Master's Thesis

Monetary Policy, Financial Deepening and the Market for Ideas

Chair of Economic Theory

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Abstract

In this thesis we create a framework which allows to analyse the issues faced by young Entrepreneurs that want to build start-up companies and Investors seeking to invest in such companies. Therefore, we build an overlapping-generations model with a production function that requires knowledge and labour as inputs. By analysing the equilibrium outcome of the model, we find that real money holdings of Investors are important in determining whether the efficient solution can be reached. This friction can't be overcome by an active monetary policy, as a growing or shrinking money stock only drives the equilibrium further away from the efficient solution. In the absence of full commitment, however, a sufficiently high inflation rate is necessary to prevent the economy from breaking down.

Contents

1 Introduction

In any economy, it is crucial that people with ideas get access to the funds they need to realize these ideas. While financing for listed companies works pretty smooth nowadays thanks to financial markets which allow agents to trade all kinds of risk or shares of a company, start-ups and other small enterprises do not get access to these financial markets and are therefore facing much more problems when trying to raise funds. However, such start-up companies are a very important part of the economy since they are often more innovative and dynamic than listed companies, which makes them an important factor in the process of economic growth.

The main reason why it is difficult for a young Entrepreneur with a good idea to find Investors for his start-up company is that he has nothing to offer except a promise. He has not yet build up a reputation, he typically has no funds of his own to invest or to use as collateral and he does not yet have a running business to prove how well his idea is working in reality. Because of this, such young Entrepreneurs can not get their funds from stock markets or financial intermediaries, and therefore they have to rely on private Investors who have the knowledge to determine whether or not this business idea has the potential to be profitable in the future.

The goal of this master's thesis is to build a model that incorporates features able to reflect the issues described in the first two paragraphs. The model used is based on an overlapping-generations (OLG) structure and works pretty straightforward, but is able to capture some important features of the typical problems faced by a young Entrepreneur and produces some interesting results. The building blocks for the model stem partly from the typical OLG literature (see for example [Wallace, 1980]). Apart from that, the model also incorporates some features of the search literature, from which especially [Kiyotaki and Wright, 1989] as a starting point and [Lagos and Wright, 2005] as a basic model for monetary economics are important in this context.

This thesis builds on another master's thesis with the title "An Overlapping

Generations Model of the Search Market for Ideas" [Bünter, 2013]. That thesis was written by Noemi Bünter in spring 2013 under the supervision of Professor Aleksander Berentsen. In Bünter's thesis, the Entrepreneurs need knowledge from Investors in order to be able to produce consumption goods. In this thesis I reproduce Bünter's model with different assumptions on the bargaining protocol. This allows to show that in such a model, the market outcome is equal to a social planner's solution. As an extension, labour is introduced as a second input in the production function besides knowledge. This requires a monetary investment from the Investors because it is assumed that Workers only accept cash-in-advance payments. With this extension, it is revealed that production can only be sustained if the Investors do not hold all the bargaining power. If the Entrepreneurs hold the bargaining power, they want to choose the efficient solution, but it might not be feasible to achieve it due to the cash-in-advance constraint. In the extended model, the effects of monetary policy and limited commitment are also analysed. It is shown that monetary policy is no means to overcome the friction caused by the cash-in-advance constraint, and that a constant money supply is the best policy the monetary authority can choose. In the absence of full commitment, however, the incentive to cheat for Entrepreneurs is so high that Investors are not willing to invest in the Entrepreneur's businesses. In such an environment, a sufficiently high inflation rate can eliminate the issue by making cheating Entrepreneurs subject to the inflation tax and therefore reducing their incentives to cheat.

Important work on the field of venture capital and start-up companies has already been done amongst others by Rafael Silveira and Randall Wright. In their paper "Search and the market for ideas" [Silveira and Wright, 2010], they distinguish between Innovators and Entrepreneurs, which is close to what is done here but the focus in their paper lies more on the implementation of a business idea instead of the financing process. In an earlier paper [Silveira and Wright, 2007], they model the so-called "Venture Capital Cycle". Another important paper which focuses on the effects of monetary policy on the innovation sector is [Berentsen et al., 2012].

This thesis proceeds as follows: The model developed by Bünter will be presented in section 2, and the extended model is covered in section 3. In section 4, the effects of monetary policy are analysed and in section 5 the assumption of full commitment is dropped. Finally, section 6 contains the conclusion.

2 The basic Model

Let us now turn to the model. In this economy, time is discrete and continues forever. Agents live for three periods. After each period, a new generation of agents is born. The number of agents born in a period stays constant over time and is normalised to 1. In the first period of their life, agents are Entrepreneurs E who can produce goods. In the second period, they are Investors I who can assist Entrepreneurs in producing goods by providing their knowledge. In the last period of their life, agents are Retirees R that want to consume. At the end of the third period of their life, agents die without having an option to leave a bequest to any other agent.

The agents get a utility of zero from consumption in the first and the second period of their life, so for an agent born in period t, $u(y_t) = 0$ and $u(y_{t+1}) = 0$ for all y_t and $y_{t+1} \in \mathbb{R}$. In the third period of life, utility of consumption is linear in the consumption good: $u(y_{t+2}) > 0$ for all $y_{t+2} > 0$, with $u'(y_{t+2}) = 1$. Only Entrepreneurs are able to produce consumption goods, but they need knowledge provided by Investors in order to produce. Thus, the production function is $f(e) = y$, with $f'(e) > 0$ and $f''(e) < 0$, where e denotes knowledge provided by Entrepreneurs, and to provide this knowledge Entrepreneurs face the cost $c(e)$, with $c'(e) > 0$ and $c''(e) > 0$. The consumption good is non-storable. Each agent discounts future periods with a factor $\beta < 1$. There is a total amount of money available in this economy which is denoted by M. For the moment, we assume that this money stock is fixed.

Each period is divided into a decentralized market (DM) and a centralized market (CM).[1] In the DM, Entrepreneurs are matched randomly with Investors.

[1] Among the first to use this distinction were [Lagos and Wright, 2005].

The probability of a match is denoted σ_E for Entrepreneurs. σ_I denotes the probability of being in a match for an Investor if he was already matched in the period before (as an Entrepreneur). If an agent was not in a match as an Entrepreneur, he will not be matched in any future period of his life either.[2] In the CM, Entrepreneurs can sell the goods they produced to the Retirees in a Walrasian market and will get some amount of money m_t in return. Since this market is centralized, there is no random matching, and each Entrepreneur that was able to produce y_t will get m_t. After the CM has taken place but before the period ends, Entrepreneurs get the chance to pay some amount of money x_t to the Investors to compensate them for the cost they faced by providing their knowledge.

2.1 Social planner's solution

Before we turn to the market outcome, let us consider what a benevolent social planner would do in the economy described. The goal of such a planner is to maximize the lifetime utility of all agents. Therefore, he would match all Entrepreneurs with Investors and then choose the knowledge input according to

[2] This assumption is crucial to get steady states. Assume for a moment that all agents in the second period of their life could be matched with an Entrepreneur, meaning those with knowledge as well as those without. First of all, this would mean that $\sigma_E = \sigma_I$ because the number of matched Investors has to be equal to the number of matched Entrepreneurs, and since their total population is equal, the fractions of matched agents in each group has to be equal as well. Clearly, this assumption would lead to a number of unproductive matches since those second-period agents without knowledge could not help the Entrepreneurs in any way. The interesting fact here is that this number of unproductive matches would be ever increasing, simply by mathematical logic. That is true even if we assume that in the original period, all the Investors had knowledge. This would lead to σ productive matches in this original period. In period two, however, there would only be σ Investors with knowledge (those that were matched as an entrepreneur) and only σ of them would be matched, meaning that there would be even less Investors with knowledge in the upcoming third period. This line of thought leads to the conclusion that the number of productive matches in a given period t is equal to σ^t, which is decreasing in t. In a steady state, however, it is required that the number of productive matches stays constant across periods.

the following maximization problem:

$$\max_{y,e} \beta^2 u(y) - \beta c(e) \tag{1}$$

$$s.t. \qquad y = f(e) \tag{2}$$

Note that we only look at steady states here, which means there is no need for time indices in the above equations. Equation 1 is the lifetime value of an agent, which equals his utility from consumption as a Retiree (discounted twice) minus his disutility of providing knowledge as an Investor (discounted once). Equation 2 is a feasibility constraint.

To solve the maximization problem, we can replace y in equation 1 with the result from equation 2 and then take the first derivative. Note that we can simply drop the utility function because we assumed a marginal utility of 1 for all consumption levels. This leads to the following result:

$$\beta f'(e) = c'(e) \tag{3}$$

Equation 3 states that it is optimal to choose the knowledge level e in such a way that the discounted marginal productivity of knowledge equals the marginal cost of providing knowledge. We will use this result later on as a benchmark to compare the market outcomes to the efficient solution.

