Master's Thesis

High Inflation Currencies

An Analysis of the Rate-of-Return Dominance Puzzle

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Abstract

In most high inflation countries, foreign currencies with low inflation rates are available. Despite this fact, the high inflation domestic currency remains in circulation. This contradicts a result in currency competition which predicts that in frictionless markets low inflation currencies drive out high inflation currencies. The above contradiction is known by the name "rate-of-return dominance puzzle". This thesis will provide explanations for the occurrence of this puzzle.

Contents

1 Introduction

The competition between means of payment has always been a topic of great interest and significance for all participants in an economy: For residents who need a means of payment to trade and sometimes store wealth, for central banks whose power depends on the demand for their supplied means of payment and for governments whose independence and power might suffer if the national currency is abandoned.

When comparable means of payment exist in an economy, competition and replacements of currencies are observed. In several leading industrial countries gold drove out other means of payment in the second half of the 19th century resulting in the classical gold standard (Bordo and Redish, 2013, pp.6-9). The break-down of the Bretton Woods System in 1973 definitely ended the era of the gold standard and "the world shifted [...] to a managed floating exchange rate regime" (Bordo and Redish, 2013, p.18). Since then, central banks were less forced to follow a certain, possibly restrictive policy, instead they were able to act more independently. This led to relatively different growth rates of money supply across countries and resulted in varied inflation rates.
Different inflation rates imply different costs of holding money. These costs represent the negative rate-of-return from holding fiat money. The rate-of-return of a currency is equal to the opportunity cost caused by holding it, i.e. neither using it for current consumption nor investing it in an interest-bearing asset.[1]

The literature in currency competition is divided in its answer to the question which currency is used as a means of payment if two currencies (fiat money) with different rates-of-return are available.

One long-held view in the field is that in the absence of any restrictions on currency choice, the currency with the highest return is preferred. Prominent economists who describe this outcome of currency competition are Hayek, Kareken and Wallace. Hayek (1976) argues that the logical outcome of currency competition is that the lowest inflation currency is preferred and is circulated most widely. Kareken and Wallace (1981) demonstrate that with unrestricted currency portfolio choice and free access to currency markets, two currencies can only coexist in an economy if the rates-of-return are identical. They call this result of currency competition the "dominance result" (Kareken and Wallace, 1981, p.211).

[1] Thus, the return of money is equal to $-i$ where i is equal to the nominal interest rate. According to Fisher $i \approx r + \pi$, where π is the inflation rate and r the real interest rate.
It is here assumed that r is equal for all currencies. This assumption implies that the rate-of-return of a currency depends only on inflation. The currency with the lowest inflation rate therefore has the highest rate-of-return.

However, the above views can not be the final word in the theory of currency competition because in many countries domestic high inflation currency is not fully substituted by low inflation foreign currencies. So why do people have preferences for a high inflation currency, if a lower inflation currency is available? This question is the heart of the rate-of-return dominance puzzle related to currencies.
The discussion of the rate-of-return dominance puzzle started when Hicks (1935) asked why capital goods with positive rates-of-return do not drive out fiat money as means of payment. Hicks nominated the rate-of-return dominance puzzle as the central issue in the pure theory of money and advised not to evade the puzzle in macroeconomic models (Hicks, 1935, p.5). He said:

> "Of course the great evaders would not have denied that there must be some explanation of the fact. But they would have put it down to frictions, and since there was no adequate place for frictions in the rest of their economic theory, a theory of money based on frictions did not seem to them a promising field of research. This is where I disagree. I think we have to look the frictions in the face..." (Hicks, 1935, p.6).

According to Hicks different kinds of frictions can explain the rate-of-return dominance puzzle. In this thesis the advice of Hicks is followed – we "look the frictions in the face" (Nosal and Rocheteau, 2011, pp.251-252).
Thus, the objective of this thesis is to answer the question which kind of frictions can explain the rate-of-return dominance puzzle related to currencies. The thesis provides a literature survey of different models that examine how currencies with different returns can coexist.

The models presented in this thesis adopt different assumptions. However throughout the thesis, two currencies as a means of payment are available. Good trades are anonymous, meaning agents are not able to identify their trading partners, this motivates the role for money as a means of payment in every trade (Berentsen et al., 2007, p.5). It exists a domestic and foreign central bank, which issues its own fiat money. The domestic currency has higher inflation than the foreign currency. The inflation rates represent the changes in price level $\frac{P_1-P_0}{P_1}$. An increase in price level implies positive costs of holding money, since after depreciation one can afford less goods with the same amount of money than before.

The thesis proceeds as follows: an introduction in Section (1), followed by a discussion of currency competition in Section (2). This section includes a presentation of the dominance result and the rate-of-return dominance puzzle by using the benchmark model of divisible money by Nosal and Rocheteau (2011). Section (3), (4) and (5) present different macroeconomic models that examine the effect of frictions on the coexistence of competing currencies. Section (3) presents models that show how government interventions can keep a low return domestic money in circulation. The discussed interventions are prohibition, legal restrictions on adherence of fixed exchange rates and the imposition of taxes payable in domestic currency. Subsequently, in Section (4), the impact of transaction costs on currency competition is examined. In Section (5) the rate-of-return dominance puzzle is analysed by assuming different information problems connected to the foreign currency. The information problems discussed in this thesis are: Difficulty authenticating foreign currency, insecurity about the level of foreign inflation[2] and higher default probability of debt denominated in foreign currency. Section (6) presents the conclusion.

[2]This idea for explaining the rate-of-return dominance puzzle has been suggested to me by Professor Berentsen. To the best of my knowledge this approach has not yet been recognized in the literature of competition between two currencies.

2 Currency Competition

Monetary Theory offers different approaches to explain which currency is used in an economy if two currencies with different rates-of-return are available. In this section the discussion of the dominance result and the rate-of-return dominance puzzle is based on the work of Nosal and Rocheteau (2011).[3] The framework in Nosal and Rocheteau (2011) is based on the divisible money model by Lagos and Wright (2005). This framework is used since "it provides a micro foundation for money demand and it allows us to introduce heterogeneous preferences for consumption and production, while keeping the distribution of money balance analytical tractable" (Berentsen et al., 2007, p.5).

2.1 Dominance Result

Some economists claim that in frictionless markets low inflation currency drives out high inflation currency. By using the benchmark model of divisible money by Nosal and Rocheteau (2011), it can be demonstrated that if domestic and foreign currency have the same liquidity properties and no other frictions apply, the currency with the higher return is preferred and the dominance result applies.
The dominance result can also be observed in practice in developing economies, where high return foreign currencies (e.g the United-States (U.S.)-Dollar) circulate widely. Residents often prefer foreign currencies because of its lower holding costs.

2.1.1 Environment

In the framework of Nosal and Rocheteau (2011), time is discrete and continuous. Every period is divided into two sub-periods – day and night. During the day, trading takes place in decentralized markets (DM). At night, trade occurs in centralized markets (CM). Agents discount between the night and the next day with the discount factor $\beta = \frac{1}{1+r} \in (0,1)$. Any agent can be a buyer or seller. The measure of buyers and sellers are equal.
In DM sellers can produce the DM good q, but they do not consume it. Buyers want to consume q, but cannot produce it. Buyers and sellers meet during the day with probability α. In every match the buyer makes a take-it-or-leave-it offer to the seller. The buyer has all the bargaining power. Hence, the seller gets nothing of the trade surplus, therefore the seller is indifferent to trading or not, although it is assumed he trades.

[3] Nosal and Rocheteau (2011) analyse dual currency payment systems in Chapter 10.

If there is a match between a buyer and seller in the DM, the seller produces q units of the DM good for the buyer and the buyer transfers an amount of money d. At the end of the day, all matches are broken up.
At night, both agents can produce and consume the CM good x. At the end of the CM, after trading, again all matches are destroyed.
(Nosal and Rocheteau, 2011, pp.14-18)

Nosal and Rocheteaus's (2011) benchmark model is extended here by the following assumptions. Agents have domestic and foreign currency available as means of payment. In every match both currencies can be used to trade. One unit of domestic currency buys $\phi_{domestic}$ units of the CM good. One unit of foreign currency buys $\phi_{foreign}$ units of the CM good. In this thesis it is assumed in the CM in Period t is $\phi_{domestic,t} = \phi_{foreign,t}$. But between periods, the stock of domestic money grows faster than the stock of foreign money. The change of the stock of money represents the inflation rate $\frac{M_{t+1}}{M_t} = \frac{\phi_t}{\phi_{t+1}}$. Thus, in Period $t+1$ is $\phi_{domestic,t+1} < \phi_{foreign,t+1}$. To consume a certain amount of goods (q) in DM_{t+1} relatively more of the domestic currency – compared to the foreign currency – has to be brought from CM_t into DM_{t+1}.

2.1.2 Choice of Money

By using the DM and CM value function of the buyer and rearranging terms one realises the buyer's maximization problem (Appendix (A)). This determines which currency a buyer will hold and use in trading in the next DM $(\hat{m}_1, \hat{m}_2)$.[4]

$$\max_{\hat{m}_1,\hat{m}_2\geq 0} \hat{m}_1\phi_{1,t+1}\left[-\overbrace{\left(\frac{\frac{\phi_{1,t}}{\phi_{1,t+1}}}{\beta}-1\right)}^{\text{costs of holding money 1} = i_{m_1}}\right] + \hat{m}_2\phi_{2,t+1}\left[-\overbrace{\left(\frac{\frac{\phi_{2,t}}{\phi_{2,t+1}}}{\beta}-1\right)}^{\text{costs of holding money 2} = i_{m2}}\right] + \alpha[u(q)-c(q)] \tag{2.1}$$

The buyer maximizes his expected surplus in the DM, net of the cost of holding domestic and foreign money. According to Fisher, the costs of holding money are equal to i, since $\frac{\frac{\phi_t}{\phi_{t+1}}}{\beta} = (1+\pi)(1+r)$, which is equal to $\pi + r \approx i$. By assuming that the real interest rate r is equal in both countries, different returns of money correspond to different inflation rates. Positive costs of money holdings are assumed. This means $\frac{\phi_t}{\phi_{t+1}} > \beta$. This condition implies that buyers do not bring enough money balances in

[4]To shorten notation, all domestic variables are labelled by 1 (e.g. domestic money = m_1). Foreign variables are denoted by 2 (e.g. foreign money = m_2).

the DM to buy the efficient quantity of the DM good q^*.[5] (Nosal and Rocheteau, 2011, pp.67-69)
The first order conditions of the buyer's maximization problem (2.1) are:

$$-i_{m1} + \overbrace{\alpha \left[\frac{u'(q)}{c'(q)} - 1 \right]}^{\text{liquidity factor } \lambda(q)} = 0 \tag{2.2}$$

$$-i_{m2} + \alpha \left[\frac{u'(q)}{c'(q)} - 1 \right] = 0 \tag{2.3}$$

where λ captures the marginal benefit of using the currency in DM trades and buying q.[6] As λ is in Equation (2.2) and (2.3) equal, the choice of money depends on the money holding costs. The inflation rate of the domestic currency is higher than the inflation rate of the foreign currency, thus the cost of holding domestic currency is higher: $i_{m1} > i_{m2}$. If both currencies are equally accepted in trades (with probability α), the currency with the higher return – in our case the foreign currency is preferred by all agents, because of its lower costs of holding.

2.1.3 Empirical Evidence

The dominance result is a common outcome of currency competition as shown in the empirical study Bernholz (2003) covering several past advanced inflation and hyperinflation economies. He found that if the increase of the price level is greater than the increase of the domestic money supply, the real stock of domestic money $\frac{M}{P}$ decreases and currency substitution[7] progresses (Bernholz, 2003, pp.74-76). People try to spend their domestic currency as soon as possible, since they expect a further and larger loss of purchasing power as the currency depreciates. As a consequence, prices rise faster than the money supply. This reduces the real stock of high inflation domestic money. (Bernholz, 2003, p.75)

The empirical literature shows if inflation of the domestic currency is continuously increasing, while the inflation rate of the available foreign currency stays relatively constant, currency substitution often occurs.

