R*beta*(1-n))/(((1-(n*beta*R))*delta0l)))
bl = ql/(beta*R*fdl)
wl = (((1-n)*(log(ql)-ql)) + ((beta*R)-1)*bl)/(1-beta)
zl0l = (beta*R*bl/delta0l)
zm0l = ql-zl0l

fplot(q, [-0.01, 0.004])
grid on
hold on
fplot(ql, [-0.01, 0.004])
legend('show')

% Welfare costs
delta01 = (1+(L1))/(1+D1)
fd01 = (1/delta01)*(1+(((1-n)*(delta01-1))
/(1 + (beta*n*((delta01-1)) - (delta01/R)))))
q01 = ((R*beta*(1-n))/((1-(n*beta*R))*delta01))
b01 = q01/(beta*R*fd01)
w01 = ((((1-n)*(log(q01)-q01))+ ((beta*R)-1)*b01) /(1-beta))
delta01 = (1+(L1))/(1+D1)
zl01 = (beta*R*b01*delta01)
```

```
zm01 = q01 - zl01

delta02 = (1+L2)/(1+D2)
fd02 = ((1/delta02)*(1+(((1-n)*(delta02-1))
/(1 + (beta*n*(delta02-1))- (delta02/R)))))
q02 = ((R*beta*(1-n))/((1-(n*beta*R))*delta02))
b02 = q02/(beta*R*fd02)
w02 = ((((1-n)*(log(q02)-q02)) + ((beta*R)-1)*b02) /(1-beta))
zl02 = (beta*R*b02*delta02)
zm02 = q02-zl02

delta03 = (1+L3)/(1+D3)
fd03 = ((1/delta03)*(1+(((1-n)*(delta03-1))
/(1 + (beta*n*(delta03-1))- (delta03/R)))))
q03 = ((R*beta*(1-n))/((1-(n*beta*R))*delta03))
b03 = q03/(beta*R*fd03)
w03 = ((((1-n)*(log(q03)-q03)) + ((beta*R)-1)*b03) /(1-beta))
zl03 = (beta*R*b03*delta03)
zm03 = q03-zl03

syms m
Answer = solve([((1-n)*(log(q01*m)-q01) +
(((beta*R*(q01*m/(beta*R*fd01)))-b01)))
/(1-beta) == ((1-n)*(log(q02)-q02) + ((beta*R)-1)*b02)
/(1-beta)], m, 'ReturnConditions', true)
Answer.m
double(Answer.m)
1-double(Answer.m)


syms k
Answer = solve([((1-n)*(log(q01*k)-q01) +
(((beta*R*(q01*k/(beta*R*fd01)))-b01))) /(1-beta)
== ((1-n)*(log(q03)-q03) + ((beta*R)-1)*b03) /(1-beta)],
k, 'ReturnConditions', true)
```

```
Answer.k
double(Answer.k)
1-double(Answer.k)

col1 = (1-n)*((1/q01)*(1/(beta*R*delta01)) - (beta*R))
col2 = (1-n)*((1/q02)*(1/(beta*R*delta01)) - (beta*R))
col3 = (1-n)*((1/q03)*(1/(beta*R*delta01)) - (beta*R))

syms m
Answer = solve([((1-n)*(log(q01*m)-q01) /(1-beta)
== ((1-n)*(log(q02)-q02) + /(1-beta)], m,
'ReturnConditions', true)
Answer.m
double(Answer.m)
1-double(Answer.m)

syms k
Answer = solve([((1-n)*(log(q01*k)-q01)+
(((beta*R*(q01*k/(beta*R*fd01)))-b01))) /(1-beta)
== ((1-n)*(log(q03)-q03) +
((beta*R)-1)*b03) /(1-beta)],
k, 'ReturnConditions', true)
Answer.k
double(Answer.k)
1-double(Answer.k)

%Trade in the Money Market
%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Consumption of deposit rate (constant corridor)
syms qh(d)
qh(d) = ((1-nh)/(nh))*((1+d)/(1+(d+0.0056))*
(n*beta*R/(1-(n*beta*R))))
syms ql(d)
ql(d) = ((1-nl)/(nl))*((1+d)/(1+(d+0.0056))*
```

```
(n*beta*R/(1-(n*beta*R))))

syms d
delta = ((1+(d+0.0056))/(1+d))
deltahat = (delta)/((n*beta*R*(1 - (delta))) + ((1+delta)))
zm = (deltahat/(deltahat-1))*((deltahat - 1)
*((sigmal*(1-nl))*ql + (sigmah*(1-nh)*qh)) -
(epsilon*sigmal*sigmah*(ql-qh)*deltahat)
*(R*(deltahat-1)))
zh= - sigmal*(ql-qh)*(deltahat/(((1+(deltahat-1)))))
betarb = (sigmah*qh) + (sigmal*ql) - zm
b = (((sigmah*qh) + (sigmal*ql) - zm)
/(beta*R))*((1+d)/(1+(d+0.0055)))
im = (d+0.0055) - (n*beta*R*0.0056)
gamma = (1+im)/R
zl = (sigmah*(ql-qh))*((deltahat/(deltahat-1))*sigmal)
phi = ((R*deltahat) - delta)/((R*(delta - 1))+((1-n)*deltahat))