2.2 Steady states

Knowing the properties of the efficient solution for this economy, we can turn to the market outcome. As we will only consider steady states in this thesis, we first need to specifiy the necessary conditions required on the matching probabilities in order to allow for steady states. In a steady state, all variables take the same value in each period, i.e. for example $y_t = y_{t+1} = y_{t+2} = y$. The existence of a steady state in this model depends on the assumption that the amount of money m_t paid by Retirees to Entrepreneurs stays constant over time. This assumption is only valid if the following equation holds:

$$\sigma_E m_t = \sigma_E(m_{t-2} - x_{t-2}) + \sigma_E \sigma_I x_{t-1} \tag{4}$$

The left hand side of equation 4 shows the total amount of money that Entrepreneurs will hold after the transaction in the CM has taken place, but before they pay the Investors for the provision of their knowledge. The right hand side of equation 4 shows the total amount of money held by Retirees. The Retirees will spend all their money to buy goods in the CM because they have no other use for it, so equation 4 must always hold. In a steady state, the amounts of money m_t and x_t have to be the same in each period, so we can drop the time subscripts and equation 4 simplifies to:

$$m = m - x(1 - \sigma_I) \tag{5}$$

Obviously, this equation only holds if either $x = 0$ or $\sigma_I = 1$. The case of $x = 0$ would imply that Investors get nothing in return for their provision of knowledge, but since they face a cost to provide knowledge, they would never agree to such a deal. Consequently, this case is ruled out. This only leaves the case of $\sigma_I = 1$ as an option, and this also makes sense intuitively. If not all of the Investors who already were in a match as an Entrepreneur would be matched, this would mean that the total amount of possible matches would be lower than σ_E. In that case, σ_E would be decreasing with each period, which is not consistent with steady states.

2.3 Equilibrium in the CM

As we have seen in section 2.2, agents in the model are either matched two times (once as an Entrepreneur and once as an Investor) or never. This means that there are only two types of Retirees, namely the ones without money which are not able to participate in the economy, and those that were matched in the earlier periods of their life. All of those who were matched now hold the same amount of money, which is $m + x - x = m$. In the CM, trade takes place in a Walrasian market so the goods market always clears. This means that in equilibrium, supply and demand of goods has to be equal. So we can determine the value of money in period t, ϕ_t by setting supply and demand equal and solving the resulting equation for ϕ_t. The total supply of goods in the CM is:

$$\sigma_E f(e) = \sigma_E y \tag{6}$$

while the total demand in real terms equals:

$$\phi_t \sigma_E m \tag{7}$$

Setting equations 6 and 7 equal, we get:

$$\sigma_E y = \phi_t \sigma_E m \tag{8}$$

Equation 8 shows that the value of money stays constant over time (which allows to drop the time subscript) and that each Retiree with money buys the whole production of consumption goods y of a matched Entrepreneur. This means that the value of money ϕ is defined as:

$$\phi = \frac{y}{m} \tag{9}$$

2.4 Equilibrium in the DM

Before we can analyse the Equilibrium in the DM, we need to state the value functions of an Entrepreneur and an Investor at the beginning of the DM.[3] In general, the value functions of the three types of agents look like this:

$$V_E = \sigma_E \beta V_I(n) + (1 - \sigma_E)\beta V_U(0) \tag{10}$$

$$V_I(n) = \sigma_I(\beta V_R(o) - c(e)) + (1 - \sigma_I)\beta V_R(n) \tag{11}$$

$$V_R(o) = u[\phi(m_{t-2} - x_{t-2} + x_{t-1})] \tag{12}$$

The state variables n and o in the value functions stand for the amounts of money an agent carries into a period. Therefore, $n = m - x$ and $o = n + x$. V_U is the value function of an agent that is in the second period of his life but can't act as an Investor. The value function of such an agent equals 0.

[3]The solving techniques in this chapter as well as in the sections 3.3 and 4.1 are partly borrowed from chapter 4 of the book "Money, Payments, and Liquidity" written by Ed Nosal and Guillaume Rocheteau [Nosal and Rocheteau, 2011].

Now we can insert the value functions of Retirees and Investors into that of the Entrepreneur to get:

$$V_E = \sigma_E\{\beta^2 u[\phi(m_t - x_t + x_{t+1})] - \beta c(e_{t+1})\} \tag{13}$$

This equation states the utility the Entrepreneur gets from consuming the goods he can buy with his money balances, minus the cost he has to face in the next period of life if he provides a future Entrepreneur with his knowledge.[4] Everything is multiplied by σ_E because the Entrepreneur only gets this value if he finds a match in his first period of life. Equation 13 does not only state the value for the Entrepreneur if he agrees in a match with the Investor, but also his total surplus from an agreement because his utility equals zero without agreement.

If we replace V_R in equation 11 with equation 12 (and make use of the fact that $\sigma_I = 1$), we get the fully written out value function of an Investor:

$$V_I(n) = \beta u[\phi(m_{t-1} - x_{t-1} + x_t)] - c(e_t) \tag{14}$$

Again, this equation states the utility an Investor gets from consumption minus the cost he faces to provide the Entrepreneur with knowledge. If there is no agreement in a match between an Investor and an Entrepreneur, the utility of an Investor is:

$$V_I(n) = \beta u[\phi(m_{t-1} - x_{t-1})] \tag{15}$$

Now if we subtract Equation 15 from Equation 14, we get the surplus from an agreement for the Investor:

$$\begin{aligned} S_I &= \beta u[\phi(m_{t-1} - x_{t-1} + x_t)] - c(e_t) - \beta u[\phi(m_{t-1} - x_{t-1})] \\ &= \beta u(\phi x_t) - c(e_t) \end{aligned} \tag{16}$$

Note that the simplification is possible because of the assumption of linear utility. Now that we know the surplus for both the Investor and the Entrepreneur

[4] Although we know that in equilibrium, x_t and x_{t+1} will be equal, we have to keep them as seperate variables in the equation since one of them is a decision variable in the bargaining process while the other is taken as an exogenous value by the Entrepreneur at the moment.

from agreement, we can look at the equilibrium. We will assume take it or leave it offers from both agents as the bargaining protocol, starting with the Investor as the agent who can make the offer.

2.4.1 Take it or leave it offer from Investors

If the Investor can make the take it or leave it offer, he tries to maximize his utility while ensuring that the Entrepreneur accepts his offer. This means that his offer solves the following problem:

$$\max_{x_t, e_t} \beta u(\phi x_t) - c(e_t) \tag{17}$$

$$s.t. \qquad \beta^2 u[\phi(m_t - x_t + x_{t+1})] - \beta c(e_{t+1}) \geq 0 \tag{18}$$

$$x_t \leq m_t \tag{19}$$

Equation 17 states the maximization problem, while equation 18 is the constraint that the Entrepreneurs surplus from an agreement must be at least zero. Equation 19 says that the Investor can not take more money from the Entrepreneur than he earns in the CM. The Investor wants to choose the decision variables x_t and e_t such that the participation constraint of the Entrepreneur holds with equality. By rearranging terms in equation 18, we get:

$$\beta[\beta u(\phi m_t - \phi x_t) + \beta u(\phi x_{t+1}) - c(e_{t+1})] \geq 0 \tag{20}$$

Again, the separation of the terms in the utility function is only possible because we assumed a linear utility function. The second part of equation 20 is exactly the same equation as the maximization problem of the Investor, and it consists only of variables that can't be affected in this period. Also, the second part of the equation can never be below zero because if it were, the Entrepreneur would just not act as an Investor in his next period of life. This means that the Investor has to choose the decision variables in a way that the first part of the equation is zero or even lower in order to make the participation constraint of the Entrepreneur hold with equality. However, he can't choose a value of x_t higher than m_t because of equation 19. This means that in equilibrium, the Entrepreneur's surplus will still be positive because the second part of equation 20 will be positive in equilibrium, but the Investor gets all the money the Entrepreneur earns.

Now that we know that equation 19 holds with equality, we can replace x_t with m_t in equation 17. Furthermore, from equation 9 we know that ϕm_t equals y_t, and since $y_t = f(e_t)$ (equation 6), we can write the Investor's maximization problem as follows:

$$\max_{e_t} \beta u(f(e_t)) - c(e_t) \tag{21}$$

And by taking the derivate and setting it equal to zero, we get:

$$\beta f'(e_t) = c'(e_t) \tag{22}$$

Equation 22 is the same as 3, which is the social planner's solution. This means that the production is at the optimal level if the Investor gets to make a take it or leave it offer.