[5] Appendix (A), Terms of Trade in the DM: With positive costs of holding money buyers do not hold more money than they spend in the DM ($d_i = \hat{m}_i$) and $\phi_{i,t+1}\hat{m}_i < c(q*)$, where $i = 1, 2$. Since the participation constraint of the seller holds, q is given by $q = c^{-1}(\phi_{i,t+1}\hat{m}_i) < q*$.

[6] Since money is costly to hold, buyers are only able to afford $q < q^*$. Therefore a buyer values an additional asset, because an additional asset increases his surplus and $u'(q) > c'(q)$ – the liquidity premium is positive (Nosal and Rocheteau, 2011, p.68).

[7] In this thesis the term currency substitution means the substitution of the high inflation domestic currency by the low inflation foreign currency.

2.1.4 Implications

The availability of two currencies with equal liquidity properties but different returns leads to a substitution of low return currency by higher return currency. One can speak of an unplanned currency reform (Bernholz, 1989, p.467).

For example during the hyperinflation in Zimbabwe 2007/2008 "good money" (foreign currencies) has driven "bad money" (Zimbabwe Dollar) out of circulation. Foreign currencies obtained on the black currency market floated against the Zimbabwe dollar and replaced it. The U.S. dollar, South African rand, Botswana pula, Zambian kwacha and Mozambican metical all became increasingly popular because of its lower holding costs. (Hanke and Kwok, 2009, p.358)

In the following sections it will be seen however that moderate differences in inflation rates can be consistent with a rate-of-return dominance puzzle.

2.2 Rate-of-Return Dominance Puzzle

In most high inflation countries, foreign currencies with low inflation rates are available. Despite this fact, the high inflation domestic currency remains in circulation. This contradicts the traditionally predicted outcome of currency competition. The thesis will provide explanations for the rate-of-return dominance puzzle by analysing models that introduce frictions which influence people to hold high inflation domestic currency in spite of the availability of low inflation foreign currency.

To understand how two currencies with different returns can coexist, the relationship between rates-of-return and liquidity properties, i.e. the ability to use the currencies in trading, is important. The dominance result occurs as long as the currencies have the same liquidity properties and no other frictions apply. By assuming distinct liquidity properties of currencies, one can easily break "the curse of Kareken and Wallace" (Gomis-Porqueras et al., 2014, p.2). By making currencies imperfect substitutes, currencies with different rates-of-return can coexist.

2.2.1 Liquidity Frictions

In this section the framework from Nosal and Rocheteau (2011) is used as in Section (2.1). The probability that a buyer meets a seller who accepts domestic and foreign currency in the DM is α. The model presented in Section (2.1.1) is extended by the assumption that with probability α_{m1}, the buyer meets a seller who only accepts domestic currency. It is assumed that $\alpha_{m1} > 0$ and $\alpha > 0$, which implies that the domestic currency is more liquid than the foreign currency since it is more often accepted in trades. (Lagos et al., 2014, p.53)

The buyer's maximization problem is as follows (Appendix (A)):

$$\max_{\hat{m}_1,\hat{m}_2\geq 0} \hat{m}_1\phi_{1,t+1}\left[\overbrace{-\left(\frac{\frac{\phi_{1,t}}{\phi_{1,t+1}}}{\beta}-1\right)}^{\text{costs of holding money } 1 = i_{m1}}\right] + \hat{m}_2\phi_{2,t+1}\left[\overbrace{-\left(\frac{\frac{\phi_{2,t}}{\phi_{2,t+1}}}{\beta}-1\right)}^{\text{costs of holding money } 2 = i_{m2}}\right] + \alpha[u(q)-c(q)] + \alpha_{m1}[u(q)-c(q)] \tag{2.4}$$

The buyer maximizes his expected surplus in the DM, net of the cost of holding both domestic and foreign currency.
The first order conditions of the buyer's maximization problem (2.4) are:

$$-i_{m1} + \alpha\left[\frac{u'(q)}{c'(q)}-1\right] + \alpha_{m1}\left[\frac{u'(q)}{c'(q)}-1\right] = 0 \tag{2.5}$$

$$-i_{m2} + \alpha\left[\frac{u'(q)}{c'(q)}-1\right] = 0 \tag{2.6}$$

Equation (2.5) and (2.6) show that as long as the return difference between the domestic and foreign currency $i_{m1} - i_{m2}$ is equal to $\alpha_{m1}\left[\frac{u'(q)}{c'(q)}-1\right]$ a buyer is indifferent between holding the two currencies. If the return difference $i_{m1} - i_{m2}$ is smaller than $\alpha_{m1}\left[\frac{u'(q)}{c'(q)}-1\right]$ all agents prefer the domestic currency. The return difference equals the cost of liquidity, that is the return an agent foregoes for the utility of using the domestic currency more regularly in trading. This clearly illustrates that the return of a currency depends on its inflation rate and on its degree of liquidity.

2.2.2 Implications

This section highlights that liquidity properties of currencies are important for determining which currency is preferred. But the model of Nosal and Rocheteau (2011) does not deliver answers why liquidity properties of currencies can differ. The following three sections ((3),(4),(5)) present models that introduce frictions to explain preferences for the high inflation domestic currency (where strong preferences for domestic currency explain its acceptance and widespread circulation in an economy).

3 Government Frictions

Governments of developing economies often impose restrictions on the use of foreign currency in order to try to prevent the replacement of the domestic currency. By introducing restrictions governments are able to change liquidity properties of currencies in favour of the high inflation domestic currency.

Many models that analyse the effect of legal restrictions on currency competition have been developed. For example Curtis and Waller (2000) analyse what kind of government restrictions can lower the value of foreign currency and drive it out of the economy. Or Li and Wright (1998) assume that the government is represented by a subset of agents in the economy. These agents are randomly matched with private agents and trade, but only accept domestic currency. If government is large enough, foreign currencies will be driven out from circulation.
(Craig et al., 2000, pp.9-10)

This section analysis three types of government restrictions: Prohibition of foreign currencies, fixed exchange rate in favour of the domestic currency and imposition of taxes only payable in domestic currency.

3.1 Prohibition of Foreign Currencies

Governments of a high inflation countries can prohibit the use of foreign currencies and introduce severe punishments for violating the law. Several Latin American countries, for example Bolivia in 1982 or Peru in 1985, tried to "de-dollarize" their economy by forcing a conversion of foreign currencies into domestic currency (Vegh and Calvo, 1992, p.8). Laws which prohibit foreign currencies are able to increase the liquidity of domestic currency and decrease the liquidity of foreign currency.

3.2 Fixed Exchange Rate

A government may define a fixed exchange rate between a high inflation domestic and low inflation foreign currency, that overvalues the domestic currency. Government introduces fines to enforce this exchange rate. During the French Revolution the following law applied:[8]

[8]During the French Revolution Assignats (paper money) were used as a means of payment.

> "Each Frenchman convicted of having refused the payment of assignat money, or to have it taken or given at a discount, will be fined 3000 livres and be imprisoned for six months the first time. In case of reversion the fine will be doubled and he will be condemned to 20 years in prison in chains." (Bernholz, 2003, p.67)

The effect of such legal restrictions on the competition between currencies is analysed by using the model of Bernholz (1989). The working of Gresham's Law and the reversed Gresham's Law (Bernholz calls it Thier's Law) are illustrated in this model. Gresham's Law states that bad money drives out good money and is named after Sir Thomas Gresham, who was inter alia the financial advisor of Queen Elizabeth I of England in the 16th Century. He noticed that when debased (undervalued) coins circulate together with coins of proper weight and value, people hoard the good money and use the bad for trading. (Mundell, 1998, Introduction)
A fixed exchange rate in favour of domestic currency can cause that only the bad money will circulate since people spend it to get rid of it. The good money is hoarded or spend abroad and thus disappears from circulation.

3.2.1 Model

Bernholz (1989) presents a model with four periods, in which the coexistence and replacement mechanisms of two currencies are described.[9] The following scenario is given: There is a small home country and foreign country (which may be thought of as the rest of the world) which both use gold coins as means of payment. The gold coins as a means of payment represent the higher return currency.
In the home country it is assumed the government faces a budget deficit D, constant in every period and exogenously given.
The domestic government wants to finance the deficit by issuing a domestic paper money M, representing the low return currency. Agents of the home country hold both gold coins and paper money. In the foreign country only gold coins are used. The domestic currency is issued in exchange for gold at a certain fixed exchange rate w (price of gold expressed in paper money). The monetary authority itself holds gold reserves G^B and is not independent from the government. The fixed exchange rate w equals the parity q as long as the gold reserves of the monetary authority G^B cover a certain percentage f of the issued domestic paper money M, ($fM < G^B w$). The percentage f is exogenously given and can be seen as the "credibility frontier", a minimum coverage ratio that prevents devaluation of the paper money. (Bernholz, 2003, pp.118-121)

[9] The paper Bernholz (1989) is contained in Bernholz (2003), Chapter 6, pp.118-132.

Period One: Introduction of Domestic Fiat Money

The government introduces a domestic paper money at a fixed exchange rate to the foreign currency to finance the budget deficit, D. This deficit arises from a balance of payments[10] deficit B (imports > exports). To ensure the newly issued domestic paper money is accepted by local residents, the paper money needs to be superior to the gold money in certain features. As gold coins are metallic money, larger payments imply heavy transaction and transportation costs. With paper money carrying the money around is easier. The public is willing to substitute part of the gold money for this advantage. In this period, the public takes all the issued paper money so the supply of paper money is smaller than demand for it. The gold stock held by the public G shrinks about the value of the nominal budget deficit D. The government uses the gold traded for paper money to cover the budget deficit D which is equal to the balance of payments deficit B. In Period One $B = -D = w\Delta G = \Delta M$ applies, where ΔG is the change of the gold stock held by the public and ΔM is the change of the domestic paper money stock.[11] The gold stock initially held by the monetary authority (G^B) stays constant $\Delta G^B = 0$. (Bernholz, 2003, p.121-122)

Period Two: Loss of Official Reserves

It is assumed now the public's demand for paper money is saturated. Consequently, the stock of gold in the hands of the public remains unchanged ($w\Delta G = 0$). However the government needs to finance its budget deficit D, so it now draws on the monetary authority's gold reserves. The gold reserves of the monetary authority shrinks ($\Delta G^B < 0$). As in Period One gold flows out of the country since the budget deficit is equal to the balance of payment deficit that needs to be financed. The gold outflow in Period Two is equal to the balance of payments deficit, the budget deficit and the decrease in the monetary authority's gold reserves ($B = -D = w\Delta G^B$).[12]

Since the government wants to keep the fixed exchange rate w equal to the parity q, the monetary authority's gold reserves wG^B need to be larger than fM. (Bernholz, 2003, pp.122-123)

Period Three: Gresham's Law

At the point where a monetary authority's gold reserves have reached $wG^B = fM$,

[10]Note that the balance of payments is the sum of current account (CA) + private capital account (KA) + the official reserve transaction (ORT). It is equal to zero according to the accounting identity (Caves et al., 2007, p.283). The term "balance of payments" used by Bernholz (1989) means only the actual current account (= exports – imports).

[11]The following two equations apply: $\Delta M = D + w\Delta G^B$ and $B = w(\Delta G + \Delta G^B)$. By rearranging terms and setting $\Delta G^B = 0$, the result is $\Delta M - D = B - w\Delta G$, hence the change in money stock is equal to the change in the gold stock held by the public.