%Constant lending rate
syms qhl(d)
qhl(d) = ((1-nh)/(nh))*((1+d)/(1+iL)*(n*beta*R/(1-(n*beta*R))))
syms qll(d)
qll(d) = ((1-nl)/(nl))*((1+d)/(1+iL)*(n*beta*R/(1-(n*beta*R))))

syms d
deltal = (1+iL)/(1+d)
deltahatl = ((deltal)/((n*beta*R*(1 - ((deltal))) + ((deltal)))))
zml = (deltahatl/(deltahatl-1))*((deltahatl - 1)
*((sigmal*(1-nl))*qll + (sigmah*(1-nh)*qhl)) -
(epsilon*sigmal*sigmah*(qll-qhl)*deltahatl)*
(R*(deltahatl-1)))
zhl= - (sigmal*(qll-qhl)*(deltahatl/(deltal)-1))
betarbl = (sigmah*qhl) + (sigmal*qll) - zml
bl = ((((sigmah*qhl) + (sigmal*qll) - zml)/(beta*R))*(deltal))
iml = (iL) - (n*beta*R*(iL-d))
```

```
gammal = (1+iml)/R
zll = -(sigmah/sigmal)*zhl

%Plots
fplot(qh, [-0.01, 0.004])
grid on
hold on
fplot(qhl, [-0.01, 0.004])
legend('show')

fplot(ql, [-0.01, 0.004])
grid on
hold on
fplot(qll, [-0.01, 0.004])
legend('show')

%Welfare costs
qh1 = ((1-nh)/(nh))*((1+D1)/(1+(L1))*(n*beta*R/(1-(n*beta*R))))
ql1 = ((1-nl)/(nl))*((1+D1)/(1+(L1))*(n*beta*R/(1-(n*beta*R))))
delta1 = (1+L1)/(1+D1)
deltahat1 = (delta1)/((n*beta*R*(1 - delta1)) + delta1)
zm1 = (deltahat1/(deltahat1 - 1))*((deltahat1 - 1)
*((sigmal*(1-nl))*ql1 + (sigmah*(1-nh)*qh1)) -
(epsilon*sigmal*sigmah*(ql1-qh1)*deltahat1)
*(R*(deltahat1 - 1)))
zh1= - sigmal*(ql1-qh1)*(deltahat1/(((1+(deltahat1-1)))))
betarb1 = (sigmah*qh1) + (sigmal*ql1) - zm1
b1 = (((sigmah*qh1) + (sigmal*ql1) - zm1)/(beta*R))*((1+D1)/(1+(L1)))
im1 = (L1) - (n*beta*R*(L1-D1))
gamma1 = (1+im1)/R
zl1 = (sigmah*(ql1-qh1))*((deltahat1/(deltahat1-1))*sigmal)
qhql1 = (ql1/qh1)
phi1 =(((R*deltahat1) - delta1)/(R*(delta1 - 1)))+((1-n)*deltahat1)
frac1 = phi1 + ((sigmah/sigmal)*(deltahat1-1)
*(1-n))+(sigmah*epsilon)/(phi1 - ((deltahat1-1)
```

```
*(1-n))+(sigmah*epsilon))
frac2 = phi1 - ((deltahat1-1)*(1-n)) +
((sigmal*sigmah*epsilon*(1-(qh1/ql1)))
/((sigmah*(qh1/ql1))+sigmal))
frac3 = phi1 / (deltahat1*(sigmah*(qh1/ql1))+sigmal)
wh1 = ((1-n)*(log(qh1)-qh1) + (beta*R)*b1) /(1-beta)
wl1 = ((1-n)*(log(ql1)-ql1) + (beta*R)*b1) /(1-beta)

qh2 = ((1-nh)/(nh))*((1+D2)/(1+(L2))*(n*beta*R/(1-(n*beta*R))))
ql2 = ((1-nl)/(nl))*((1+D2)/(1+(L2))*(n*beta*R/(1-(n*beta*R))))
delta2 = (1+L2)/(1+D2)
deltahat2 = (delta2)/((n*beta*R*(1 - delta2)) + delta2)
zm2 = (deltahat2/(deltahat2 - 1))*(((deltahat2 - 1)
*((sigmal*(1-nl))*ql2 + (sigmah*(1-nh)*qh2))
- (epsilon*sigmal*sigmah*(ql2-qh2)*deltahat2)))
*(R*(deltahat2 - 1))
zh2= - sigmal*(ql2-qh2)*(deltahat2/(((1+(deltahat2-1)))))
betarb2 = (sigmah*qh2) + (sigmal*ql2) - zm2
b2 = (((sigmah*qh2) + (sigmal*ql2) - zm2)/(beta*R))*((1+D2)/(1+(L2)))
im2 = (L2) - (n*beta*R*(L2-D2))
gamma2 = (1+im2)/R
zl2 = (sigmah*((ql2)-(qh2))*((deltahat2/(deltahat2-1))*sigmal))
qhql2 = (ql2/qh2)
phi2 = ((R*deltahat2) - delta2)/(R*(delta2 - 1))+((1-n)*deltahat2)
frac12 = phi2 + ((sigmah/sigmal)*(deltahat2-1)
*(1-n))+(sigmah*epsilon)/(phi2 - ((deltahat2-1)
*(1-n))+(sigmah*epsilon))
frac22 = phi2 - ((deltahat2-1)*(1-n)) +
((sigmal*sigmah*epsilon*(1-(qh2/ql2)))
/((sigmah*(qh2/ql2))+sigmal))
frac32 = phi2 / (deltahat2*(sigmah*(qh2/ql2))+sigmal)
wh2 = (1-n)*(log(qh2)-(qh2)) + (beta*R)*b2 /(1-beta)
wl2 = (1-n)*(log(ql2)-(ql2)) + (beta*R)*b2 /(1-beta)