2.4.2 Take it or leave it offer from Entrepreneurs

Now let us consider the case where the Entrepreneur has all the bargaining power and can make an offer. The Entrepreneur solves the following problem:

$$\max_{x_t, e_t} \beta^2 u[\phi(m_t - x_t + x_{t+1})] - \beta c(e_{t+1}) \tag{23}$$

$$s.t. \qquad \beta u(\phi x_t) - c(e_t) \geq 0 \tag{24}$$

$$x_t \leq m_t \tag{25}$$

Equation 23 is the maximization problem of the Entrepreneur, equation 24 is the participation constraint for the Investor and equation 25 states again that the maximum payment from the Entrepreneur to the Investor can be all the money holdings of the Entrepreneur.

The Entrepreneur's goal is to make the Investor's participation constraint hold with equality. If we set equation 24 equal to zero, rearrange terms and drop the utility function - which can be done because linear utility is assumed - we get:

$$\phi x_t = \frac{c(e_t)}{\beta} \tag{26}$$

This states that the real amount of money the Investor receives has to be equal to the cost of providing his knowledge times the inverse of the discount factor. To see whether this solution also satisfies equation 25, we can replace ϕ with $\frac{f(e_t)}{m}$ and rearrange terms to get:

$$m_t = x_t \frac{\beta f(e)}{c(e)} \tag{27}$$

This shows that equation 25 is satisfied as long as $\frac{\beta f(e)}{c(e)} \geq 1$. We will see later that this always holds in equilibrium, so all the constraints are satisfied with this solution.

Substituting 26 into the maximization problem and replacing ϕm_t with $f(e_t)$ (which we can because of equations 6 and 9) gives us:

$$\max_{e_t} \beta^2 u[(f(e_t) - \frac{c(e_t)}{\beta} + \phi x_{t+1})] - \beta c(e_{t+1}) \tag{28}$$

Now we can move all elements independent of the decision variable e_t in front of the maximization problem to get:

$$\beta^2 u(\phi x_{t+1}) - \beta c(e_{t+1}) + \max_{e_t} \beta^2 u[(f(e_t) - \frac{c(e_t)}{\beta})] \tag{29}$$

The maximization problem also confirms that $\frac{\beta f(e)}{c(e)} \geq 1$ indeed holds in equilibrium, because the statement is true as long as the result of the maximization problem is at least equal to zero. If the solution of the maximization problem was lower than zero, the Entrepreneur would not produce anything, because his utility from producing and consuming nothing is zero. Furthermore, this shows that x_t will be lower than m_t as long as the surplus of the Entrepreneur is positive, so the value of x_t in the scenario where the Entrepreneur has all the bargaining power will in general be different from the value in the scenario where the Investor has all the bargaining power.

By taking the derivate of the maximization problem and setting it equal to zero, we get:

$$\beta f'(e_t) = c'(e_t) \tag{30}$$

This is the same result as in equation 22, which means that if the Entrepreneur gets to make a take it or leave it offer, production will also be at the optimal level.

The distribution of the bargaining power only affects the amount of money x_t paid by the Entrepreneur, which in general will be lower when the Entrepreneur has all the bargaining power compared to the situation when the Investor has all the bargaining power.

3 The extended Model

The first part of this thesis described the model as originally developed by Noemi Bünter for her Master's Thesis [Bünter, 2013]. We will now make some extensions to the original model to better reflect the issues faced by a young Entrepreneur which were described in the introduction.

So far, the only input in the production function was knowledge provided by Investors. Typically though, an Entrepreneur needs cash to be able to realize his idea. To model this, we add a second input into the production function which is labour. The production function is therefore $y = f(e, l)$ for the remainder of this thesis, where l stands for the amount of labour, measured in hours, used in the production process. The new production function is assumed to be concave in knowledge and in labour, so $f_e(e, l) > 0$, $f_{ee}(e, l) < 0$, $f_l(e, l) > 0$ and $f_{ll}(e, l) < 0$.[5] Additionally, some amount of both inputs is needed to produce anything, which means $f(0, l) = f(e, 0) = 0$.

Labour is provided by Workers, so from now on we assume that a fraction $1 - \delta$ of the new-born agents are Workers, while the remaining δ agents are born as Entrepreneurs. Each Worker lives for only one period. We assume that Workers get the same utility from consuming goods as Retirees. A Worker can provide an infinite amount of labour, and he offers each hour of work for the real wage rate $\phi w = \eta$. This means that labour supply is perfectly elastic, and η can be seen as either the outside option of Workers or as their (linear) utility of leisure. Both of these explanations imply that no Worker can be hired for less than the

[5]The notation for derivatives has changed because there are now several input variables in one function. From now on, the lower case letter indicates the variable after which the function has been derived, so $\frac{\partial f(e,l)}{\partial e} = f_e(e, l)$.

real wage rate, because then the Workers strictly prefer to enjoy leisure or work for someone else respectively. On the other hand, Workers will never be paid a real wage rate higher than η because competition between Workers drives down real wages to η.[6]

The production process takes place between the DM and the CM of a period, and since Workers live only for one period they want to consume the good y in the CM of the same period as they provide their work. Therefore, the Workers need to be paid before the CM takes place.[7] Because the Entrepreneurs only earn money in the CM, they now need not only knowledge provided by Investors, but also money to pay the Workers. Given that the Entrepreneurs have money, we assume that they hire Workers right after the end of the DM, pay them instantly and then the Workers provide the labour they have been paid for.

3.1 Social planner's solution

Again, let us first consider what a social planner would do before turning to the market outcome. As our main interest in this thesis is on the agents that live for

[6]Instead of assuming a perfectly elastic labour supply, a perfectly inelastic labour supply with a fixed endowment of hours per Worker could also be assumed. We will do this in section 3.4, where we will see that the assumption of a perfectly elastic labour supply is more interesting in terms of the implications of the model. Additionally, the idea of perfectly elastic labour supply is closer to reality if we think about the situation for small startup companies, because such companies only make up a relatively small share of the entire economy and therefore have to take the real wage rate as an exogenous variable. Of course the idea of startup companies making up only a small share of the economy is not modeled here, but having that idea in mind the assumption of an exogenous real wage rate can be well justified.

[7]One could argue that instead of paying the Workers with money, the Entrepreneurs could just give them some of the proceedings from the production process because the Workers consume that good anyway. Admittedly, this is a shortfall of this thesis, but we could assume that there is a commitment problem which makes it necessary that the Entrepreneurs pay the Workers in advance, which would make the use of money necessary. The more realistic story is that the goods which the Entrepreneurs produce are actually heterogenous, and the Workers are not interested in the specific good which they helped producing. However to model this, we would need a [Kiyotaki and Wright, 1989] style search environment in the CM instead of a Walrasian market, which would make this thesis much more complex mathematically.

three periods, we assume that the social planner only maximizes their lifetime utility while just giving the Workers enough goods to make them provide their labour. Therefore, the social planner now maximizes the following function:

$$\max_{y,e,l} \beta^2 u(y - l\eta) - \beta c(e) \tag{31}$$

$$s.t. \qquad y = f(e, l) \tag{32}$$

Equation 31 states the lifetime value of an agent. Note that a Retiree can only consume an amount of goods lower than y because a part of the production now goes to Workers. A Worker is ready to provide his labour if he gets paid η goods per hour he works, therefore this amount has to be subtracted from the consumption of a Retiree. Equation 32 is again the feasibility constraint. Now we can put equation 32 into the maximization problem and then take the first derivatives after both input variables. This yields:

$$e^* : \quad \beta f_e(e, l^*) = c_e(e) \tag{33}$$

$$l^* : \quad f_l(e^*, l) = \eta \tag{34}$$

Equation 33 states that it is still optimal to use the knowledge level for which the marginal productivity of knowledge is equal to the marginal cost of providing knowledge, and equation 34 states that it is optimal to use the amount of labour that makes the marginal productivity of labour equal to the reservation wage of the Workers. We will use this result later to compare it to the market outcomes.

3.2 Equilibrium in the CM

Having defined the new features of the model and the social planner's solution, we can turn to the analysis of the equilibrium in the CM. The matching mechanism for the Entrepreneurs and Investors is almost the same as in the previous version of the model, with the difference that we now only have δ agents in each period that are Entrepreneurs or Investors. This means that a total of $\delta\sigma_E$ Entrepreneurs will be in a match, and since each of them produces y units of the consumption good, the total supply of consumption goods is:

$$\delta\sigma_E f(e,l) = \delta\sigma_E y_t \tag{35}$$

On the demand side, we now have two groups of agents that want to purchase consumption goods. One of these groups are the Retirees. In the original version of the model, their money holdings were $m_{t-1} - x_{t-1} + x_t$, but now Investors need to give some amount of money to the Entrepreneurs in order to enable them to pay the Workers. We will call this amount of money k_t. In total, there are $\delta\sigma_E$ Retirees and therefore the total money holdings of Retirees are now:

$$\delta\sigma_E(m_{t-1} - x_{t-1} - k_t + x_t) = \delta\sigma_E(m_{t-1} - k_t) \tag{36}$$

The simplification here is possible because we are still assuming steady states. Knowing what the money holdings of the Retirees are, we can now turn to the Workers. We will see in the analysis of the events in the DM that each Entrepreneur will hire a total amount of labour l, which means that each Entrepreneur pays wl amounts of money to Workers. As there are $\delta\sigma_E$ matched Entrepreneurs in each period, the total amount of money paid to Workers is:

$$\delta\sigma_E wl \tag{37}$$

This means that the amount of money each Worker holds will be $\frac{\delta\sigma_E wl}{1-\delta}$, but to find the equilibrium we need the total amount of money held by Workers, so equation 37 is more helpful.