[12]Since $\Delta M = D + w\Delta G^B$, $\Delta M = 0$, $\Delta G = 0$ and $B = w(\Delta G + \Delta G^B)$ apply, by rearranging terms one can get $-D = B = w\Delta G^B$.

the confidence in the paper money diminishes. Without government interventions a devaluation of the domestic paper money results, since convertibility becomes difficult or is even suspended. To prevent a devaluation of paper money, the government makes the domestic paper money legal tender and introduces punishments for deviating from the fixed exchange rate w.
At w gold is undervalued. In the home country only the high inflation domestic paper money is used as a means of payment as it is overvalued by law. No resident will use gold as a means of payment in the home country, they will choose to spend it abroad. The public reduces its stock of gold money as gold leaves the country, thus $B = w\Delta G$. The public's stock of gold money in the home country drives down to zero. As no one exchanges gold money for paper money at w in the home country, the government prints uncovered paper money and can balance expenses in the home country ($D = \Delta M$). Gresham's Law applies, the high inflation paper money drives out the low inflation foreign (gold) money. (Bernholz, 2003, 123-125)

Period 4: Thier's Law
The gold stock held by the public has reached zero as gold has left the country. Given the government's continuous issuance of money, the exchange rate can no longer be fixed in spite severe punishments ($wG^B < fM$). The exchange rate increases, bad paper money devalues as the good gold money appreciates. The domestic price level increases faster than the money supply. Currency substitution can not be prevented (Section (2.1.3)). The demand for paper money decreases and approaches asymptotically zero, as the cost of holding paper money inflates (Bernholz, 1989, p.481). In contrast, the demand for foreign gold money approaches asymptotically the total demand for money. The depreciation of the domestic money leads to a balance of payments surplus ($B = w\Delta G > 0$), since increasing exports allow gold to flow into the country and the public substitutes domestic paper money by foreign (gold) money. The low inflation foreign money drives out the high inflation domestic money. The dominance result (Thier's Law) applies. (Bernholz, 2003, pp.126-131)

3.2.2 Implications

Gresham's Law is analysed to explain the rate-of-return dominance puzzle. A fixed exchange rate which overvalues the domestic currency as compared to the foreign currency is introduced. The foreign currency vanishes from circulation leaving domestic currency alone to circulate in the home country. Agents prefer spending the overvalued domestic currency in the home country.[13] Bernholz (2003) describes that it is only a matter of time until high inflation domestic currency is driven out of circulation by a lower inflation foreign currency.

This model may show a potential scenario for Greece, assuming that the country leaves the Euro-zone and reintroduces its domestic currency. In this case two currencies could coexist in Greece. It might be difficult to convince the public to substitute part of the Euro with the new Greek paper money. Because Period One probably will not occur and Period Two – the outflow of official good money reserves until a minimum – has almost finished. The process of currency competition will most likely start in Period Three: The Greek money will circulate and the Euro is hoarded or spend abroad. The occurrence of Period Four (dominance result) will depend on the extent of money stock growth.

3.3 Tax Frictions

The subsequent section discusses further government intervention to strengthen the circulation of high inflation domestic currency. Compared to the previous section, no direct restrictions on the exchange rate between currencies are introduced. Instead, government intervention is analysed by the Matsui model (1998) on how the imposition of taxes on production revenues payable in domestic money, can explain the rate-of-return dominance puzzle.
If an economy has no taxes on the revenue of production and no other frictions exist, different inflation rates imply all agents choose to hold the strong currency (featured by a low inflation rate), since the cost of holding foreign currency is smaller (Matsui, 1998, p.316). If the domestic government imposes taxes on its own residents, payable in its high inflation currency, agents will have to sell part of their production in exchange for the domestic currency in order to pay taxes.[14] Consequently, demand for the high inflation domestic currency increases.

[13] An other model that describes Gresham's Law is developed by Velde et al. (1999). They assume that "some sellers have imperfect information and can not determine which currency they are trading for. This creates a "lemons" problem – uninformed sellers are not willing to produce a sufficient amount of the commodity for the good money, since they are afraid of getting the bad money in return.[...] Holders of the good money will not trade with these uninformed sellers, since they undervalue the good money" (Craig et al., 2000, pp.8-9). Thus, like in Bernholz (1989) an undervaluation of the high return foreign currency leads to the circulation of the low return domestic currency.

[14] Matsui (1998) does not allow for direct currency exchange.

3.3.1 Model

Matsui (1998) developed a two-country model (home (H) and foreign (F) country) in which two currencies (domestic and foreign money) with different inflation rates ($\pi^H > \pi^F$) compete. Agents can freely choose which currency to use in trades. (Matsui, 1998, p.316)
In both countries exist two types of agents $k = 1, 2$ and k goods. Agent 1 produces Good 1 and only gets utility from consuming Good 2. The vice versa applies for agent 2. No barter trade market $(1, 2)$ exists, i.e. agents use fiat money to trade goods. A cash-in-advance constraint applies.

The government of each country issues its fiat money, where 0 represents foreign currency and 0^* domestic currency (all variables for the home country are further denoted with *). The government can buy Good 1 from Agent 1 in exchange for freshly printed domestic currency and can so flood the economy with its currency. The government does not consume Good 2. The government is able to enforce taxes on production, for all its domestic residents, payable in domestic currency.

Each agent can participate in four markets $(0, 1); (0, 2); (0^*, 1); (0^*, 2)$. To give an example, Agent 1 sells his goods on the foreign market $(0, 1)$ to get foreign currency 0, then he has to wait one period to use 0 to buy Good 2 in the foreign market $(0, 2)$.[15]
Positive costs of holding money $(1 + \pi > \beta)$ are assumed. (Matsui, 1998, p.314)
The following figure presents the trading pattern in two countries, where the arrows show the flow of money.

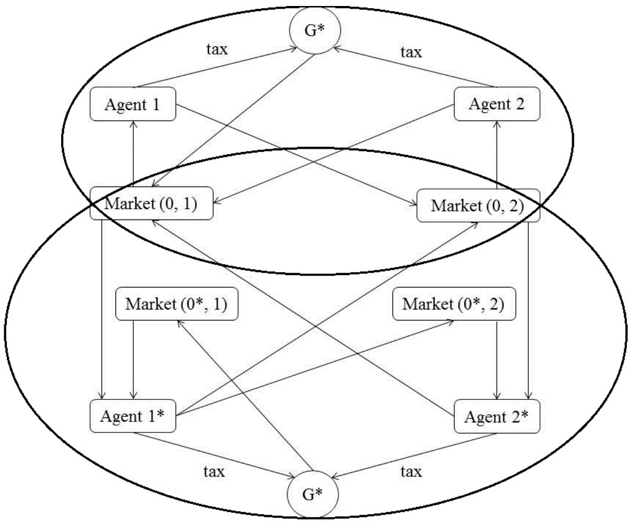

Figure 1: Trading Pattern
(Matsui, 1998, p.319)

[15] In this case the taxes are levied using the price of production in domestic currency.

The upper ellipse in Figure (1) illustrates in a foreign country only foreign currency is used as a means of payment. Foreigners have no need to use the high inflation domestic currency, since they pay production taxes in foreign currency.

The lower ellipse in Figure (1) presents the home country where Agent 2^* sells his goods in the home and foreign market. He sells a (small) share of his production in the domestic market $(0^*, 2)$ to receive high inflation domestic currency to pay taxes to the government (G^*). He sells the rest of his production in the foreign market to receive low inflation currency and buy Good 1 in the next period. Agent 2 only holds the minimum high inflation domestic money necessary for the taxes because of the higher holding costs. Agent 1^* can sell his goods to the government (G^*) over the $(0^*, 1)$-market and foreign market $(0, 1)$. The Agent's 1^* demand for high inflation currency exceeds the amount of taxes he has to pay. Agent 1^* holds more domestic currency than Agent 2^*. Agent 2^* needs a certain amount of domestic currency to pay his taxes while Agent 1^* is the only agent who can supply domestic currency to Agent 2^*, since government G^* does not trade with Agent 2^*. Agent 1^* sells his goods on domestic market $(0^*, 1)$ and spends the received domestic currency on $(0^*, 2)$ market, that is equal to the supply of domestic currency. Agent 1^* needs to be indifferent between markets $(0, 1), (0^*, 1)$ and $(0, 2), (0^*, 2)$. Thus, real relative prices for Goods 1 and 2 have to be equal in both foreign and domestic markets.[16] The following equation has to hold (Matsui, 1998, p.317):

$$\frac{1}{1+\pi^*}\frac{p_1^*}{p_2^*} = \frac{1}{1+\pi}\frac{p_1}{p_2} \tag{3.1}$$

Agent 2^* is willing to offer Good 2 on the domestic market at a real price equal (or cheaper than) the price of Good 2 in the foreign country. The dependence of Agent 2^* on Agent 1^* generates "a positive value for domestic money" for the latter. (Matsui, 1998, p.318)

Figure (1) also shows that the low inflation currency circulates as an international means of payment and if agents from different countries trade they will use foreign currency. Consequently, foreign governments have three sources of tax revenues – production tax revenues from its F-agents, seigniorage collected from its F-agents and seigniorage collected from the agent of the home country (Matsui, 1998, p.318). In contrast, the domestic government only gets tax revenues from production and its own seigniorage. The home country may be incentivised to set the rate of inflation just below that of the foreign country to obtain seigniorage from agents of the latter (Matsui, 1998, p.308).

[16] The relative price of Good 1 in the domestic country ($\frac{p_1}{p_2}$) is higher than the foreign country's, since $\pi > \pi^*$.

3.3.2 Implications

To summarize, the imposition taxes causes the high inflation domestic currency to circulate as a means of payment in the home country, even if there is lower inflation foreign currency available. This in an important issue to study, as almost all economies have governments that enforce its residents to pay taxes in domestic currency. Government also supports the circulation of domestic currency for example by purchasing goods with domestic currency and pay civil servants with domestic currency.

3.4 Problems of Government Restrictions

When a local government finances part of its expenditures by increasing the stock of domestic currency, this leads to inflation. But a relatively high acceptance of domestic currency in trades can prevent currency substitution and thus may prevent a hyperinflation, since a significant percentage of the country's residents regularly use domestic currency. Thus, already small inflation creates big inflation tax revenues (Chang, 1994, p.911).

But improving domestic money's liquidity properties by government restrictions as described in Section (3.1) and in Period 3 of Section (3.2) often create false incentives. They trigger capital flight and simply increase currency transactions in the black market (Vegh and Calvo, 1992, p.8).
Rostowski (1992) describes that government restrictions can lead to a situation in which no attractive means of payment is available in the economy. This has negative impacts on trades and on output of the economy. (Vegh and Calvo, 1992, p.7)

From now on, it is assumed that the government does not intervene in the competition of currencies. Instead, the domestic currency has different advantages compared to the foreign currency, which lead to varying liquidity properties of the currencies.

4 Transaction Costs

Theoretical literature on currency competition highlights transaction costs that allow low return currencies to circulate, in spite of the existence of a high return currency. By introducing transaction costs, the assumption of frictionless markets is relaxed and the currencies are no longer perfect substitutes (Engineer, 2000, p.119). In this section, the transaction costs represent the inconveniences associated with using foreign currency as opposed to domestic currency, for example agents may need to verify the authenticity of foreign currency, undertake costs of obtaining information about the actual exchange rate, spend time and effort dealing with foreign exchange traders (Chang, 1994, p.906).

It is assumed that domestic and foreign currency as means of payment are available. In contrast to the foreign currency, the domestic currency faces lower or no transaction costs. If both currencies have the same return, it is clear agents will use the domestic currency with lower transaction costs. Now, suppose domestic inflation increases, agents will face the following trade-off – if the anticipated depreciation of domestic currency for the period during which they expect to hold their reserves is smaller than the transaction costs of foreign currency, agents will choose to hold the high inflation domestic currency, despite the presence of lower inflation foreign currency (Tullock, 1975, p.493).

In the following two subsections two models with different kinds of transaction costs are considered. First, fixed transaction costs are discussed followed by a study of proportional transaction costs. The different assumptions on transaction costs lead to varying explanations for the rate-of-return dominance puzzle.

4.1 Fixed Transaction Costs

Fixed transaction costs imply that rich agents accept these costs in contrast to poor agents since for rich the costs are relatively smaller. Consequently, rich agents hold the low inflation foreign currency (i.e. they choose to evade the domestic inflation tax) and poor agents choose to hold the high inflation domestic currency (i.e. pay the inflation tax).
Chang (1994) presents an overlapping generation model with fixed transaction costs.

4.1.1 Environment

In Chang (1994) time is discrete and periods are indexed by $t = 1, 2, ...$ At the beginning of each period, a new generation of N rich and n poor agents are born. Each agent born in $t \geq 1$, lives for two periods. In the first period they are young and in the second period they are old.