qh3 = ((1-nh)/(nh))*((1+D3)/(1+(L3))*(n*beta*R/(1-(n*beta*R))))
```

```
ql3 = ((1-nl)/(nl))*((1+D3)/(1+(L3))*(n*beta*R/(1-(n*beta*R))))
delta3 = (1+L3)/(1+D3)
deltahat3 = (delta3)/((n*beta*R*(1 - delta3)) + delta3)
zm3 = (deltahat3/(deltahat3-1))*((deltahat3 - 1)*
((sigmal*(1-nl))*ql3 + (sigmah*(1-nh)*qh3)) -
(epsilon*sigmal*sigmah*(ql3-qh3)*deltahat3)
*(R*(deltahat3-1)))
zh3 = - sigmal*((ql3-qh3)*(deltahat3/(((1+(deltahat3-1))))))
betarb3 = (sigmah*qh3) + (sigmal*ql3) - zm3
b3 = (((sigmah*qh3) + (sigmal*ql3) - zm3)/(beta*R))*((1+D3)/(1+(L3)))
im3 = (L3) - (n*beta*R*(L3-D3))
gamma3 = (1+im3)/R
zl3 = (sigmah*(ql3-qh3)*((deltahat3/(deltahat3-1))*sigmal))
qhql3 = (ql3/qh3)
phi3 = (((R*deltahat3) - delta3)/(R*(delta3 - 1)))+((1-n)*deltahat3)
frac13 = phi3 + ((sigmah/sigmal)*(deltahat3-1)
*(1-n))+(sigmah*epsilon)/(phi3 -
((deltahat3-1)*(1-n))+(sigmah*epsilon))
frac23 = phi3 - ((deltahat3-1)*(1-n))
+ ((sigmal*sigmah*epsilon*(1-(qh3/ql3)))
/((sigmah*(qh3/ql3))+sigmal))
frac33 = phi3 / (deltahat3*(sigmah*(qh3/ql3))+sigmal)
wh3 = (1-n)*(log(qh3)-qh3) + (beta*R)*b3 /(1-beta)
wl3 = (1-n)*(log(ql3)-ql3) + (beta*R)*b3 /(1-beta)

syms s
Answer = solve([((1-n)*(log(qh1*s)-qh1) +
(((beta*R*((((sigmah*qh1*s) + (sigmal*ql1*s) - zm1)
/(beta*R))*((1+D1)/(1+(L1))))))-b1)) /(1-beta)
== ((1-n)*(log(qh2)-qh2) + ((beta*R)-1)*b2) /(1-beta)]
, s, 'ReturnConditions', true)
Answer.s
double(Answer.s)
1-double(Answer.s)
```

```
syms t
Answer = solve([((1-n)*(log(qh1*t)-qh1) +
(((beta*R*((((sigmah*qh1*t) + (sigmal*ql1*t) - zm1)
/(beta*R))*((1+D1)/(1+(L1))))))-b1)) /(1-beta) ==
((1-n)*(log(qh3)-qh3) + ((beta*R)-1)*b3) /(1-beta)]
, t, 'ReturnConditions', true)
Answer.t
double(Answer.t)
1-double(Answer.t)

%yk
fplot(zh, [-0.01, 0.004])
grid on
hold on
fplot(zl, [-0.01, 0.004])
hold on
fplot(zhl, [-0.01, 0.004])
hold on
fplot(zll, [-0.01, 0.004])
legend('show')

fplot(zh, [-0.5, 0.5])
grid on
hold on
fplot(zl, [-0.5, 0.5])
hold on
fplot(zhl, [-0.5, 0.5])
hold on
fplot(zll, [-0.5, 0.5])
legend('show')
```

Plagiatserklärung

Ich bezeuge mit meiner Unterschrift, dass meine Angaben über die bei der Abfassung meiner Arbeit benutzten Hilfsmittel sowie über die mir zuteil gewordene Hilfe in jeder Hinsicht der Wahrheit entsprechen und vollständig sind.
Ich habe das Merkblatt zu Plagiat und Betrug vom 22. Februar 2011 gelesen und bin mir der Konsequenzen eines solchen Handelns bewusst.

15.6.2016