Now that we have defined all the money holdings of consumers in the CM, the equilibrium can be found by setting supply and demand equal:

$$\delta\sigma_E y_t = \phi[\delta\sigma_E(m_{t-1} - k_t) + \delta\sigma_E wl] \tag{38}$$

Solving this equation for the value of money ϕ in steady states yields:

$$\phi = \frac{y}{m - k + wl} \tag{39}$$

This equation is fairly similar to the result from the original model (see equation 9). In fact, the result is exactly the same as we will assume from now on that

$k = wl$. This is done because the only reason for the Investor to give money to the Entrepreneur is to enable him to pay the Workers, and by assuming that the two amounts of money are the same we ensure that the Entrepreneur really uses all the money to pay workers. This means that the two terms cancel out in equation 39 and the value of money is still defined as $\phi = \frac{y}{m}$.

3.3 Equilibrium in the DM

After having learned the properties of the CM equilibrium in the expanded version of the model, we can now turn to the DM. Therefore, we first need to have a look at the value functions of the agents at the beginning of the first and second period of their life. For the Entrepreneur, the value function at the beginning of the first period looks as follows:[8]

$$V_E = \sigma_E\{\beta^2 u[\phi(m_t - x_t - k_{t+1} + x_{t+1})] - \beta c(e_{t+1})\} \tag{40}$$

The difference to the value function in the original model (see equation 13) is that the Entrepreneur now has to give up some of his money holdings in his second period of life to enable the agent matched with him to pay the workers. This is reflected by the term $-k_{t+1}$ in the value function. The same is true for the Investor, so his new value function is now:

$$V_I(n) = \beta u[\phi(m_{t-1} - x_{t-1} - k_t + x_t)] - c(e_t) \tag{41}$$

Equation 41 shows the value for the Investor if he finds an agreement with his matched Entrepreneur. If he doesn't find an agreement, his value function reduces to:

$$V_I(n) = \beta u[\phi(m_{t-1} - x_{t-1})] \tag{42}$$

This means that an Investor's surplus from an agreement is:

$$\begin{aligned} S_I &= \beta u[\phi(m_{t-1} - x_{t-1} - k_t + x_t)] - c(e_t) - \beta u[\phi(m_{t-1} - x_{t-1})] \\ &= \beta u[\phi(-k_t + x_t)] - c(e_t) \end{aligned} \tag{43}$$

[8]In this section, we work directly with the fully written out value functions. The derivation for these is similar to the one we did in section 2.4.

With this new insights and assumptions, we can again check the trading outcomes in the DM in the two cases where either the Investor or the Entrepreneur has all the bargaining power.

3.3.1 Take it or leave it offer from Investors

To find the offer that solves the Investor's maximization problem, we have to look at the new value functions and keep some additional constraints in mind. So if the Investor has all the bargaining power, his offer solves the following problem:

$$\max_{x_t,e_t,k_t,l_t} \beta u[\phi(-k_t + x_t)] - c(e_t) \tag{44}$$

$$s.t. \quad \beta\{\beta u[\phi m_t - \phi x_t] + \beta u[-\phi k_{t+1} + \phi x_{t+1}] - c(e_{t+1})\} \geq 0 \tag{45}$$

$$x_t \leq m_t \tag{46}$$

$$k_t \leq m_{t-1} - x_{t-1} \tag{47}$$

$$x_{t+1} \leq f(e_{t+1}, \phi\frac{m_t - x_t}{w}) \tag{48}$$

$$wl_t = k_t \tag{49}$$

$$\phi w = \eta \tag{50}$$

Equation 44 states the surplus from an agreement for an Investor, which we derived in equation 43. Equation 45 states that the surplus from an agreement for an Entrepreneur, which equals his lifetime value after being matched as stated in equation 40, has to be at least equal to zero. Note that equation 45 is really the same as equation 40 (except for the term σ_E since we only consider matched entrepreneurs), but the equation looks slightly different just to highlight which terms are influenced by the bargaining decision and which are independent. Equation 46 states that the Entrepreneur can't give the Investor more money than he earned in the CM. Equation 47 states that the Investor can't give more money to the Investor than he currently owns and equation 48 shows that the Entrepreneur needs some positive amount of money balances to act as an Investor in the next period (and get some reward x_{t+1} for it). Equation

49 states that the Entrepreneur spends the money he gets from the Investor on hiring workers, and finally, 50 states the condition on the real wage rate.
As in the basic model, the Investor tries to make equation 45 binding. By choosing $x_t = m_t$, the first part of the equation equals zero. As we know, he can't go any lower because this would violate equation 46. But because of equation 48, the choice of $x_t = m_t$ leads to $x_{t+1} = 0$, which means that the Entrepreneur can not act as an Investor in the next period and therefore equation 45 is binding. But because we are only considering steady states, we know that the choice of the Investors is the same in each period and therefore we can conclude that the economy breaks down in the extended model if the Investor has all the bargaining power. The reason for this is that it is in the Investor's personal interest to extract all the money from the Entrepreneur, but doing so makes it impossible for the Entrepreneur to produce anything in the next period.

3.3.2 Take it or leave it offer from Entrepreneurs

Now that we have seen the result in the situation when the Investor has all the bargaining power, we can turn to the other case with all the bargaining power on the Entrepreneur's side. Under these circumstances, the Entrepreneur's offer will maximize the following function under several constraints:

$$\max_{x_t, e_t, k_t, l_t} \beta^2 u[\phi(m_t - x_t - k_{t+1} + x_{t+1})] - \beta c(e_{t+1}) \tag{51}$$

$$s.t. \qquad \beta u[\phi(-k_t + x_t)] - c(e_t) \geq 0 \tag{52}$$

$$x_t \leq m_t \tag{53}$$

$$k_t \leq m_{t-1} - x_{t-1} \tag{54}$$

$$x_{t+1} \leq f(e_{t+1}, \phi \frac{m_t - x_t}{w}) \tag{55}$$

$$w l_t = k_t \tag{56}$$

$$\phi w = \eta \tag{57}$$

The Entrepreneur's problem is very similar to the Investor's problem, since five of the constraints are the same in both problems and the first constraint of the Entrepreneur's problem is simply the maximization problem in the Investor's problem and vice versa. The results are different, however.

As always, the Entrepreneur wants to set the Investor's surplus from an agreement to zero, so equation 52 is binding. If we rearrange equation 52 and replace k_t with wl_t (which comes from equation 56), we get:

$$\phi x_t = \frac{c(e_t)}{\beta} + \phi w l_t \tag{58}$$

This shows that the Entrepreneur's payment has to compensate the Investor for the money he gives away and for the cost he bears.

Because of equation 55, the choice of x_t now also influences x_{t+1}. Since x_{t+1} is a positive variable in the maximization problem and the value of x_{t+1} is inversely related to x_t by equation 55, it is the Entrepreneur's goal to set x_t as low as possible. This is already satisfied by equation 58.