The model uses a single, perishable consumption good. The domestic price of consumption is p_t. Each young agent $h = r, p$, where r stands for born rich and p for born poor are endowed with an amount of consumption goods e_h, where $e_r > e_p$. Old agents receive no endowment to finance old age consumption. Thus, to finance old age consumption, young agents must sell part of their endowment and acquire non-perishable assets. Non-perishable assets can be taken along periods. Two kinds of non-perishable assets are available:[17] domestic and foreign currency. In order to hold and trade with the foreign currency a domestic agent bears a fixed transaction cost $\delta \in (0, e_p)$.

The government consumes $G \geq 0$ units of the consumption good. It is assumed the government finances its expenditures by printing domestic currency. Consequently, $M_t - M_{t-1} = p_t G$ applies, where M_t is the quantity of domestic currency in period t and the fiscal deficit G is constant over time. The fiscal constraint becomes[18]

$$\left[\frac{\pi}{(1+\pi)}\right] Q = G, \text{ where } \pi = \frac{p_{t+1}}{p_t} - 1 \text{ and } Q = \frac{M_t}{p_t} \tag{4.1}$$

Equation (4.1) illustrates that the fiscal deficit is financed by inflation taxes imposed on the real quantity of domestic money holdings, which forms the base for the tax. (Chang, 1994, p.907)

4.1.2 Choice of Money

Old agents spend all their money on consumption. Young agents choose consumption (c_h^1) and holdings of domestic or foreign money (m and f) in order to consume when they are old in Period Two (c_h^2). Young agents maximize their utility $u(c_h^1, c_h^2) = \alpha \log c_h^1 + (1-\alpha) \log c_h^2$ subject to budget constraints. In Period One when agents are young, the value of the endowment needs to be equal or larger than the value of consumption, domestic and foreign money holdings as well as transaction costs. In Period Two it is required that the value of consumption needs to be smaller or equal to foreign respectively domestic money holdings.[19]

To solve Agent h's maximization problem Chang (1994) differentiates between two cases. If $\pi \leq \pi^*(e_h)$ that means the actual inflation rate is smaller than the inflation rate at which Agent h is indifferent between holding domestic and foreign money. Agent h consumes some of his endowment in Period One and invests the rest in domestic money for consumption in Period Two.

[17] We simplify the model and reduce to the choice between two assets (instead of three like in Chang (1994)).
[18] Appendix (A)
[19] Appendix (A)

Alternatively, if $\pi \geq \pi^*(e_h)$, Agent h consumes some of his endowment in Period One and invests the rest in foreign money, which he uses for consumption in Period Two. He bears the transaction costs δ to evade the cost of domestic inflation. (Chang, 1994, p.908)
The threshold value $\pi^*(e_h)$ decreases in e_h, richer agents choose to evade the inflation tax at lower inflation rates than poor agents. The fixed transaction costs with holding foreign money are relatively lower for wealthier agents.[20] (Chang, 1994, p.909)

4.1.3 Currency Substitution Equilibria

Chang (1994) differentiates between several currency substitution (CS) - equilibria.

Non CS - Equilibrium: No agent, rich or poor, has any incentive to hold the foreign currency. This requires that the inflation rate must be not greater than π_r^*. Since the inflation threshold value π^* is decreasing in wealth, it applies that $\pi_r^* \leq \pi_p^*$. Solving Equation (4.1) for π one obtains:

$$\pi = \frac{G}{(Q - G)} \leq \pi_r^* \tag{4.2}$$

If Equation (4.2) applies, all agents hold the domestic money where the domestic money holdings are:

$$Q_{ns} \equiv (1 - \alpha)\,(Ne_r + ne_p) \tag{4.3}$$

Now G_{ns} is defined by plugging in Q_{ns} into Equation (4.1):

$$G_{ns} \equiv \frac{\pi_r^*}{(1 + \pi_r^*)} Q_{ns} \tag{4.4}$$

For any $G \in [0, G_{ns}]$, a non-CS-equilibrium results. G_{ns} is the upper bound at which rich agents do not substitute domestic currency by foreign currency. (Chang, 1994, p.910)

CS - Equilibrium: Rich agents only hold foreign currency while poor agents hold domestic currency. This equilibrium shows how high and low inflation currencies can coexist; it can explain the rate-of-return dominance puzzle. To achieve this equilibrium the inflation rate must lie between π_r^* and π_p^*. Hence, rich agents hold the foreign currency and poor agents hold domestic currency. Since only poor agents hold the domestic currency, the domestic money holdings and the base for the inflation tax is:

$$Q_{cs} = (1 - \alpha)ne_p \tag{4.5}$$

[20] Appendix (A)

Only the poor agents pay inflation tax, so in this equilibrium an increase of the inflation rate makes the poor worse off.
For any $G \in [\underline{G}_{cs}, \overline{G}_{cs}]$ there is a CS - equilibrium regime which explains the rate-of-return dominance puzzle, where

$$\underline{G}_{cs} \equiv \frac{\pi_r^*}{(1+\pi_r^*)} Q_{cs} \text{ and } \overline{G}_{cs} \equiv \frac{\pi_p^*}{(1+\pi_p^*)} Q_{cs} \tag{4.6}$$

If $G < \underline{G}_{cs}$, the inflation tax is too low for rich agents to pay the transaction costs and evade the tax. If $G > \overline{G}_{cs}$ the inflation tax is so high that even poor people evade the tax and pay transaction costs. Consequently, all agents use the foreign currency and the government cannot obtain any revenue from inflation; G can not be financed. (Chang, 1994, pp.910-911)

Multiple Equilibria: The two results above imply if $G \in [\underline{G}_{cs}, Min\{\overline{G}_{cs}, G_{ns}\}]$ both equilibria with and without CS could appear. $\underline{G}_{cs}$ is strictly less than G_{ns} and $\overline{G}_{cs}$; thus the interval is non-empty. Whether $\overline{G}_{cs}$ or G_{ns} is smaller depends on Q and π. For example, assume $G_{ns} < \overline{G}_{cs}$. This is possible if Q_{ns} is not too big relative to Q_{cs} and π_p^* is sufficiently larger than π_r^*. Then, in the range $(\underline{G}_{cs}, G_{ns})$ a CS or a non-CS equilibrium can arise. The following figure clarifies:

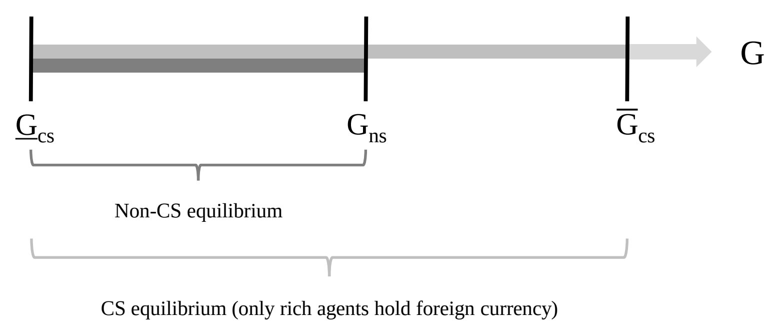

Figure 2: Budget Deficit

In the range $(\underline{G}_{cs}, G_{ns})$ varying expectations of inflation determine which of the two possible equilibrium arises. The following cases are possible:

- If the public expects high inflation, rich agents will hold foreign currency. The base for the inflation tax becomes only the savings of the poor. The inflation rate to finance G becomes larger, confirming expectations of the high inflation rate.

- If the public expects inflation to be low, rich agents will continue to hold domestic money, enlarging the base for inflation tax. The inflation rate needed to finance G turns out to be as low as expected.

This means that CS can emerge even if the fundamentals of the economy – the deficit, are consistent with the equilibria without CS. This is because expectations are self-fulfilling. Thus, inflation causes CS, just as CS causes inflation. (Chang, 1994, p.911)

4.1.4 Implications

The analysis of fixed transaction costs shows that there exists an equilibrium where poor agents hold domestic currency whereas at the same time rich agents hold foreign currency, since the transaction costs are relatively higher for poor agents than for rich agents.
Chang (1994) demonstrates that the non-CS equilibria dominates the CS-equilibria, in the case where multiple equilibria are possible. For rich agents it is more advantageous to hold the foreign currency. The base for the inflation tax shrinks and poor agents, who hold domestic currency, suffer from an increasing inflation. If rich agents would hold domestic currency, inflation would be low enough to justify this choice. Both types of agents will be better off. (Chang, 1994, 912)

Chang's model (1994) assumes the transaction costs are independent of the quantity of money exchanged. By introducing proportional costs into Chang's model (1994), all agents would choose the same currency, because they face the same marginal costs (Engineer, 2000, p.116). Engineer (2000) develops a model in which proportional transaction costs can explain the rate-of-return dominance puzzle.

4.2 Proportional Transaction Costs

Engineer (2000) shares the same assumptions of Chang (1994), that foreign currency has a higher transaction cost and a lower inflation rate. In contrast to Chang (1994), Engineer (2000) also assumes stochastic consumption shocks – in some periods consumption demand is low in others high (Engineer, 2000, p.118). Further, he assumes agents to hold both currencies as means of payment and he considers proportional transaction costs (proportional to the amount of money purchased) in order to hold foreign currency.

Engineer (2000) argues in high inflation countries, the foreign currency substitutes the domestic currency as a store of value, meaning foreign currency is used for large and occasional purchases. However, domestic currency is always used in small daily transactions. Agents do not use foreign currency in small transactions in order to avoid transaction costs. For daily transactions agents prefer the domestic currency since the turn over is very high, thus the loss of real value due to inflation is small (hot potato effect). The low inflation foreign currency is hoarded to make occasional large purchases, since the cost of holding foreign currency due to inflation is small and turn-over rate of large amounts of money is low. Generally, the higher the velocity of a currency, the lower the losses per agent due to inflation. This leads to an increase of demand of domestic currency. Consequently, high velocity leads to the circulation of high inflation domestic currency since the transaction costs of low inflation foreign money are higher.

Engineer (2000) shows the availability of foreign currency has a negative effect on the value of domestic currency (Engineer, 2000, p.130). However, the burden of inflation is reduced to inelastically everyday small transactions. Savings increase as a low inflation currency is available leading to an increase in elastic and occasional large transactions. (Engineer, 2000, p.131).

In Engineer (2000)'s model the rate-of-return dominance puzzle is explained by transaction costs exceeding inflation losses in daily transactions.

4.3 Network Externalities

High level of domestic inflation leads to an increasing use of foreign currency. First foreign currency is used as a store of value and as domestic inflation reaches very high levels, foreign currency is also used as a means of payment.

Uribe (1997) explains that the costs of using foreign currency as a means of payment are decreasing in the "economy's accumulated experience" in using it. Here, network externalities describe a reduction of transaction costs as a consequence of the widespread use of the foreign currency.
Thus, if currency substitution reaches a sufficiently high level during a period of high inflation, people learn how to trade with foreign money. Due to the network externalities the level of currency substitution will persist even if inflation declines (hysteresis-effect). (Uribe, 1997, p.196)

5 Information Frictions

In Section (5.1), (5.2) and (5.3) domestic currency is preferred since more informations about the currency are available. Domestic currency can be better checked for authenticity, domestic inflation is more predictable and debt denominated in domestic currency gives lenders the information of a lower default probability.

5.1 Counterfeit Ability

This section highlights that the ability to counterfeit a currency is important for the discussion of the rate-of-return dominance puzzle. It is assumed that the higher return foreign currency can be counterfeited at a fixed cost and the decision of counterfeit is private information. We revisit the model by Nosal and Rocheteau (2011) introduced in Section (2.1.1).[21] The impact of counterfeit currency is also discussed by Gomis-Porqueras et al. (2014).[22]

5.1.1 Environment

The environment is described in Section (2.1.1). New is the assumption that a buyer can produce any amount of counterfeit foreign currency by incurring a fixed real cost of $\kappa > 0$ between the CM_t and DM_{t+1}. The domestic currency can not be counterfeited.[23] The counterfeits can only be used in trades in the DM_{t+1} since the government detects and confiscates it at the end of DM_{t+1}. Thus, a seller never accepts counterfeit currency as it will be worthless at night.

