$a \in \mathbb{R}$

$$f(x,y) = \frac{1}{2}x^2 - x + ay(x-1) - \frac{1}{3}y^3 + a^2y^2$$

$$\text{I} \quad f_x = x - 1 + ay = 0 \iff x-1 = -ay$$

$$\text{II} \quad f_y = a(x-1) - y^2 + 2a^2y = 0$$

$$f_{xx} = 1$$

$$f_{yy} = 2a^2 - 2y \qquad H_f = \begin{pmatrix} 1 & a \\ a & 2a^2-2y \end{pmatrix}$$

$$f_{xy} = a$$

$$\text{I in II} \quad -a^2y - y^2 + 2a^2y = 0$$

$$\iff (a^2 - y)\cdot y = 0$$

$$x = 1 - a^3 \leftarrow y = a^2 \qquad y = 0 \rightarrow x = 1$$

$$\underline{\underline{(1-a^3, a^2)}} = \vec{x}_1 \qquad \underline{\underline{(1,0)}} = \vec{x}_2$$

<u>Fall</u> $a \neq 0$

$(1,0)$ $\quad H_f(1,0) = \begin{pmatrix} 1 & a \\ a & 2a^2 \end{pmatrix}$ pos. def.

$\rightarrow$ Minimum

$(1-a^3, a^2)$ $\quad H_f = \begin{pmatrix} 1 & a \\ a & 0 \end{pmatrix}$ indefinit

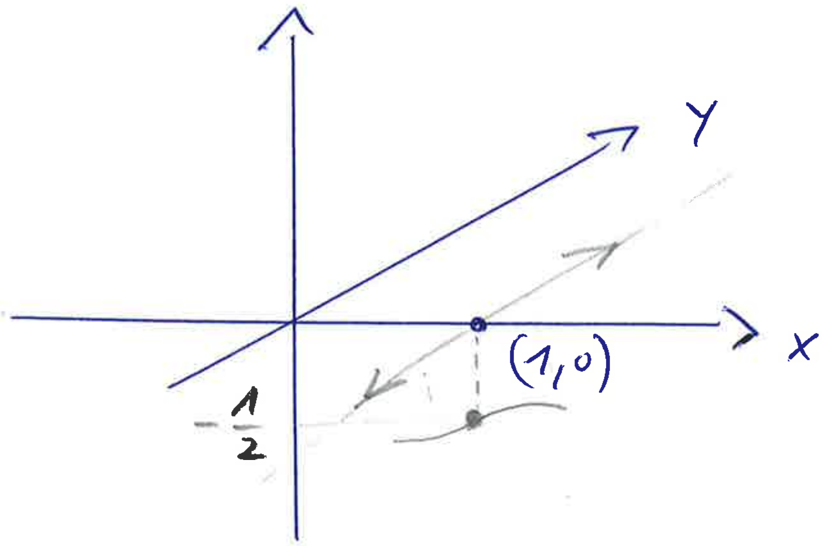
$\rightarrow$ Sattel

Fall $a = 0$ $\quad \vec{x}_1 = \vec{x}_2 = (1,0)$

$$H_f(1,0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$ pos. semi def. ?

$$f(x,y) = \frac{1}{2}x^2 - x - \frac{1}{3}y^3$$

Untersuche f auf $\begin{pmatrix}1\\0\end{pmatrix} + t\begin{pmatrix}0\\1\end{pmatrix}$!



$$f(1,0) = -\frac{1}{2}$$

$$f(1,t) = -\frac{1}{2} - \frac{1}{3}t^3 \begin{cases} < -\frac{1}{2} & t > 0 \\ > -\frac{1}{2} & t < 0 \end{cases}$$

Sattel !