We now have three remaining constraints, namely 53, 54 and 57. We will be able to show that 53 always holds and for 54 we will make a case distinction in the end, and we will use 57 later to specify the equilibrium. Now we can proceed to put the result from 58 into equation 51 to get:

$$\max_{e_t, l_t} \beta^2 u[f(e_t, l_t) - (\frac{c(e_t)}{\beta} + \phi w l_t) - \phi k_{t+1} + \phi x_{t+1}] - \beta c(e_{t+1}) \tag{59}$$

Note that we have replaced ϕm_t in the above equation with the production function to include the two decision variables e_t and l_t. Now we can move the elements which are independent of the maximization problem in front of it to get a clearer picture:

$$\beta^2 u(-\phi k_{t+1} + \phi x_{t+1}) - \beta c(e_{t+1}) + \max_{e_t, l_t} \beta^2 u[f(e_t, l_t) - \frac{c(e_t)}{\beta} - \phi w l_t)] \tag{60}$$

At this point, we can prove that equation 53 always holds. If we multiply both sides of 53 with ϕ, we can replace the left hand side with our solution for ϕx_t from equation 58, and the right-hand side can be replaced with the production function. This yields:

$$\frac{c(e_t)}{\beta} + \phi w l_t \leq f(e, l) \tag{61}$$

And if we rearrange this expression, we get:

$$f(e,l) - \frac{c(e_t)}{\beta} - \phi w l_t \geq 0 \tag{62}$$

Which is exactly the same statement as we found in the maximization problem (equation 60). This means that equation 53 holds whenever the solution to the maximisation problem of the Entrepreneur is non-negative, and because we know that the Entrepreneur will never choose a negative solution for the maximisation problem, we can conclude that the constraint is always satisfied.[9] With this knowledge, we can now solve the maximization problem, which yields the following results:

$$e_t^* : \quad \beta f_e(e_t, l_t^*) = c_e(e_t) \tag{63}$$

$$l_t^* : \quad f_l(e_t^*, l_t) = \phi w = \eta \tag{64}$$

This shows that if the Entrepreneur has all the bargaining power, the economy can reach the efficient level, because equations 63 and 64 are the same as 33 and 34 (the planner's solution). But we still have to take care of equation 54. While equation 63 is always feasible, equation 64 depends on the wage level and could therefore violate the constraint from equation 54. We can rearrange equation 64 to get $w = \frac{f_l(e_t^*, l_t^*)}{\phi}$, and rearranging equation 54 after having replaced k_t with wl_t gives $w \leq \frac{m_{t-1} - x_{t-1}}{l_t^*}$. Now we can replace w with our result to get:

$$f_l(e_t^*, l_t^*) \leq \frac{\phi(m_{t-1} - x_{t-1})}{l_t^*} \tag{65}$$

This states that the marginal productivity of labour at the efficient input level has to be smaller or equal to the real money balances of an Entrepreneur divided by the efficient amount of labour input. If this condition holds at the optimal choice of labour l_t^*, the result we derived in equations 63 and 64 is feasible. But if the wage rate and the marginal product of capital are such that the Entrepreneur would like to get more hours of work than he can pay for, equation 54 binds and the labour used in the production process is determined by:

$$l_t = \frac{m_{t-1} - x_{t-1}}{w} \tag{66}$$

[9] The Entrepreneur can always choose to get zero by setting all decision variables equal to zero.

Which means that the Entrepreneur gets all the money holdings of the Investor and buys as much labour as he can with it. The optimal choice of knowledge then becomes:

$$\beta f_e(e_t, \frac{m_{t-1} - x_{t-1}}{w}) = c_e(e_t) \tag{67}$$

In such a situation, the market outcome leads to an inefficiently low production level compared to the social planner's solution. This is due to the cash-in-advance constraint we assumed on Worker's payments.

There are a couple of interesting results in this section. First of all, the economy will in general not break down if the Entrepreneurs have all the bargaining power. This means that in the extended model, the distribution of the bargaining power matters and it is beneficial for everyone if the bargaining power is in the hands of Entrepreneurs. Second, the liquidity of Investors matters in determining whether or not the efficient solution is feasible. Both of these findings stem from the cash-in-advance friction on Worker's payments, which makes the liquidity of the Investor important in determining whether or not the efficient solution can be reached. This result is in line with the findings of [Berentsen et al., 2012], who showed that reducing inflation increases the liquidity of Investors, which also leads to more efficient solutions.

3.4 Alternative assumption about the labour market

Now that we have seen the results of the extended model, let us again think about the assumptions we made for the labour market. As stated at the beginning of this section, an alternative assumption would be a perfectly inelastic labour supply. To get an inelastic labour supply, the endowment of hours for each Worker has to be fixed, so we assume that each Worker has a total amount of h hours during which he can work. As there are $1 - \delta$ Workers overall, there is a total amount of labour supply of $H = h(1 - \delta)$. Because the labour supply is perfectly inelastic, Workers are ready to provide labour for any positive wage rate, which means that labour demand will now determine the wage rate. Now we can analyse how these different assumptions about the labour market change the equilibrium outcome.

From the equilibrium analysis, we know that the optimal amount of labour used in production sets the real wage rate and the marginal product of labour equal. This is expressed in equation 64. But with our new assumptions, the amount of labour which can be used is limited, while the wage rate is endogenous. As the marginal product of labour is always positive, Entrepreneurs want to hire all available Workers and use all of their endowments, so the equilibrium wage rate w^* is determined according to the following equation:

$$\phi w^* = f_l(e^*, \frac{H}{\delta\sigma_E}) \tag{68}$$

This states that the real wage rate is equal to the marginal product of labour of the total amount of hours each Entrepreneur uses in the production. As the total amount of hours available in a given period is H, and there are $\delta\sigma_E$ productive Entrepreneurs, each of them can hire $\frac{H}{\delta\sigma_E}$.

As we know, there is another constraint for the Entrepreneurs who want to hire workers, depending on the money holdings of Investors. Each Investor holds an amount $\phi(m-x)$ of real money, which means that in total, productive Entrepreneurs have an amount of $\delta\sigma_E\phi(m-x)$ of real money available to spend on Workers, and they are willing to pay an amount of $\phi w^* H$ to workers. Now let us think about what happens when we are in the following situation:

$$\delta\sigma_E\phi(m-x) < \phi w^* H \tag{69}$$

Basically, if that is the case, then Entrepeneurs do not have enough (real) money balances to pay for the efficient amount of labour. We have seen this already in the other version of the model, as stated in equation 65. Before, this meant that the Entrepreneurs could not hire as much labour as they wanted to. However, since the wage rate is now endogenous, things look different. Because all Workers want to work during all their hours available, competition between them will drive down the wage rate until the labour market clears again, and the wage rate will then be determined by:

$$w = \frac{\delta \sigma_E (m-x)}{H} \tag{70}$$

To get this expression, we have to set the two sides of 69 equal and solve for w. Again, there are two possible outcomes, and the real money holdings of Investors decide which of them is achieved. With these new assumptions however, the only thing affected is the real wage rate for the Workers, while production always stays at the same level. The only constraint on production now is the amount of labour available in the economy, while before, the constraint on production came from the money holdings of Investors. This gives another justification of the assumption of a perfectly elastic labour supply, because total labour supply seems not to be a constraint for start-up companies in reality, while the liquidity of Investors certainly matters a lot.

4 Monetary Policy

So far, we have assumed that the total amount of fiat money in the economy is constant. In this section, we will allow for an increasing or decreasing stock of money.[10] We will model this through the assumption of a constant growth rate of money z, such that that the money stock changes according to $M_{t+1} = zM_t$. Values of z larger than 1 imply that the money stock is growing, while $z < 1$ signifies a shrinking money stock. The monetary policy works through lump-sum transfers τ_t, which all Retirees that were matched as an Entrepreneur receive at the beginning of the period.[11]

We will now first analyse how a changing monetary stock influences the production level in the DM, before we turn to the CM to find the effects of a changing

[10]The solving techniques in this section are partly borrowed from chapter 3 of the book "Modeling Monetary Economies" written by Bruce Champ, Scott Freeman and Joseph Haslag [Champ et al., 2011].

[11]If we assumed that all Retirees receive lump-sum transfers, we could not model a shrinking money stock because only the matched agents hold money as Retirees and to model deflation, the lump-sum transfers have to be negative.

money stock on the value of money.

4.1 Equilibrium in the DM

As always when analysing the Equilibrium in the DM, we first need to specify the value functions of the agents. For the Entrepreneur, the value function is now:[12]

$$V_E = \sigma_E\{\beta^2 u[\phi_{t+2}(m_t - x_t - k_{t+1} + x_{t+1} + \tau_{t+2})] - \beta c(e_{t+1})\} \tag{71}$$

There are two changes compared to the value function in the extended model (see equation 40). First, the lump-sum transfer τ_{t+2} is now part of the money holdings of the Entrepreneur. Second, we have to clearly specify the value of money which matters for the given agent. For the Entrepreneur, this is ϕ_{t+2}, because he wants to consume two periods from now and therefore cares about the value which his money holdings will have in that period.