A seller is unable to distinguish between genuine and counterfeit currency, therefore the decision to counterfeit currency is private information of the buyer. However a seller anticipates that a buyer will offer genuine currency if the cost of counterfeiting (κ) is higher than the "cost of the accumulation of the foreign money" d_2 at the price

[21] The analysis of the impact of counterfeiting ability on the rate-of-return dominance puzzle is contained in Chapter 10.4. of the work Nosal and Rocheteau (2011). They use their benchmark model to analyse how fiat money and government bonds can coexist, when both can be used to settle trading and bonds can be counterfeited.

[22] Gomis-Porqueras et al. (2014) present a search-theoretic model of two currencies to study the determinacy of exchange rates (coexistence of currencies) when agents face the problem of information asymmetry regarding the decision of counterfeiting currency. Both currencies can be counterfeited at a fixed cost. Nosal and Rocheteau (2011) in contrast work with the simplifying assumption that only bonds can be counterfeited. The simpler model of Nosal and Rocheteau (2011) is presented here. The conclusions in both approaches concerning counterfeit ability are the same.

[23] This assumption is justified as agents are familiar with the domestic currency and can identify whether it is genuine. In contrast, agents have more problems identifying whether foreign currency is counterfeited.

ϕ_2. The no-counterfeiting constraint looks as follows:

$$\phi_2 d_2 \leq \kappa \tag{5.1}$$

Every take-it-or-leave-it offer by the buyer must satisfy the no-counterfeiting constraint (5.1), since the seller understands the buyer's incentives and adjusts his rule for acceptance. The inequality (5.1) is an endogenous liquidity constraint arising from the information asymmetry problem. The constraint specifies an upper bound on the quantity of foreign currency accepted from the seller in DM trades. The constraint is relaxed if the cost to counterfeit increases. (Nosal and Rocheteau, 2011, p.275)

5.1.2 Choice of Money

In the CM_t a buyer decides which currency he will use in the DM_{t+1} in order to maximize his trade surplus, net of the cost of holding different currencies. Depending on whether the no-counterfeiting constraint binds or not, agents make different choices regarding their money holdings:

1. Suppose κ is so large that no buyer has the incentive to counterfeit the foreign currency – the no-counterfeiting constraint does not bind.
   Because counterfeiting does not occur, the currency with the lower holding cost is preferred. Since it is assumed that $i_1 > i_2$, only the foreign currency will circulate in the economy. Both currencies only coexist if the return of the currencies are equal. The dominance result applies.

2. Suppose κ is so small that the no-counterfeiting constraint binds, meaning the low inflation foreign currency can easily be counterfeited. In this case, the two currencies coexist as long as the cost of holding domestic currency do not exceed the sum of the cost of holding foreign currency and the probable disutility from getting counterfeits. When a seller accepts foreign currency in trades he can not be sure that the currency is genuine. This potential disutility from accepting foreign currency allows for a higher inflation rate of the domestic currency and thus can explain preferences for high inflation domestic currency. The information asymmetry of counterfeit foreign currency "breaks the curse of Kareken and Wallace" (Gomis-Porqueras et al., 2014, p.31) and explains the rate-of-return dominance puzzle.

5.1.3 Implications

The result presented in the above section implies that the value of fiat money and the extent of its circulation depends on its own characteristics as well as on the physical properties (example the counterfeit ability) of competing means of payment.

The model developed in Nosal and Rocheteau (2011) implies that a monetary authority should be interested in a difficult-to-counterfeit domestic currency. As this lowers the relative advantage of the foreign currency, it allows for a higher inflation rate of the domestic currency without driving it out of circulation. By making the domestic currency relatively difficult to counterfeit, a country can generate a valuable advantage of the domestic currency over the foreign currency and stimulate money demand for domestic currency. This is an interesting implication regarding the problem of dollarization in developing countries e.g. in some South American countries since the U.S. dollar is said to be relatively easy counterfeited.

But like in the Section (4) the network effect in the adoption of foreign currency in times of high domestic inflation can cause hysteresis in money demand (Uribe, 1997, p.196). In periods of very high domestic inflation more and more agents use foreign currency in trades, agent's experience in trading with foreign currency increases – they learn to recognize if foreign currency is counterfeited – this can lead to a permanent high level of currency substitution even if domestic inflation decreases or domestic currency gets more difficult to counterfeit.

5.2 Inflation Variability

Inflation variability is important for the discussion of the rate-of-return dominance puzzle. Inflation variability means different inflation rates are possible in the future. This creates uncertainty about the future's effective inflation rate, which can have a negative impact on agents' utility. Thus, if domestic inflation is predictable and foreign inflation is variable, agents might prefer the domestic currency even if domestic inflation is higher than foreign inflation.

5.2.1 Expected Utility Theory

A classic concept for decision-making under uncertainty is Expected Utility Theory. This theory states, the subjective value associated with a decision by an individual is the weighted sum of the utilities of the different possible outcomes. If an agent can

choose between a certain payment[24] and a real lottery he will choose the alternative which maximizes his expected utility.
The Expected Utility Theory was initiated by Daniel Bernoulli[25] in the 18th Century and was formally developed by John von Neumann and Oscar Morgenstern (1944)[26] in their book "Theory of Games and Economic Behaviour".
The Expected Utility Theory is used in both approaches to determine which currency an agent will prefer.

5.2.2 Flexible Prices and Risk Aversion

This section explains the rate-of-return dominance puzzle by assuming that agents are risk-averse to losses.

5.2.2.1 Assumptions

In this section the following assumptions hold: There exists a domestic and a foreign central bank. The domestic central bank keeps its inflation rate constant over time. The foreign central bank follows a different policy: with different probabilities p_i,[27] different inflation rates can occur. As different inflation rates are possible for the future, the domestic future inflation rate is uncertain. It is assumed the average inflation of the foreign currency is lower than the inflation of the domestic currency.
Two time periods $t = 1, 2$ are considered. Between these periods the money depreciates under the inflation rate. Suppose all agents take some amount of domestic or foreign currency into the next period as they want to ensure future consumption. Agents can freely choose which currency they want to hold and bring into the next period.
In this section no price rigidities exist, meaning all prices are perfectly flexible.

[24] A lottery with an outcome of probability 1 is called a degenerate lottery.

[25] Bernoulli explains the value of something must not be based on its price, but rather the utility it yields. The price is equal for everyone, the utility depends on individual preferences (Bernoulli, 1954, p.24). The following example illustrates why the Expected Utility Theory should be used to calculate the value of a lottery and not the Expected Value Theory. An agent can toss a coin until it lands on "heads". The agent can continue to toss the coin until it lands on "tails" then the game ends. If it lands on "heads" on the first throw the agent gets one dollar, on the second throw he gets two dollars, on the third throw he gets four dollars, on the fourth throw he gets eight dollars and so on... The expected value of this gamble is infinitely great, $\sum_{i=1}^{\infty} \frac{1}{2^i} * 2^{i-1} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + ...$, where i denotes the number of tosses. But no agent is willing to purchase this game for an infinitely high price (Bernoulli, 1954, p.31). This is called the St. Petersburg Paradox. Bernoulli proposed the following solution: Suppose an agent's utility is described by a square root function, so that $\sum_{i=1}^{\infty} \frac{1}{2^i} * \sqrt{2^{i-1}} = \frac{1}{2-\sqrt{2}}$ (Appendix (A)) (Bernoulli, 1954, p.34). By including the expected utility a price for the lottery which is more plausible than the one implied by the expected value can be calculated.

[26] Von Neumann and Morgenstern state if certain axioms of rationality are satisfied, an agent can assign utilities to each outcome of a lottery, such that they choose the gamble which maximizes their expected utility. (Von Neumann and Mogrenstern, 1953, Chapter 3: The Notion of Utility)

[27] $\sum_{i=1}^{n} p_i = 1$ and $p_i > 0$, where n denotes the number of outcomes.

5.2.2.2 Choice of Money

By using the Expected Utility Theory, one can explain why agents who are risk-averse to losses hold the higher inflation domestic currency in spite of the availability of variable but on average lower inflation foreign currency. To follow this approach the thesis will develop a simple numerical example.

It is assumed that the domestic currency has a constant inflation rate of 5% over time. The foreign monetary policy leads to a variable inflation rate of 1% or 9% with equal probability ($p = 1 - p = 0.5$). The outcome of the actual foreign inflation rate can be seen as the result of a simple lottery, whereas the domestic inflation rate is the outcome of a degenerate lottery.
The expected value of the inflation rates of both currencies are equal (Figure (4)). However, it is essential that agents base their decision of which currency to hold on the expected utility and not on the expected value. So for every possible inflation outcome a specific utility is assigned. Note that in this numerical example expected utilities are negative, so agents prefer the currency with the lowest disutility. The sum of the weighted utilities equals the expected utility of a currency:

$$\begin{aligned} E[U(\epsilon)] &= prob_1 \times U(\epsilon_1) + prob_2 \times U(\epsilon_2) + prob_3 \times U(\epsilon_3) + ... \\ &= \sum_{i=1}^{n} prob_i U(\epsilon_i) \end{aligned} \tag{5.2}$$

In equation (5.2), i is the number of outcomes and ϵ is equal to $\frac{1}{1+\pi} - 1$, where π denotes the inflation rate. ϵ therefore represents the return of a currency. For example, if the inflation rate is 5%, then in the next period after depreciation, your money holdings will have a reduced real value of $95.23\% = (\frac{100}{1.05})$ of its original real value in Period 1, so the resulting rate-of-return of money is -4.76%. The return of money always negative, since positive inflation reduces the real value of money. The smaller the inflation rate π, the higher is ϵ (closer to zero) and the lower the disutility. The currency with the lowest disutility is preferred. An "inflation-in-the-utility function" model is constructed, this is a short cut for getting inflation directly valued.

The preferences of agents are represented by $u(\epsilon) = 4 * (\epsilon^3)$. For negative ϵ, this utility function is concave (Figure (3)). Agents with this preference relation are risk-averse to losses.

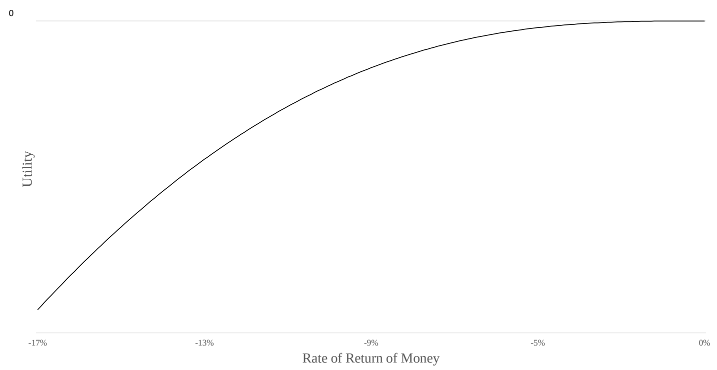

Figure 3: Utility function

The calculations of the expected utilities of the two currencies are presented below:

	Inflation rate	Rate-of-Return of Money	Utility	Probability	Expected Inflation rate	Expected Rate-of-Return of Money	**Expected Utility**
Domestic Money	5%	-4.76%	-0.000432	100%	5%	-4.76%	**-0.0004**
Foreign Money	1%	-0.99%	-0.000004	50%	5%	-4.62%	**-0.0011**
	9%	-8.26%	-0.002252	50%			

Figure 4: Expected Utility

First, one can see that a risk-averse agent decides to hold the domestic currency if the expected values are equal, since the expected utility of the domestic currency is greater than the expected utility of the foreign currency.[28] With informations from Figure (4) it can be calculated by how much the domestic inflation rate can increase until agents are indifferent between the two currencies. Thus, the thesis looks for the domestic inflation rate $\hat{\pi}$ (certainty equivalent) for which the expected utility of the domestic currency equates the given expected utility of the foreign currency. By solving the following equation for $\hat{\epsilon}$ and then using $\hat{\epsilon} = \frac{1}{1+\hat{\pi}} - 1$ one obtains

$$-0.0011 = 4 * \hat{\epsilon}^3 \Rightarrow \hat{\pi} = 0.07 = 7\% \tag{5.3}$$

[28]In contrast, preferences of agents, who are risk neutral to losses, are represented by linear utility functions. These agents are indifferent between the two currencies. If the expected value is equal, the expected utility is equal.
For agents who are risk-loving to losses, preferences are represented by a convex utility function, therefore the foreign currency is preferred – if expected values are equal, then the expected utility of the riskier money is greater.

In this example, domestic inflation can exceed foreign inflation by 2 percentage points. If the domestic inflation rate exceeds foreign inflation by less than 2 percent, only the domestic currency would circulate in the home country. Agents accept a lower return of domestic currency since they are risk-averse and prefer inflation certainty, implying no consumption variability. Agents pay a risk premium (RP) to minimize loss.

This approach is illustrated in the following figure:

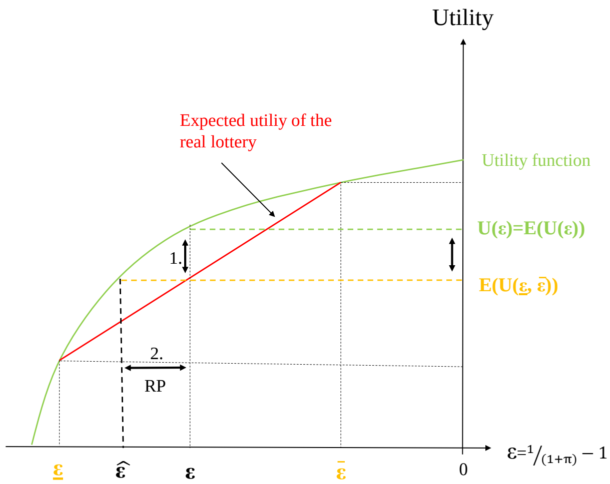

Figure 5: Risk Aversion

High inflation is represented by a highly negative ϵ and leads to a high disutility. On the x-axis $\bar{\epsilon}, \underline{\epsilon}$ represent the two possible returns of the foreign currency and ϵ is the return of the domestic currency. The expected inflation of the real lottery $(\underline{\epsilon}, p_1; \bar{\epsilon}, p_2)$ is equal to the expected inflation of the degenerate lottery $(\epsilon, p = 1)$. The y axis indicates the utility associated with the returns of money. First (1.), one can see that for a risk-averse agent, the (expected) utility from a certain inflation π is higher than the expected utility of variable inflation, when the expected values of the two alternatives are equal. Second (2.), a risk-averse agent accepts a higher domestic inflation, maximum $\hat{\pi}$, which is represented by the return of money $\hat{\epsilon}$, under the condition that the money has no inflation variability. Agents pay a risk premium when they accept higher domestic inflation, since they are risk-averse to losses and want to avoid consumption variability.

5.2.3 Nominal Price Rigidities and Loss Aversion

In this section we re-analyse inflation variability. The presence of nominal price rigidities (New Keynesian View) and loss aversion of agents can explain the rate-of-return dominance puzzle when domestic inflation is predictable and foreign inflation is uncertain.

5.2.3.1 Nominal Price Rigitities

Since Friedman (1977) and Fischer (1977), the negative effect of inflation variability (or inflation uncertainty) on economic variables due to price rigidities has been a topic of considerable interest. Friedman (1977) mentioned higher inflation variability reduces the efficiency of the price system and hinders long-term contracting. This may rise unemployment. As a consequence output is reduced. Fischer (1977) emphasised the problem of inflation variability arises from the presence of nominal price rigidities. Many wages and prices are set in nominal terms and can not be readjusted for a certain period of time. If there is a change in monetary policy, nominal rigidities can lead to real distortions. Fischer (1977) considers nominal wage contracts, "nonindexed labour contracts". Wage contracts are long-term contracts, since the costs of frequent contract negotiations and wage settings are high (Fischer, 1977, p.198).

In the presence of nominal price rigidities, monetary policy has an impact on the economy's output.[29] Consequences of uncertainty about future inflation for economic variables is a popular topic of empirical studies. To give an example, Grier and Tullock (1989) show in a broad sample of countries from 1951 to 1980 that inflation variability has a negative effect on output growth (Grier and Tullock, 1989, p.265). As measure for inflation uncertainty they use volatility, the standard deviation of the mean inflation rate (Grier and Tullock, 1989, p.262).[30]

5.2.3.2 Assumptions

To study the competition of currencies in the presence of nominal price rigidities and to explain the rate-of-return dominance puzzle the following assumptions are made: High inflation domestic currency as well as low inflation foreign currency are available. The inflation rate of domestic currency is assumed to be predictable (in this thesis the

[29] In contrast, if wage contracts can be continuously adjusted monetary policy is irrelevant for output, as money is neutral (Fischer, 1977, p.197).

[30] The problem of using volatility is that inflation may be variable and predictable, so volatility probably systematically overstates the level of uncertainty (Grier and Grier, 2006, p.481).

assumed rate is 10%), contrary to foreign currency, where agents expect several inflation rates to be possible (in this thesis it is 2% and 8% with equal probability).[31] The model consists of two time periods (t_1, t_2) between which the currencies depreciate. There are two types of agents – employers and employees that meet in wage negotiations in Period One (t_1) to fix the nominal wage. In t_1 the real wage is equal to the nominal wage, which is 100 and the price level is 1 ($p_1 = 1$). The price level will increase due to inflation until Period Two (t_2). Agents take this into account resulting in a higher nominal wage for t_2. In t_2 wage payment and consumption proceed.

5.2.3.3 Wage Setting

The goal of employers and employees is to minimize the deviation of expected from effective inflation. For an employee it is important he does not underestimate inflation, since underestimation leads to decreased consumption in t_2 – the real wage would decrease. For an employer it is essential that inflation is not overestimated in wage negotiations. This would lead to higher costs in terms of too high wage payments – the real wage of an employee would increase.

A numerical example follows to illustrate the wage setting process.
If agents choose domestic currency, the calculation of the nominal wage paid in t_2 is simple. As inflation is predictable and known to be 10%, agents will agree on a nominal wage in t_2 of 110.[32] Neither employees nor employers will profit or lose.
If agents decide to fix the nominal wage using foreign currency, the calculation of the nominal wage rate becomes more difficult. Agents expect the inflation rate to be 2% and 8% with equal probability. The expected inflation rate is 5%, but employers will not agree on a nominal wage rate of 105, because a nominal loss of 3 occurs in the case of $P_2 = 1.02$ equals -2.94 in real terms while a profit of 3 at $P_2 = 1.08$ only equals 2.78 in real terms.[33] The increase of the nominal wage must on average compensate for the purchasing power loss, which is equal to the expected rate of return of the foreign money. Nominal wage must rise by 4.91%.[34] Figure (6) shows that at a nominal wage of 104.91 neither employees nor employers are worse off.

[31] From the point of view of an investor who uses money to store value, holding foreign money is obviously the better choice.
[32] The increase of the nominal wage is equal to $\frac{1}{1+(-0.09)} - 1 = 0.1 = 10\%$.
[33] $\left|\frac{-3}{1.02}\right| > \frac{3}{1.08}$
[34] If inflation is 2%, the return of money is $\frac{1}{1.02} - 1 = -1.96\%$, if inflation is 8% the return is $\frac{1}{1.08} - 1 = -7.41\%$. Thus, the expected return of foreign money equals -4.68%. The nominal wage that compensates for this negative average return is 104.91, since $\frac{1}{1-4.68} - 1 = 4.91\%$.

Real Payoffs at a nominal wage of 105

	Employee	Employer
$P_2 = 1.02$	2.94	-2.94
$P_2 = 1.08$	-2.78	2.78

Real Payoffs at a nominal wage of 104.91

	Employee	Employer
$P_2 = 1.02$	2.86	-2.86
$P_2 = 1.08$	-2.86	2.86

Figure 6: Wage Setting

5.2.3.4 Loss Aversion

To explain why the high inflation domestic currency is preferred as a means of payment, the expected utility theory from (5.2.1) is considered and loss aversion is assumed. Loss aversion means that agents' disutility coming from the loss of an amount x is greater than their utility from a profit of the same amount x. Loss aversion is part of the Prospect Theory developed in Kahneman and Tversky (1979) and can be illustrated as follows:

$$u(x) = \begin{cases} x^{\alpha} & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -\lambda(-x)^{\alpha} & \text{if } x < 0 \end{cases} \tag{5.4}$$

$\lambda > 1$ states an agents is loss averse.

In the numerical example preferences are represented by the following value function:

$$u(x) = \begin{cases} x^{\frac{1}{3}} & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -4(-x)^{\frac{1}{3}} & \text{if } x < 0 \end{cases} \tag{5.5}$$

Equation (5.5) expresses risk aversion to profits and risk love for losses. However, risk preferences (risk aversion, risk loving, risk neutral) are not crucial to the choice of money, instead loss aversion ($\lambda > 1$) matters.

The function is represented in the following figure:

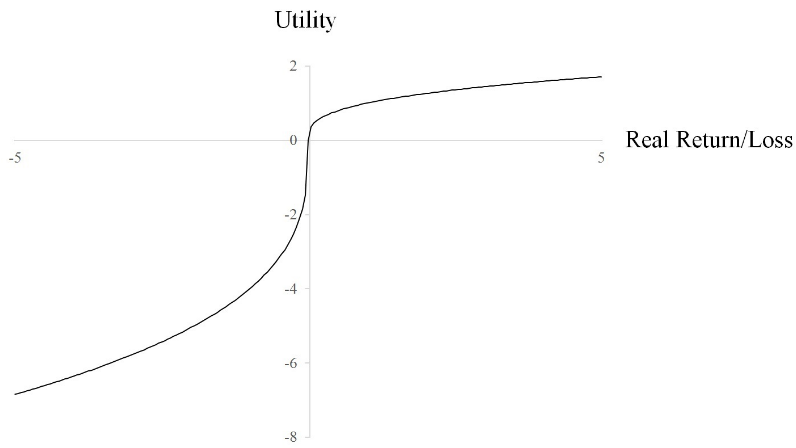

Figure 7: Loss Aversion

5.2.3.5 Choice of Money

Figure (8) illustrates why an employee will always choose the high inflation domestic currency.[35] The figure shows agents have no profit or loss when using domestic currency as means of payment and therefore no utility. If foreign currency is used agents would have asymmetric payoffs.[36]

	Inflation rate	Price Level t_2	Nominal wage t_2	Real Wage t_2	Probability	Profit / Loss *[x]*	Utility *[u(x)]*	**Expected Utility**
Domestic Money	10%	1.10	110.00	100.00	100%	0.00	0.00	**0.0000**
Foreign Money	2%	1.02	104.91	102.86	50%	2.86	1.42	**-2.12848**
	8%	1.08		97.14	50%	-2.86	-5.68	

Figure 8: Loss Aversion

Since all agents are loss averse, the expected utility of domestic currency is higher than foreign currency. In this model utility and disutility depend on the deviation of the expected from the effective inflation rate. The presence of nominal price rigidities

[35] For the employer expected utility is equal, but the signs in the columns Profit/Loss and Utility are reversed.

[36] When employees earn 104.91 at a price level of only 1.02 their salary is too high about $104.91 - 102 = 2.91$ which is in real terms $\frac{2.91}{1.02} = 2.86$. That is equal to the loss of the employers. The reverse applies for a price level of 1.08.

gives the high inflation domestic currency value as its inflation rate is predictable. Low inflation foreign currency can create profits and losses as it is assumed agents cannot exactly predict the inflation rate and therefore estimate it by weighting different outcomes.

5.2.4 Implications

Inflation variability contributes to the understanding of the rate-of-return dominance puzzle. To explain the puzzle, it is assumed that the domestic inflation is high but predictable and foreign inflation is low but variable. The assumption of higher uncertainty of foreign inflation is reasonable as residents are familiar with their economy and monetary policy, thus they can precisely estimate domestic inflation. But they are less able to identify macro shocks or understand the consequences of government elections in a foreign country, implying more uncertainty associated with foreign monetary policy. However, this assumption is controversial as some empirical studies illustrate that a positive relationship between inflation and the degree of inflation uncertainty exists. For example, Ball (1992) shows that high inflation raises inflation uncertainty.