For the Investor, things look very similar:

$$V_I(n) = \beta u[\phi_{t+1}(m_{t-1} - x_{t-1} - k_t + x_t + \tau_{t+1})] - c(e_t) \tag{72}$$

Again we have the additional term τ_{t+1} and an exact specification of the value of money as the differences compared to equation 41. To get the Investors surplus from trade, we need to subtract his value function for the case when he doesn't find an agreement with the Entrepreneur, which is:

$$V_I(n) = \beta u[\phi_{t+1}(m_{t-1} - x_{t-1} + \tau_t)] \tag{73}$$

And if we subtract equation 73 from equation 72, we get:

$$\begin{aligned} S_I &= \beta u[\phi_{t+1}(m_{t-1} - x_{t-1} - k_t + x_t + \tau_{t+1})] - c(e_t) \\ &\quad - \beta u[\phi_{t+1}(m_{t-1} - x_{t-1} + \tau_{t+1})] \\ &= \beta u[\phi_{t+1}(-k_t + x_t)] - c(e_t) \end{aligned} \tag{74}$$

Now that we know the value functions of the two agents, we can turn to the maximization problem. We will only look at the case when the Entrepreneur

[12]Again, the value functions we get after replacement are used here.

has all the bargaining power this time, because we have already seen in section 3.3.1 that the economy breaks down if the Investors hold all the bargaining power, which still holds true with our new assumptions.
Here is the maximization problem the Entrepreneur has to solve:

$$\max_{x_t,e_t,k_t,l_t} \beta^2 u[\phi_{t+2}(m_t - x_t - k_{t+1} + x_{t+1} + \tau_{t+2})] - \beta c(e_{t+1}) \tag{75}$$

$$s.t. \qquad \beta u[\phi_{t+1}(-k_t + x_t)] - c(e_t) \geq 0 \tag{76}$$

$$x_t \leq m_t \tag{77}$$

$$k_t \leq m_{t-1} - x_{t-1} \tag{78}$$

$$x_{t+1} \leq f(e_{t+1}, \phi_{t+1}\frac{m_t - x_t}{w_{t+1}}) \tag{79}$$

$$w_t l_t = k_t \tag{80}$$

$$\phi_t w_t = \eta \tag{81}$$

The problem is very similar to the one we analysed in the extended model, but the difference is that all variables now possibly change from period to period, so we have to be careful about time indices. To find the solution, we start as always by setting the first constraint (equation 76) equal to zero, replace k_t with $w_t l_t$ (see equation 80) and solve for x_t. This yields:

$$x_t = \frac{c(e_t)}{\beta\phi_{t+1}} + w_t l_t \tag{82}$$

As we have already seen in section 3.3.2, this solution also satisfies the constraints 77 and 79, so the only constraint left is now equation 78, and we will take care of this by making a case distinction at the end.
Now that we know the value for x_t, we can put it into the maximization problem to find the solution. we can also replace m_t in the maximization problem with $\frac{f(e_t,l_t)}{\phi_t}$, and this gives us:

$$\begin{aligned}\max_{e_t,l_t} \beta^2[&\frac{\phi_{t+2}}{\phi_t} f(e_t, l_t) - \frac{\phi_{t+2}}{\beta\phi_{t+1}} c(e_t) - \phi_{t+2} w_t l_t \\ &-\phi_{t+2}(k_{t+1} - x_{t+1} - \tau_{t+2})] - \beta c(e_{t+1})\end{aligned} \tag{83}$$

And if we ignore all the terms that are not part of the maximization problem, we are left with:

$$\max_{e_t, l_t} \beta^2 [\frac{\phi_{t+2}}{\phi_t} f(e_t, l_t) - \frac{\phi_{t+2}}{\beta \phi_{t+1}} c(e_t) - \phi_{t+2} w_t l_t] \tag{84}$$

Maximization then yields the solutions for optimal effort level and optimal labour input, which are:

$$e_t^* : \quad \beta \frac{\phi_{t+1}}{\phi_t} f_e(e_t, l_t^*) = c_e(e_t) \tag{85}$$

$$l_t^* : \quad f_l(e_t^*, l_t) = \phi_t w_t = \eta \tag{86}$$

From equation 85 we see that monetary policy has an effect on the output level. Compared to the situation in the extended model, there is an additional factor apart from the discount rate that makes the agents value the revenue from production different to the cost from providing effort. If we are in an inflationatory regime, the fraction $\frac{\phi_{t+1}}{\phi_t}$ will be smaller than one, which means that production is lower than in the extended model without money growth. On the other hand, a deflation results in a higher production. However, if we compare this result to the social planner's solution (section 3.1), we see that it is optimal to keep the value of money constant.

As we are still only considering steady states, we assume that the value of money decreases (or increases) at a constant rate across periods. If this is the case, we learn from equation 85 that the production level stays constant over time.

We still have to take care of equation 78, but since equation 86 looks exactly the same as equation 64, we know what happens when this constraint binds from the extended model and there is no need to reproduce the derivation. Instead we can just look at the result, which is:

$$l_t = \frac{m_{t-1} - x_{t-1}}{w} \tag{87}$$

and:

$$\beta \frac{\phi_{t+1}}{\phi_t} f_e(e_t, \frac{m_{t-1} - x_{t-1}}{w}) = c_e(e_t) \tag{88}$$

So if the real money holdings of the Investor are not enough to pay for the efficient amount of labour, the Entrepreneur will just hire as much hours of labour as he can with the money holdings available, and the effort level is then chosen accordingly. As in the unconstrained case, the change in the value of money from period to period matters in determining the production level.

4.2 Equilibrium in the CM

Now that we have shown how the change in the value of money from period to period affects the equilibrium production level, we also have to check how the money growth affects the value of money. An important insight from the DM equilibrium in this context is that production stays at the same level across periods, and we will use this fact in order to determine the changes in the value of money.

The value of money in a given period can be found by setting the supply of goods in the CM equal to the money holdings of consumers (which are Retirees and Workers in the extended model). Total production hereby equals $\delta\sigma_E y_t$ as we know from the DM equilibrium. Money holdings of the consumers are now:

$$\delta\sigma_E(m_{t-2} - x_{t-2} - k_{t-1} + x_{t-1} + \tau_t) + \delta\sigma_E w_t l_t \tag{89}$$

In this equation, the first summand represents the money holdings of the Retirees while the second summand gives the total money holdings of Workers. Unlike in the extended model, this equation can not be simplified, because all variables now differ over time. But instead of working with this complex equation, we can take advantage of something else: We know that in a given period, the total money stock is M_t, and we also know how this money stock changes over time. Additionally, we know that the money stock can only be distributed in two different ways: If we are in a situation where it is optimal to spend all of the Investor's money holdings on labour, the money stock will be completely in the hands of the consumers before trade in the CM happens. If it is not optimal to spend all the money holdings of the Investors on labour, there will be some share of the money stock in the hands of the Investors, while the rest is in posession of the consumers. Let us first consider the simpler case where the Investors hold no money after the DM. In that case, the value of money is determined by:

$$\phi_t = \frac{\delta \sigma_E y_t}{M_t} \tag{90}$$

But more important than the absolute value of money is the relative value of money in a period compared to the period before, which is the rate of return on money. To get this ratio, we also need the value of money in period $t+1$, which is:

$$\phi_{t+1} = \frac{\delta \sigma_E y_{t+1}}{z M_t} \tag{91}$$

And the ratio between equations 91 and 90 gives us the rate of return on money, which is:

$$\frac{\phi_{t+1}}{\phi_t} = \frac{\frac{\delta \sigma_E y_{t+1}}{z M_t}}{\frac{\delta \sigma_E y_t}{M_t}} \tag{92}$$

From the analysis of the DM equilibrium, we know that the production level stays constant from period to period, so we can simplify 92 to get:

$$\frac{\phi_{t+1}}{\phi_t} = \frac{1}{z} \tag{93}$$

Which shows that the rate of return on money develops according to the inverse of the money growth rate.

This holds true for the case when the consumers hold all the money supply, but we also have to check what happens when some parts of the money stock are in the hands of the Investors. Let us now assume that it is not optimal to spend all the money holdings of Investors on labour, and therefore the Investors will still have positive money balances after the matching in the DM took place. Therefore, the Investors hold a part of the old money stock M_{t-1} which we denote with $1-\alpha$, while the Consumers own the rest of the old money stock plus the newly created money which the Retirees receive through lump-sum transfers. The newly created money in a given period equals:

$$M_t - M_{t-1} = z M_{t-1} - M_{t-1} = (z-1) M_{t-1} \tag{94}$$

And this means that the total money holdings of consumers are:

$$\alpha M_{t-1} + (z-1) M_{t-1} \tag{95}$$

Now we can again solve for the value of money in a given period by setting the supply of goods equal to the money holdings of the consumers, and this yields:

$$\phi_t = \frac{\delta \sigma_E y_t}{\alpha M_{t-1} + (z-1)M_{t-1}} \tag{96}$$

And if we do the same for the value of money in period $t+1$, we get:

$$\phi_{t+1} = \frac{\delta \sigma_E y_t}{\alpha z M_{t-1} + (z-1)zM_{t-1}} \tag{97}$$

Again, we are primarily interested in the rate of return on money, so by taking the ratio of equations 97 and 96, we get:

$$\frac{\phi_{t+1}}{\phi_t} = \frac{\frac{\delta \sigma_E y_t}{\alpha z M_{t-1} + (z-1)zM_{t-1}}}{\frac{\delta \sigma_E y_t}{\alpha M_{t-1} + (z-1)M_{t-1}}} = \frac{\alpha M_{t-1} + (z-1)M_{t-1}}{z(\alpha M_{t-1} + (z-1)M_{t-1})} = \frac{1}{z} \tag{98}$$

Which is exactly the same statement as we had in equation 93, so it doesn't matter for the rate of return on money whether the consumers hold the complete stock of money.