The Expected Utility Theory is used in this thesis since it demonstrates how agents make decisions under uncertainty. In Section (5.2.2) flexible prices and risk aversion to losses are assumed. It is illustrated why it is economically rational for agents to prefer the high inflation domestic money. The explanation for the rate-of-return dominance puzzle only holds for limited average inflation differences.
In Section (5.2.3) prices rigidities and loss aversion are assumed. Inflation itself is not problematic for agents, it is the inflation variability which brings disadvantages, since variability can lead to real price distortions. This is why monetary policies should reduce inflation volatility and seek price stability, even if the level of inflation is relatively high. If the monetary authority needs to change its policy, it should be announced in advance, so wage setters can take future inflation into account when setting wages.

The actual modelling of these two approaches in the Setting of Lagos and Wright (2005) is an interesting work for further research.

5.3 Enforcement Frictions

This section presents an economy with financial intermediation, where the high inflation of the domestic currency can itself be the source of why it circulates more widely than an available lower inflation foreign currency (Breu, 2012, p.3)

In this section money and credit coexist. This works as it is assumed that financial intermediaries such as banks, who accept deposits and make loans have a record-keeping technology to keep track of financial history. But, there is no record-keeping of anonymous good trades. The existence of financial record-keeping allows bilateral credit between an agent and bank but does not eliminate the need for money as a means of payment for trades between agents in the goods market. (Berentsen et al., 2007, p.2)

Inflation may function as a commitment device, since the gain of defaulting is decreasing and the cost of defaulting is increasing with inflation. The low incentive to default when debt is denominated in high inflation currency reduces the actual cost of holding high inflation currency, since the relaxation of the buyer's borrowing constraint allows for high deposit interest rates (Breu, 2012, p.2).

This approach is developed in Breu (2009) and (2012). The model of Breu is an extension of the model in Berentsen et al. (2007).[37] The framework is based on the divisible money model developed in Lagos and Wright (2005).

5.3.1 Environment

In Breu (2012) time is discrete and continuous. Every period is divided into two sub-periods – two competitive markets open sequentially. Agents discount across periods with $\beta \in (0, 1)$. In the economy exist a continuum of infinitely-lived agents and two types of divisible and perishable goods – market goods (produced by others) and home-made goods (produced by themselves). (Breu, 2012, p.4)

In every trade, agents can freely choose between two currencies as means of payment – a high return foreign currency with a depreciation rate of γ_f and a low return domestic currency with a depreciation rate of γ_h.[38] The domestic (or foreign) central bank

[37] Berentsen et al. (2007) present a one country- , one currency-model, where money and credit coexist. They show welfare can increase as inflation increases as credit rationing occurs in an environment with no credit enforcement. A positive inflation is optimal since it makes default less attractive to borrowers, this reduces credit rationing (Berentsen et al., 2007, p.21). Inflation reduces the incentive to default since the punishment of default (no future access to the financial market) is worse when inflation is high. Thus, increasing welfare, due to increasing inflation, arises from paying interest on deposits.
But in Breu (2009) and (2012) two currencies with different returns are available and deposits denominated in both currencies pay corresponding interests. Thus in Breu (2009) and (2012) with no enforcement, if agents default they would further only use the high return foreign currency which allows for high consumption without having access to the banking system. Therefore, domestic inflation does not alter the incentive to default if a lower inflation foreign currency is available. The rate-of-return dominance puzzle can not be explained. This is why in Breu (2009) and (2012) a limited enforcement environment is developed, in which the high inflation domestic currency is preferred because a relaxation of the borrowing constraint allows for higher deposit interest rates, such that compensation for domestic money deposits is higher than compensation for foreign money deposits. This is crucial to explain the rate-of-return dominance puzzle.

[38] In Breu (2009) two types of fiat money are available as a means of payment. In Breu (2012) the coexistence

controls the supply of the domestic (or foreign) money.[39] The depreciation across periods is $\frac{M_{+1}}{M} = \gamma_h \geq \gamma_f \geq \beta$, meaning the depreciation or inflation across periods is higher for the domestic than for foreign currency. (Breu, 2012, p.5)

5.3.2 Chronology of Events

The chronology of events is as follows:

$t-1$	t		$t+1$
	1^{st} market	2^{nd} market	
	1^{st} shock → Banks → 2^{nd} shock → Trade	Trade Repayment	Transfers

Figure 9: Timing of events
(Breu, 2012, p.6)

In the first market four things happen.
First, an idiosyncratic preference shock determines which agents are consumers and which are sellers, where the probability of being a seller is 0.5 (Breu, 2012, p.4).
Next, competitive banks, who accept deposits and make loans, open.[40] Sellers decide how much they want to deposit d_j (conditioned that $d_j \leq j$) and consumers decide how much to loan l_j (subject to the borrowing constraint), where $j = f, h$. f denotes foreign currency and h domestic currency. (Breu, 2012, p.5)
After banks close, consumers get a second preference shock – with some probability agents will only get utility from consuming home-made goods. These agents are called home-consumers. The rest of the agents only get utility from consuming the market good. These agents are called buyers. (Breu, 2012, p.5)
Trade then occurs. Sellers decide how much they produce in exchange for the low and high return currency. Consumers choose consumption quantity q, where their choice is subject to the borrowing constraint and to the condition that consumption can not exceed the amount of money and loan. Buyers get utility $u(q)$[41] from consuming market goods q. Home-consumers get utility q when they consume home-made goods q. For sellers and home consumers productions costs are $c(q) = q$. (Breu, 2012, pp.7-8)

of fiat money and a real asset as means of payment are considered. In this thesis the author analysis the coexistence of two currencies like in In Breu (2009).

[39] Agents receive lump-sum transfers $T = (\gamma_j - 1)M_{-1}$ after the second market in Period t.

[40] Banks do not issue its own money, they use money issued by the central banks.

[41] Where $u'(q) > 0, u''(q) < 0, u'(0) = \infty, u'(\infty) = 0$.

Now, the second market opens. All agents can produce both types of goods. Agents choose good x consumption,[42] working hours h for production of good x and the amount of money they want to take into the next period. This decision is constrained by their earnings which must be equal to or larger than their expenditures. (Breu, 2012, p.7)
The repayment of loans and corresponding interest rate take place in the second market: sellers receive their deposits plus interest rate $(1 + i_j^d)d_j$. Creditors may repay loans plus interest rates $(1 + i_j^l)l_j$. The disutility from producing x is h, where one unit of h generates one unit of x. (Breu, 2012, p.5)
Banks face an exogenous level of reserve requirements $r = \frac{\text{deposits}}{loans}$ (where $0 \leq r \geq 1$). For each unit loaned they must hold a proportion r in deposits. The record-keeping on borrowers by banks cause costs κ per unit of asset loaned. The non profit condition on banks is: $i_j^l - ri_j^d - \kappa = 0$, where borrowers pay interest i_j^l to the bank and depositors get ri_j^d from the bank. Banks maximize l_j in consideration of the borrowing constraint[43] and the participation constraint of borrowers[44]. (Breu, 2012, p.9)

5.3.3 Limited Enforcement

The key assumption of this model is that the enforcement of repaying loans by banks is limited. Banks can only force borrowers to repay their loan when they voluntarily enter the second market. This enforcement power can only ensure repayment temporarily, that is in the period in which the loan is issued. (Breu, 2012, pp.5-6)
Given the assumption on preferences, buyers always choose to enter the second market, since they want to consume the market good. In contrast, home-consumers do not desire to consume the market good as they consume their own production. Thus, the home-consumer can skip the second market to avoid repayment of a loan. Banks set a borrowing constraint such that these agents do not default. Banks choose l_j such that the gain of repaying the loan $v(l_j)$ is at least equal to the gain of defaulting $\hat{v}(l_j)$. The endogenous borrowing constraint $v(l_j) \geq \hat{v}(l_j)$ is given by

$$U(x) - h_c + \beta V_{+1}(f, h) \geq U(\hat{x}) - \hat{h}_c + \beta \hat{V}_{+1}(f, h) \tag{5.6}$$

The RHS of Equation (5.6) shows the pay off of a defaulter, where $\hat{h}_c$ are the working hours in the second market in the period of default and $\hat{V}_{+1}(f, h)$ is the expected lifetime value of a defaulter. The LHS shows the pay off of a non-defaulter.
(Breu, 2012, pp.11-12)

[42] Buyers (and home-consumers) and sellers get utility $U(x) = ln(x)$ from consuming market goods (respectively home-made goods) x.
[43] The pay off of a defaulter needs to be smaller or equal than the pay off of a non-defaulter: $v(l_j) \geq \hat{v}(l_j)$.
[44] The pay off of a borrower from receiving a loan needs to be at least equal to the pay off to another bank.

The gain of defaulting is given by a lower working effort in the period of default (that is, more leisure). A defaulter saves working hours in the period of default, since he does not need to work to pay the loan interest rate and can use the loan in the following period to purchase goods: $\frac{l_j}{\gamma_j}$. In a limited enforcement environment inflation reduces the incentive to default – the higher the inflation of a currency, the lower the gain of defaulting, since the loan is less valuable at the moment in which it can be used to purchase goods. (Breu, 2012, p.13)
The cost of defaulting is given by the exclusion of the banking system in the future. This implies lower utility from future consumption if inflation is high, since consumption without having access to the banking system is lower (Breu, 2012, p.13). If inflation is high, agents value access to the banks because they want the bank's insurance against inflation (Berentsen et al., 2007, p.20).

5.3.4 Choice of Money

Now domestic money equilibria, meaning equilibria where agents choose domestic money, are analysed. In order for such an equilibrium to exist when $\gamma_h > \gamma_f$, there must be a gain of using domestic money that is equal or higher than the gain of using foreign money (Breu, 2012, p.18). A domestic money equilibrium with $\gamma_h > \gamma_f$ exists if (1)Enforcement technology is limited; (2)The borrowing constraint binds; (3) The level of banks' reserve requirement is not too high and (4)Agents are neither too patient nor too impatient. (Breu, 2012, p.14)
The discussion of the conditions is as follows:

- **1. Condition – Limited Enforcement:** The equilibrium exists if banks are only able to enforce debts by agents who trade in the second market. High inflation of the currency in which the debt is denominated, lowers the value of the loan (that is, the gain of defaulting is small), since the beneficiary has to wait until he can use the loan. The decision of the buyer to default is affected.[45]
- **2. Condition – Binding Borrowing Constraint:** In general, when inflation increases, the deposit rates increase to compensate agents for the higher cost of money holdings. However, in a constrained equilibrium (binding borrowing constraint) banks worry about default, so they set the loan interest rate below the market-clearing level to prevent default, that is credit rationing (Berentsen et al., 2007, p.12). Increasing domestic inflation reduces the incentive to default

[45] In Breu (2009) an environment without enforcement and a binding borrowing constraint is discussed. As domestic inflation increases, buyers switch to foreign money. So the decision to default depends only on γ_f. Increasing domestic inflation does not relax the borrowing constraint since it does not affect the incentive of a defaulter. Thus, the dominance result applies. (Breu, 2009, p.18)

on debt denominated in domestic currency, since the gain of defaulting is small and the punishment is severe. This allows loan interest rates to be closer to the market clearing level. Thus, an additional channel through which domestic deposit interest rates increase exists. Higher loan interest rates are reflected in higher deposit rates to satisfy the zero-profit condition of banks. The borrowing constraint on loans denominated in domestic currency is less binding than the borrowing constraint on loans denominated in foreign currency. As a result, the marginal cost of holding domestic currency decreases for depositors as inflation increases, if deposit rates are taken into account. (Breu, 2012, pp.14-15)

- **3. Condition – Low Bank Reserve's Requirements:** Requiring a low r ensures that for borrowers, the depreciation of the loan across the period is sufficiently high compared to the increase in interest i_h^l (gain from defaulting). It can be verified mathematically that $\frac{\partial i_h^l}{\partial \gamma_h} > 0$, but is decreasing in r.[46]
A low r also ensures consumption is increasing in inflation, $\frac{\partial q}{\partial \gamma_m} > 0$. With higher consumption q, the expected lifetime utility and welfare increase.
(Breu, 2012, p.15)
- **4. Condition – Patience:** Agents should not be too impatient – they have to value future consumption (the access to the banking system) relative to the present cost of repayment (Breu, 2012, p.14). Agents should also not be too patient. Since if they are very patient the borrowing constraint does not bind, since they exaggerate the value of future consumption and never default (Breu, 2012, p.15).