Now that we know how the growth rate of money and the rate of return on money are linked, we can exactly specify the effect of monetary policy. Equation 85 shows that the rate of return on money has the same influence as the discount factor, so the higher z is, the lower the production level becomes. On the other hand, lowering z can actually increase production. However, a comparison with the social planner's solution shows that it is optimal to set $z = 1$, i.e. to keep the money stock constant.

This is a typical finding in overlapping-generations models (see for example [Wallace, 1980]). On the other hand, models without the overlapping-generations structure (first shown in [Shi, 1997] for some special cases and later more general in [Lagos and Wright, 2005]) often find that it is optimal to set the growth rate of money equal to the discount factor, which is the so-called Friedman rule (see [Friedman, 1969]). See also [Kocherlakota, 2005] for an overview on optimal monetary policy in different models.

To sum up, this section about monetary policy shows that it is not possible to overcome the friction caused by the cash-in-advance constraint with the help of monetary policy. Because it is optimal in this model to keep the money stock constant, monetary policy only causes additional frictions by driving the amount of production away from the efficient level.

5 Limited commitment

So far, we implicitly assumed full commitment throughout this thesis. However, in reality there is often a problem of limited commitment, which means that agents in an economy can not be sure that their trading partners will deliver on their promises. In the context of the model presented in this thesis, limited commitment is a problem for the Investors as they are giving something to Entrepreneurs while getting their return only later. If we do not assume full commitment, it would thus be possible for the Entrepreneurs to cheat on the Investors by not giving them a share of the earnings from the production.

One could argue that due to the kind of relationship between Entrepreneur and Investor in our model that goes beyond a simple exchange of goods, limited commitment is not really an issue. However, if there is a problem of limited commitment, it is especially severe in the context of our model because the Entrepreneurs have nothing to offer at the beginning of their life, as was already mentioned in the introduction of this thesis. So while in other models the presence of fiat money can help to overcome commitment problems (see e.g. [Kocherlakota, 1998]) this is not feasible in this model.

5.1 The issue

If we look at the extended model and drop the assumption of full commitment, it is obvious that the Entrepreneur has a strong incentive to cheat. If the Entrepreneur cheats, which means he does not pay x_t to the Investor, this increases his own money holdings by exactly this amount x_t, while not changing his production opportunities as this payment only takes place after production. However, if we are in a situation with limited commitment, any Investor will be aware of this cheating opportunity and change his actions accordingly. As we are assuming rational and utility-maximizing agents, the Investor knows that the Entrepreneur will always cheat if cheating makes him better off. But if the Entrepreneur cheats, the Investor's utility from trade is negative because he has a cost of providing knowledge and he has to give away a part of his money holdings to pay the Workers. So the Investor will not be ready to make an agreement unless he could somehow ensure that the Entrepreneur will not be cheating, which is only possible if the Entrepreneur has some incentive not to cheat.

5.2 Exclusion from further trade

To give the Entrepreneurs an incentive not to cheat on their trading partners, we assume that an agent that cheats is excluded from all further trade in the DM.[13] This means that an agent that cheats as an Entrepreneur is not allowed to act as an Investor in his second period of life. For such a mechanism to help overcome a problem of limited commitment, it is necessary that the continuation value of an agent is higher than his benefit from cheating. As we know, the benefit from cheating is $\beta^2 u(\phi x_t)$, because this is the utility an Entrepreneur gets from not paying the Investor his share. On the other side, the value from being able to act as an Investor is exactly an Investor's surplus from an agreement as we derived it in equation 43. From the point of view of an Entrepreneur who is still in his first period of life, this surplus is:

$$\beta^2 u[\phi(-k_{t+1} + x_{t+1})] - \beta c(e_{t+1}) \tag{99}$$

So to ensure cooperation by exluding non-cooperative agents, the following condition has to hold:

$$\beta^2 u(\phi x_t) \leq \beta^2 u[\phi(-k_{t+1} + x_{t+1})] - \beta c(e_{t+1}) \tag{100}$$

[13]This is a standard assumption in monetary theory to overcome commitment problems, see e.g. [Kehoe and Levine, 1993] or chapter 2 in [Nosal and Rocheteau, 2011].

and this expression can be simplyfied and rearranged to:

$$\phi x_{t+1} \geq \phi(k_{t+1} + x_t) + \frac{c(e_{t+1})}{\beta} \tag{101}$$

It is clear that this equation can never hold as long as $x_t = x_{t+1}$. This shows that the economy breaks down if we are in the extended model without money growth. Even in non-stationary equilibria, 101 is very unlikely to achieve because it states that the Investor's share of the total profit must get larger in every period, but we know that increasing the Investor's share of profits reduces the future Investor's money holdings which in turn reduces the amount of workers they will be able to hire. This means that the total profit will be lower in every period, and even if production can be sustained for a couple of periods, the economy will eventually break down. However, to properly analyse this we should do a thorough analysis of non-stationary equilibria, which is not addressed in this thesis.

5.2.1 Inflation

As we have seen, excluding those agents from trade that do not pay back what they promised is not enough to ensure commitment in the standard environment. But there are circumstances that allow for an Equilibrium even with limited commitment, and these circumstances are inflation or economic growth. In this section we will look at the case of inflation, while economic growth will be covered in section 5.2.2.

We have analysed inflation in section 4 already and found that inflation is generally not a beneficial monetary policy. However, one feature of an inflationary regime is that the payment from Entrepreneurs to Investors, x, nominally grows with each period. In a situation with limited commitment, we can take advantage of this fact.

We know from the previous section that agents will only agree to trade if 101 holds. If we rearrange that condition, we see that it is the difference between

x_{t+1} and x_t that has to be large enough:

$$\phi_{t+2}(x_{t+1} - x_t) \geq \phi_{t+2}(k_{t+1}) + \frac{c(e_{t+1})}{\beta} \tag{102}$$

Note that we have added the time subscript $t+2$ to ϕ, because the Entrepreneur will consume two periods from now and therefore the value of money which prevails in that period matters to him. To see how much inflation is required for condition 102 to hold, we have to find a way to get the money growth rate into the condition, and we can do this by replacing x_{t+1}. From 82 we know that x_t equals $\frac{c(e_t)}{\beta\phi_{t+1}} + w_t l_t$, and that means x_{t+1} is:

$$x_{t+1} = \frac{c(e_{t+1})}{\beta\phi_{t+2}} + w_{t+1} l_{t+1} \tag{103}$$

From section 4.1 we know that the production stays constant over all periods, which also means that all real variables are constant across periods. Therefore we only have to replace ϕ_{t+2} and w_{t+1} in equation 103. This is easy for ϕ_{t+2}, because we know that $\frac{\phi_{t+2}}{\phi_{t+1}} = \frac{1}{z}$. If we rearrange that, we get:

$$\phi_{t+2} = \frac{\phi_{t+1}}{z} \tag{104}$$

Now let us turn to w_{t+1}. From our assumption on the labour market, we know that the real wage has to be equal to η in each period (see section 3). This means that the condition $\phi_t w_t = \phi_{t+1} w_{t+1}$ always holds. Rearranging this gives us:

$$\frac{w_t}{w_{t+1}} = \frac{\phi_{t+1}}{\phi_t} = \frac{1}{z} \tag{105}$$

And then we can solve this for w_{t+1} to get:

$$w_{t+1} = z w_t \tag{106}$$

With this knowledge, we can now turn to equation 103 again and replace the two terms ϕ_{t+2} and w_{t+1} with the solutions we found:

$$x_{t+1} = \frac{zc(e_{t+1})}{\beta\phi_{t+1}} + z w_t l_{t+1} = z(\frac{c(e_t)}{\beta\phi_{t+1}} + w_t l_t) = z x_t \tag{107}$$

And now we can go back to our condition 102 and replace x_{t+1} in there. Then we get:

$$(z-1)\phi_{t+2} x_t \geq \phi_{t+2}(k_{t+1}) + \frac{c(e_{t+1})}{\beta} \tag{108}$$

And then solving this expression for z yields:

$$z \geq \frac{\phi_{t+2}(k_{t+1}) + \frac{c(e_{t+1})}{\beta}}{\phi_{t+2}x_t} + 1 \tag{109}$$

Note that all the products on the right hand side of this equation are real terms, which means that they are all constant across periods. So equation 109 gives an exact value which is needed for z to sustain production in case of limited commitment. Because the right-hand side of equation 109 is always larger than 1, it is clear that inflation is required. As inflation drives the market away from the efficient solution, the monetary authority should set z such that 109 holds with equality in order to maximize welfare. This value of z just ensures that Entrepreneurs won't cheat while keeping the economy as close as possible to the social planner's solution.