5.3.5 Implications

In Breu (2012) it becomes clear why the low return domestic currency is not driven out of the banking system: The use of the high inflation currency is a commitment device for borrowers in a limited enforcement environment. Agents anticipate that the borrowers' incentive to repay loans is higher if the loan is denominated in the higher inflation currency. It provides a higher compensation for their deposits, since the borrowing constraint is less binding on loans in terms of domestic currency than on loans in terms of foreign currency.

[46]Berentsen et al. (2007) do not require the condition on the level of reserve requirements for inflation to increase consumption because agents cannot choose between two currencies. Agents cannot switch to a lower return money. As a result in Berentsen et al. (2007), increasing inflation allows for a stronger increase in loan rates without increasing the incentive to default unlike Breu (2012).

6 Conclusion

The aim of this thesis is to understand why high inflation domestic currency is not driven out by low inflation foreign currency. The models in this thesis present different kind of frictions, which induce disadvantages using foreign currency. The frictions alter the liquidity properties in favour of the high inflation domestic currency, thus it can happen that domestic currency is preferred even if domestic inflation is higher than foreign inflation. A trade-off exists between accepting the disadvantage of domestic currency (inflation) and the disadvantages of foreign currency. Disadvantages of foreign currency can arise from legal restrictions, higher transaction costs, higher probability of counterfeit, higher uncertainty about future inflation, higher incentive of default and possibly other reasons.

High domestic inflation always involves the incentive to substitute domestic currency by foreign currency. In spite of the presented reasons that allow domestic inflation to exceed foreign inflation, the domestic monetary authority needs to be careful, since the disadvantages of using foreign currency as a means of payment tend to decrease with the "economy's accumulated experience" in using it in trades (Uribe, 1997, p.185). The best way to prevent currency substitution in the long term is an independent monetary authority that does not finance government expenditures by increasing the monetary base excessively.

To pursue the subject of this thesis further, one could expand the list of explanatory models, seek more empirical evidence or integrate the different modelling approaches consistently into a single model (e.g. into the setting of Lagos and Wright (2005)).

This thesis presented a summary of work on frictions that explain aspects of the rate-of-return dominance puzzle. The challenge of Hicks is not yet completely resolved, but substantial steps have been accomplished towards elucidating this central issue in the pure theory of money.

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A Appendix

Buyer's Maximization Problem: Derivation of Equations (2.1) **and** (2.4)

The following calculations are based on the work of Nosal and Rocheteau (2011), Chapter 4. The thesis only considers the CM and DM value functions of a buyer, the seller's CM and DM function can be looked up in Nosal and Rocheteau (2011).

CM Value Function: The expected life time utility of buyers entering the CM with portfolio (m_1, m_2), where m_1 denotes domestic money and m_2 foreign money, is given by:

$$\begin{aligned} W_t^b(m_1, m_2) &= \max_{(x,l,\hat{m}_1,\hat{m}_2)} [u(x) - l + \beta V_{t+1}^b(\hat{m}_1, \hat{m}_2)] \\ \text{s.t. } & \phi_{1,t}\hat{m}_1 + \phi_{2,t}\hat{m}_2 + x = l + \phi_{1,t}m_1 + \phi_{2,t}m_2 \end{aligned} \tag{A.1}$$

The constraint shows the buyer finances his end-of-period-money balances $(\hat{m}_1, \hat{m}_2)$ and the general good consumption x with the production l (=Working hours) and money balances brought into the market (m_1, m_2).
By plugging the budget constraint into the CM value function one obtains:

$$W_t^b(m_1, m_2) = \phi_{1,t}m_1 + \phi_{2,t}m_2 + \max_{\hat{m}_1,\hat{m}_2} [-\phi_{1,t}\hat{m}_1 - \phi_{2,t}\hat{m}_2 + \beta V_{t+1}^b(\hat{m}_1, \hat{m}_2)] \tag{A.2}$$

The value function of the buyer at the beginning of the CM is linear in m_i, where $i = 1, 2$. The linearity property allows to obtain a degenerate distribution of money holdings at the beginning of each period, despite idiosyncratic trading shocks in the DM, since the choice of money holdings $\hat{m}_i$ is independent of wealth m_i (Nosal and Rocheteau, 2011, p.63).

Terms of Trade in the DM: In the DM a buyer chooses an offer (q, d) which maximizes his expected utility subject to the seller's participation constraint (where W_t^s denotes the CM value function of a seller) and the buyer's budget constraint:

$$\begin{aligned} &\max_{q,d} [u(q) + W_t^b(m_1 - d_1, m_2 - d_2)] \\ &\text{s.t. } -c(q) + W_t^s(m_s + d_i) \geq W_t^s(m_s) \\ &\text{and } d_i \leq m_i \end{aligned} \tag{A.3}$$

Since the value function W_t^b and W_t^s are linear, one can simplify to:

$$\begin{aligned} &\max_{q,d\leq m}[u(q) - \phi_{i,t+1}d_i] \\ &\text{s.t.} \quad -c(q) + \phi_{i,t+1}d_i \geq 0 \end{aligned} \tag{A.4}$$

The assumption of the buyer's take-it-or-leave-it offer ensures the seller receives no surplus $-c(q) + \phi_{i,t+1}d_i = 0$. When positive costs of holding money are assumed, $\hat{m}_i = d_i$ this means a buyer spends all the money he takes into the the DM (Nosal and Rocheteau, 2011, p.67).

The solution for q is given by:

$$q = \begin{cases} q^* \\ c^{-1}(\phi_{i,t+1}\hat{m}_i) \end{cases} \quad \text{if } \phi_{i,t+1}\hat{m}_i \begin{cases} \geq \\ < \end{cases} \quad c(q^*) \tag{A.5}$$

The buyer can only obtain the socially efficient quantity q^* if his real money balances are large enough to compensate the seller for his disutility from production.
(Nosal and Rocheteau, 2011, pp.63-65)

DM Value Function: It is important to note for the derivation of the buyer's maximization problem Equation (2.1) in Section (2.1), one must ignore α_{m1} and set it equal to zero. $\alpha_{m1} > 0$ only applies for the derivation of the maximization problem Equation (2.4) in Section (2.2).
The expected lifetime utility of a buyer in the DM holding (m_1, m_2) is given by:

$$\begin{aligned} V_t^b(m_1, m_2) =& \alpha[u(q) + W_t^b(m_1 - d_1, m_2 - d_2)] + (1 - \alpha_b)W_t^b(m_1, m_2) \\ &+ \alpha_{m1}[u(q) + W_t^b(m_1 - d_1)] + (1 - \alpha_{m1})W_t^b(m_1) \end{aligned} \tag{A.6}$$

By multiplying and using the linearity and independence property we get:

$$\begin{aligned} V_t^b(m_1, m_2) =& \alpha[u(q) - c(q)] + W_t^b(m_1, m_2) \\ &+ \alpha_{m1}[u(q) - c(q)] + W_t^b(m_1) \end{aligned} \tag{A.7}$$

Maximization Problem: Now one can find $V_{t+1}^b(\hat{m}_1, \hat{m}_2)$ and plug it into the CM value function. The buyer's maximization problem looks as follows:

$$\begin{aligned} \max_{\hat{m}_1, \hat{m}_2} & - \phi_{1,t}\hat{m}_1 - \phi_{2,t}\hat{m}_2 + \beta[\alpha(u(q) - c(q)) + \phi_{1,t+1}\hat{m}_1 + \phi_{2,t+1}\hat{m}_2] \\ &+ \beta[\alpha_{m1}(u(q) - c(q)) + \phi_{1,t+1}\hat{m}_1] \end{aligned} \tag{A.8}$$

By rearranging terms we get Equations (2.1) and (2.4). Now we can determine the buyer's choice of money holdings in the CM.

Transaction Costs

Derivation of the Fiscal Constraint (4.1)

$$M_t - M_{t-1} = p_t G \tag{A.9}$$

Recognize that $M_t = (1+\pi)M_{t-1}$, consequently is $M_{t-1} = M_t \frac{p_t}{p_{t+1}}$. By plugging this expression into Equation (A.9) and dividing by p_t one gets:

$$\left(1 - \frac{p_t}{p_{t+1}}\right)\frac{M_t}{p_t} = G \tag{A.10}$$

Expand by $\frac{p_{t+1}}{p_{t+1}}$ and rearrange terms to obtain Equation (4.1).

The Decision Problem of a Young Agent The preferences by each agent born in $t \geq 1$ is given by a Cobb-Douglas Utility Function

$$u(c_h^1, c_h^2) = \alpha \log c_h^1 + (1-\alpha)\log c_h^2, \text{ where } \alpha \in (0,1) \tag{A.11}$$

Subject to the budget constraint in real terms, where $q = \frac{m}{p_t}$ are the domestic money holdings and f are the real foreign money holdings. c^1 is the consumption in Period 1 and c^2 of Period 2.

$$\begin{aligned} & c^1 + q + f \leq e_h - \beta \\ & c^2 \leq \frac{q}{(1+\pi)} + f \\ & c^1, c^2, q, f \geq 0 \end{aligned} \tag{A.12}$$

The solution of agent h's utility maximization problem is

- If $\pi \leq \pi^*(e_h)$; $q = (1-\alpha)e_h, f_h = 0, c_h^1 = \alpha e_h, c_h^2 = \frac{(1-\alpha)e_h}{1+\pi}$
- If $\pi \geq \pi^*(e_h)$; $q = 0, f_h = c_h^2 = (1-\alpha)(e_h - \beta), c_h^1 = \alpha(e_h - \beta)$

For the definition of the threshold value $\pi^*(e_h)$, one can solve the following equation for $\pi^*(e_h) =: \pi^*$:

$$\begin{aligned} \alpha \ln(\alpha\, e_h) + (1-\alpha) \ln\left(\frac{(1-\alpha)\ e_h}{(1+\pi^*)}\right) &= \\ \alpha \ln\big(\alpha\ (e_h - \beta)\big) + (1-\alpha) \ln\big((1-\alpha)\ (e_h - \beta)\big) \end{aligned} \tag{A.13}$$

The left side of Equation (A.13) represents the utility of an agent if he only holds domestic currency. The right hand side gives utility if an agent only uses foreign currency.

This leads to:

$$\pi^*(e_h) = \left(1 - \frac{\beta}{e_h}\right)^{\frac{-1}{(1-\alpha)}} - 1 \tag{A.14}$$

Riskaversion

Bernoulli

$$\begin{aligned} \sum_{i=1}^{\infty} \frac{1}{2^i}\sqrt{2^{i-1}} = \sum_{i=1}^{\infty} \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2^i}}{2^i} = \sum_{i=1}^{\infty} \frac{1}{\sqrt{2}} \cdot \left(\frac{1}{\sqrt{2}}\right)^i = \\ \left(\frac{1}{\sqrt{2}}\right)^2 \sum_{i=0}^{\infty} \left(\frac{1}{\sqrt{2}}\right)^i = \frac{1}{2} \cdot \frac{1}{1 - 1/\sqrt{2}} = \frac{1}{2-\sqrt{2}} \end{aligned} \tag{A.15}$$

Plagiatserklärung

Ich bezeuge mit meiner Unterschrift, dass meine Angaben über die bei der Abfassung meiner Arbeit benutzten Hilfsmittel sowie über die mir zuteil gewordene Hilfe in jeder Hinsicht der Wahrheit entsprechen und vollständig sind.
Ich habe das Merkblatt zu Plagiat und Betrug vom 22. Februar 2011 gelesen und bin mir der Konsequenzen eines solchen Handelns bewusst.

Datum:

Unterschrift