This result shows that inflation is a means to overcome commitment issues. The reason for this is that the inflation tax lowers the benefit from cheating, and therefore it is beneficial to become an Investor. Berentsen, Camera and Waller [Berentsen et al., 2007] also found that inflation can be a way to achieve better equilibria in the absence of full commitment.

5.2.2 Growth

After having analysed the situation in an inflationary monetary regime, we now turn to economic growth. Let us assume in this section that the economy gets more productive in every period. This is expressed by a growth factor $A > 1$, which influences the production function. This means that output in a given period t is equal to the following equation:

$$y_t = A^t f(e_t, k_t) \tag{110}$$

This shows that, using the same amount of inputs, more output can be produced in later periods because the economy becomes more productive. If we assume that the same inputs are used in each period, this means that the output from this period can be expressed as a function of the output produced in the previous period:

$$y_{t+1} = Ay_t \tag{111}$$

As stated before, this is only true if the inputs used in each period are constant. This will generally not be the case, as it is optimal to use more knowledge and labour if the productivity increases. For simplicity though, we will assume throughout this section that the inputs are held constant from period to period. We denote these constant amounts of knowledge and capital as e'_t and k'_t. Furthermore, we also assume in this section that we are in a situation where the money holdings of the Investors are less than what would be required to pay for the efficient amount of labour (which means that equation 65 does not hold). These assumptions allow to express the required growth rate in simple terms. At the end of this section, we will relax these assumptions and see how they influence the solution.

Additionally, we also have to make an assumption about monetary policy. In the presence of economic growth, the amount of goods sold in the CM grows with each period. If the money stock is kept constant in such a situation, the value of money has to increase with each period to clear the market. So to keep things simple here and to be able to focus solely on growth, we assume that the value of money is kept constant by the monetary authority.

From equation 101, we know the necessary condition to make exclusion from trade a punishment severe enough to ensure commitment. If we are in a growth environment, we also know that the future payment x_{t+1} can be higher than x_t as the total production is growing. Because the payment to the Investor can never be higher than the total money received, we also know that the following condition has to hold:

$$\phi x_{t+1} \leq \phi m_{t+1} = y_{t+1} = Ay_t \tag{112}$$

So to find the minimal growth rate required, we can just assume the above expression to hold with equality and then replace x_{t+1} in equation 101, which yields:

$$Ay_t \geq \phi(k'_{t+1} + x_t) + \frac{c(e'_{t+1})}{\beta} \tag{113}$$

And because we have assumed that agents spend all their money holdings on labour, we also know that $k'_{t+1} = m_t - x_t$, and if we rearrange this we can replace $\phi(k'_{t+1} + x_t)$ with y_t in equation 113 to get:

$$Ay_t \geq y_t + \frac{c(e'_{t+1})}{\beta} \tag{114}$$

And some more rearrangement then leads to:

$$(A-1)y_t \geq \frac{c(e'_{t+1})}{\beta} \tag{115}$$

This statement has the following implications: Whenever the growth rate of productivity is higher than the discounted cost of providing knowledge (at the fixed level e'_t), an agreement in presence of limited commitment is possible. If equation 115 holds, exclusion from trade is a punishment which is severe enough to ensure that agents will act as promised. Also note that if equation 115 holds with equality, it implies that Entrepreneurs have to pay all the money they earn to the Investors (Due to equation 112).

If we relax the assumptions about constant input factors, equation 115 can not be applied anymore because some steps in its derivation were only possible with these assumptions. However, relaxing the assumptions does also relax the condition required for the growth rate. As the productivity is growing, it will also pay off to use more knowledge, which in turn means that output in period $t+1$ compared to output in period t will be growing by more than just the growth rate of productivity A. Using a higher amount of knowledge obviously also increases the knowledge cost of the Investor, but total output will grow by more than his effort cost. As there is no further restriction on the payment to the Investor x_t apart from the fact that it cannot exceed the total gains from production m_t, this means that using a higher level of knowledge makes production possible in the absence of full commitment even at a lower growth level than the one we derived in equation 115.

The same is true for the assumption on capital. First of all, note that if we are in a situation in which it is efficient to use all the money holdings of the Investor to pay for labour, capital has to be constant across periods anyway so the two assumptions we made on capital actually coincide. Now let us see what happens if we are initially in a situation where the amount of money Investors are holding is higher than what the Entrepreneur needs to pay for the efficient amount of labour: In this case, $k_{t+1} < m_t - x_t$, which means that we can't derive equation 114 the way we did it. We can then only rearrange equation 113 to find the condition for the minimal required growth rate:

$$Ay_t - \phi(k_{t+1} + x_t) \geq \frac{c(e'_{t+1})}{\beta} \tag{116}$$

This expression is not very revealing, but since we know that we are now in a situation where $\phi(k_{t+1} + x_t) < y_t$, we can tell that the required growth rate in this case is smaller than the one we derived in equation 115.

So to sum up, we have seen that equation 115 states the minimal required growth rate to sustain production even if there is no commitment. We needed some assumptions to derive this growth rate, but we have seen that relaxing these assumptions is also relaxing the required growth rate, making equation 115 some kind of minimal required growth rate in the worst case scenario. Also note that even if equation 115 does not hold, it might be possible to get sustain production by lowering the effort level in each period, but then we are in a situation similar to the one described in section 5.2, so the arguments stated there also apply here.
If there is economic growth but it is not large enough to make the commitment problem disappear, the monetary authorities can still use inflation to solve the commitment problem as we have seen in section 5.2.1. Combining these two elements then of course lowers the required growth rate as found in equation 115 as well as the required rate of money growth found in equation 109 which are needed to solve the commitment problem.

6 Conclusion

In this master's thesis, I developed a model about the matching between Entrepreneurs and Investors. The goal of the thesis was to create a framework which helps to analyse the issues faced by young Entrepreneurs that want to build start-up companies and Investors seeking to invest in such companies. The model created allows for a couple of interesting insights on that issue. Probably the most important result is that a possible constraint for the efficiency of such a business arises from the real money balances of Investors, meaning that an efficient equilibrium can't be reached if money balances of the Investors ar too low. In the model, this is equivalent to saying that it is beneficial for all agents if the Entrepreneurs hold the bargaining power instead of the Investors. However, even if Entrepreneurs hold the bargaining power, an efficient solution might still not be feasible due to insufficient real money holdings of Investors. This friction stems from the cash-in-advance constraint on Worker's payments. Of course, this result about the bargaining power is heavily influenced by the OLG structure of the model. However, if we analyse the result, we see that the driving force behind it is the fact that Entrepreneurs use their gains for investments, while Investors will use it for consumption. From that perspective, we can see it as a general result that it is better for the economy when the gains from a company go to an agent that will invest such revenues. If we think of an Entrepreneur as an agent that has good ideas about how to make money, it is therefore beneficial if he gets a higher share of the gains than an Investor who can only provide money.

The monetary policy analysis also delivered some interesting results. It showed that a constant money stock is optimal and that an active monetary policy is only adding an additional friction instead of eliminating the friction caused by the cash-in-advance constraint. However, monetary policy can be beneficial in a different way: In the absence of full commitment, the economy will break down with a constant money stock and no productivity growth. But if there is a sufficiently high rate of money growth, Entrepreneurs will be prevented from cheating because the inflation tax makes cheating unattractive. This means that an inflationary monetary regime is the first-best solution in the absence of full commitment and productivity growth. Finally, we have seen that economic growth can solve the commitment problem as well, and that a combination of economic growth and inflation makes the conditions required to solve the commitment problem easier to meet.

The model as it is presented in this thesis could be developed further in a couple of directions. One approach would be to introduce uncertainty in the outcomes of the Entrepreneur's projects and also add financial intermediation, which would allow to model the investment decisions more precisely and might lead to a couple of additional insights. The other road to pursue with this model would be to integrate it into a more standard economic model as the 'innovation sector', making it an endogenous source of economic growth. With this strategy, one could get an economic model in which economic growth is based on solid microfoundations, allowing to analyse the effects of different policies on economic growth.

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Plagiatserklärung

Ich bezeuge mit meiner Unterschrift, dass meine Angaben über die bei der Abfassung meiner Arbeit benutzten Hilfsmittel sowie über die mir zuteil gewordene Hilfe in jeder Hinsicht der Wahrheit entsprechen und vollständig sind.
Ich habe das Merkblatt zu Plagiat und Betrug vom 22. Februar 2011 gelesen und bin mir der Konsequenzen eines solchen Handelns bewusst.

Datum: Unterschrift: