Economics of water resources: *Sustainability, multi-stage control, competitive usage*

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Plan of the lecture

- Basic water management framework;
- Gisser&Sanchez (1980): Competitive usage is good
- Hediger (2004): Sustainable groundwater mining: Multistage control;
- Groundwater usage in case of multiple crops.

Basic setup I

- Farmers i at the area A produce crops with the use of single input, water:

$$y_i(t) = f(q_i(t))$$

  with $y_i(t)$ being crop of farmer i at time t.
- The profit of an individual farmer is thus:

$$\pi_i(t) = p(t)f(q_i(t)) - w(t)q_i(t) - k,$$

- Profit-maximization yields the water demand:

$$\frac{\partial \pi_i}{\partial q} = 0 \rightarrow pf'_q - w = 0$$
$$\rightarrow q(w) = g^{-1}(w/p)$$

Basic setup II

- If the total area A is cultivated the total profit (quasi rent) and total demand are:

$$\Pi = A\pi,$$
$$Q = Aq(w) = Af'^{-1}(w/p)$$

- Price of water w may be defined as:
  1. **Open access**: no market price ($w = 0$), implicit price is given by (short-run) pumping marginal costs, leading to (Koundouri (2004)) overexploitation and misallocation.
  2. **Social planner**: w is set at intertemporal efficient level (co-state from optimal control)
  3. **External backup source**: farmers choose groundwater or water form the channel...

Types of dynamics

- There is no regeneration: Hotelling-type;

$$\dot{V} = -Q(t)$$

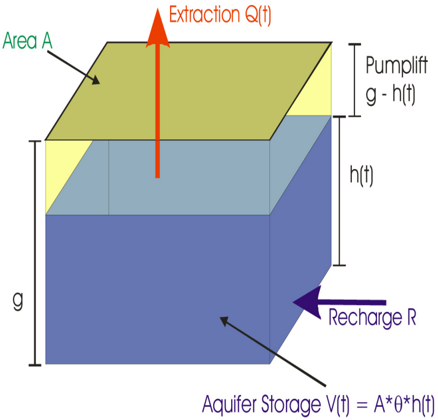
- There is (fixed) exogenous regeneration rate: two phases of usage;

$$\dot{V} = \bar{R} - Q(t)$$

- Regeneration rate is **endogenous**

$$\dot{V} = -Q(t) + \alpha(\bar{V} - V(t))$$

Schematic representation



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Technicalities

- The volume $V(t)$ changes proportionally to the *table height* $h(t)$;
- All farmers are identical and we care only about *aggregate* behavior;
- Extraction equals demand (consumption): $Q(t) = Q^D$;
- Water dynamics is obtained by summing rate of inflow R and rate of consumption $Q(t)$.

Overview

- Compares open access (competitive usage) with optimally controlled one;
- Find out there is **no difference**;
- Known as Gisser-Sanchez Effect (GSE), proven wrong only recently;
- GSE holds for some parameter constellations and not for others;
- Frequently the case with free-market protagonists.

Open access

- Under open access farmers freely pump water;
- The only cost is the marginal cost of extraction,
- Farmers equate marginal revenue and marginal costs:

$$Q^D = g + kP;$$
$$\bar{P} = C_0 + C_1 h(t)$$

- The resulting **uncontrolled** total water demand:

$$Q = d + C_1 h(t),$$

- The change in **water table height** is difference in inflow $\bar{R}$ and demand (consumption) Q^D:

$$\dot{h} = (\bar{R} - Q^D)/A\theta = (\bar{R} - d - C_1 h(t))/A\theta \qquad (1)$$

Optimal control

- Now assume farmers maximize (intertemporally) total profit:

$$\Pi \overset{def}{=} \int_0^\infty \mathrm{e}^{-rt} \{\pi(t)\}\, dt,$$

$$\pi(t) = \int_0^Q P(Q) dQ Q(t) - \bar{P} Q(t)$$

- Which results in the **optimal control problem**:

$$\max_Q \int_0^\infty \mathrm{e}^{-rt} \left\{ \frac{1}{2k} Q^2 - \frac{g}{k} Q - (C_0 + C_1 h) Q \right\} dt,$$

$$s.t.$$

$$\dot{h} = (\bar{R} - Q)/A\theta,\ h(0) = h_0 \tag{2}$$

- This takes into account intertemporal effects of water consumption for each farmer.

Intertemporally optimal solution

We follow standard steps to solve the optimal control problem above:

- Construct the Hamiltonian using (2):

$$\mathcal{H} = -\mathrm{e}^{-rt}\left[\frac{1}{2k}Q^2 - \frac{g}{k}Q - (C_0 + C_1 h)Q\right] + \lambda\left[(\bar{R} - Q)/A\theta\right]$$

- Derive first-order condition on control:

$$Q^* : \frac{\partial \mathcal{H}}{\partial Q} = -\mathrm{e}^{-rt}\left[\frac{1}{k}Q - \frac{g}{k} - (C_0 + C_1 h)\right] - \frac{\lambda}{A\theta} \quad (3)$$

- Derive the co-state equation:

$$\dot{\lambda} = -\frac{\partial \mathcal{H}}{\partial h} = -\mathrm{e}^{-rt}C_1 Q \quad (4)$$

Deriving water demand

- Use F.O.C. (3) to obtain λ:

$$\lambda = A\theta e^{-rt}\left[\frac{1}{k}Q - \frac{g}{k} - (C_0 + C_1 h)\right]$$

- Use co-state (4) to obtain water pumping dynamics:

$$\dot{\lambda} = -e^{-rt}C_1 Q = \frac{d}{dt}\left\{A\theta e^{-rt}\left[\frac{1}{k}Q - \frac{g}{k} - (C_0 + C_1 h)\right]\right\}$$

- We thus have water demand **dynamics**:

$$\dot{Q} = rQ - rkC_1 h + \left\{kC_1\bar{R}/A\theta - rg - rkC_0\right\} \tag{5}$$

Main argument: Water demand

- Under optimal control the demand is solution to (5):

$$Q^F = -\bar{R} + \left(\bar{R} + g + k(C_0 + C_1 h)\right) \mathrm{e}^{kC_1/A\theta t}$$

- Under free competition the demand is solution to (1):

$$Q^{OPT} = B\mathrm{e}^{x_2 t} - M/m,$$

where

$$x_2 = \frac{r - \sqrt{r^2 - 4nm}}{2},\ n = rkC_1, m = 1/A\theta$$

giving condition for equivalence:

$$x_2 \approx kC_1/A\theta$$

Flaws

- Equivalence is reached only for **large** $A\theta$;
- Approximation does not take into account B;
- Direct analytic solution of the **optimal system**:

$$\dot{h} = (\bar{R} - Q)/A\theta;$$
$$\dot{Q} = rQ - rkC_1 h + \left\{kC_1 \bar{R}/A\theta - rg - rkC_0\right\}$$

and comparison with solution of (1)

$$\dot{h} = (\bar{R} - g - kC_0 - C_1 h(t))/A\theta \tag{6}$$

reveals substantial differences.

Illustration: parameters as in the paper, $T = 100$

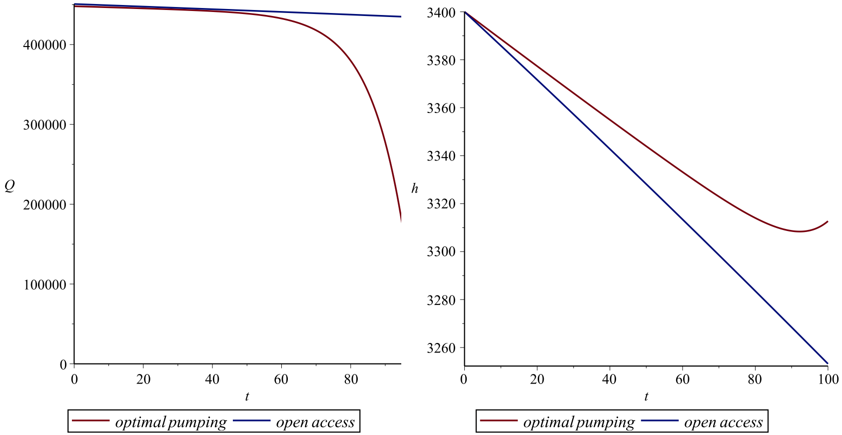


Illustration: $A\theta$ 10x as low

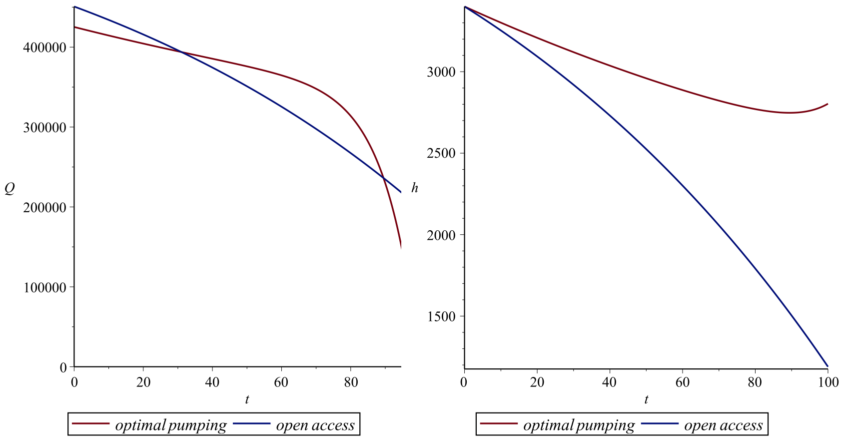
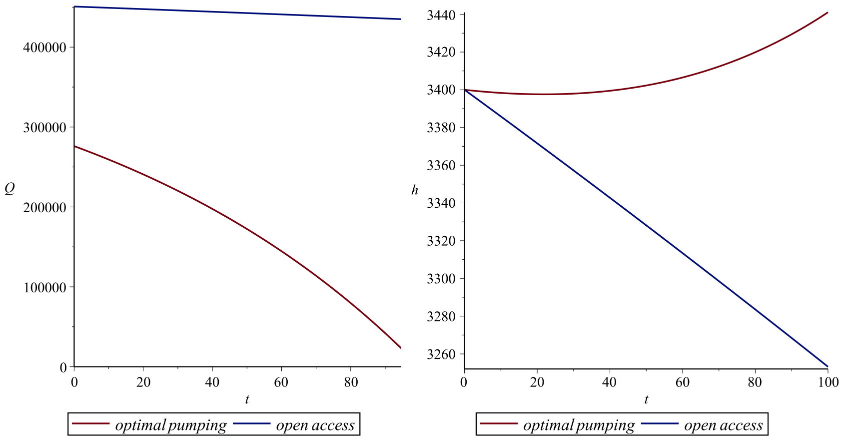


Illustration: $r = 0.01$



Remarks on GSE:

- The effect holds only in *non-generic* situation;
- If we change parameters, it disappears;
- Even for proposed range the effect vanishes a t $t \to T$ if T is finite;
- Only infinite-time, high r, large surface case supports GSE.

Social planner: Problem statement

- The social planner takes into account **intertemporal** effects:

$$\max_{Q(\bullet)} \int_0^T \mathrm{e}^{-rt} \{w - C(V)\} Q dt$$
$$s.t.$$
$$\dot{V} = \bar{R} - Q, \ V_0 = A\theta h_0, \ V_T = V_{min}, \ T = free \qquad (7)$$

- At the first stage water is pumped out till the level V_{min} is reached;
- Afterwards the water is used at the regeneration rate $\bar{R}$.

Multi-stage optimal control

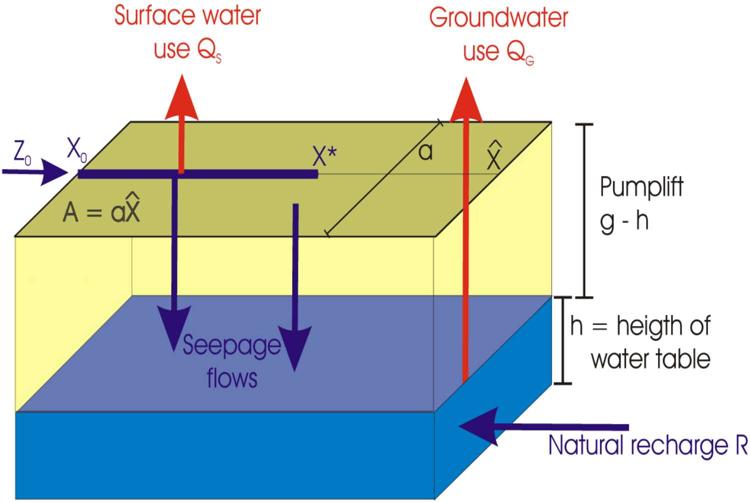
Idea of multi-stage control

- Takes into account the marginal costs of pumping and opportunity costs;
- The problem solves for optimal pricing rule before the minimum V is reached;
- Afterwards the **sustainable** solution is used, namely:

$$Q^S = \bar{R} | t \leq T : V = V_{min}. \tag{8}$$

- This is the concept of **multi-stage optimal control** in time
- It can be applied **in space** also (channel length, allocation choice)

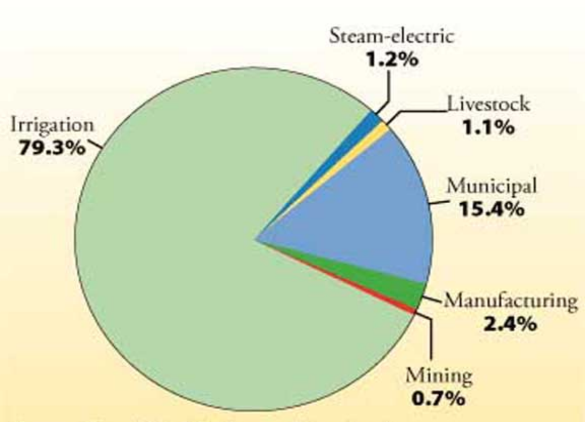
Space-time optimization case



Idea

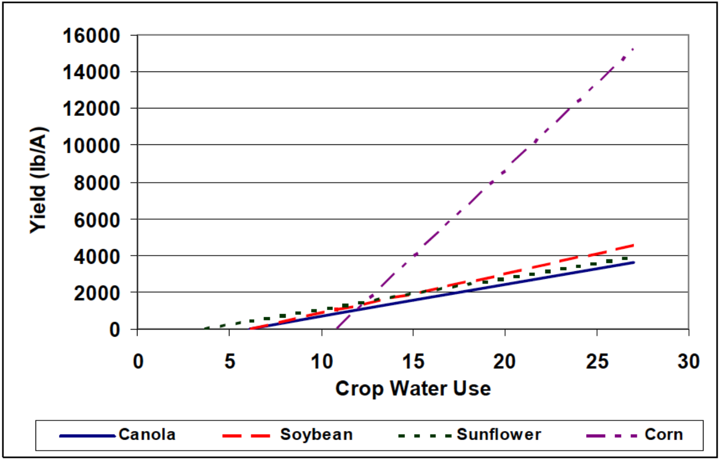
- Different crops have different water reqs and associated costs/profits;
- One water source can be used for competitive irrigation of different crops;
- The problem of which crop to cultivate where and how much water to use are thus interdependent;
- **Adaptation** of irrigation system and the spectrum of cultivated plants is important.

Structure of water usage, Texas, U.S.



Sources: Texas Water Development Board and Texas Comptroller of Public Accounts.

Crop Water Use (Aiken et al., 2011)



Questions

- Which crops one has to cultivate at what time?
- How the groundwater usage changes with adaptation?
- What is the role of scarcity/profitability in the evolving adaptation structure?

Model: (Kim et al., 1989)

- n crops with different water requirements;
- With each crop there is associated linear demand function;
- Different backstop price levels form the piecewise-linear demand for water;
- Cost of pumping are dependant on the current level of the water table;
- We compare social planner (intertemporal) and common property (aka free competition) solutions.

Some equations

Inverse demand for ith crop:

$$P = \frac{a_i}{b_i} - \frac{1}{b_i} W_i; \tag{9}$$

Marginal pumping cost function:

$$MC = k_1(SL - h) = k_2 - k_1 h; \tag{10}$$

Groundwater hydrological equation:

$$\dot{h} = \frac{R + (k_3 - 1)\sum_{i=1}^{n} W_i(t)}{AS}. \tag{11}$$

Solution concept: Planning equilibrium

- The solution procedure consists of n stages;
- At each stage the number of cultivated crops **reduces** by 1;
- Begin with ranking crops w. r. t. the choke price;
- Solve the optimal control problem for n crops for $[0; T_1]$with T_1 defined by zero 1st crop water consumption;
- Repeat the procedure for the time period $[T_1; T_2]$ for $n-1$ crops;
- Repeat till only one crop is left;
- Solve the problem with 1 crops for $[T_n; \infty)$.

Solution concept: Common property equilibrium

- Equate marginal costs of pumping with inverse demand for water for each crop i;
- Express the demand for water of each crop W_i;
- Substitute this into the **hydrological equation** to get evolution of h;
- Solve the resulting ODE to obtain phase 1 water depletion;
- Set $W_1 = 0$ and express $t = T_1$ the first switch time from the water demand;
- Repeat the procedure for $n - 1$ crops from time T_1 until 1 crop remains;
- Solve the last stage at infinity.

Solution properties

- The total groundwater height h depletes with time;
- Crops with higher water demand are used at initial phases only;
- In the given example for Texas the difference between regimes is very low;
- However the theoretical inefficiency remains: dynamic vs. static efficiency;
- The importance of the discount factor.

Solution (Kim et al., 1989) depending on disocunt factor

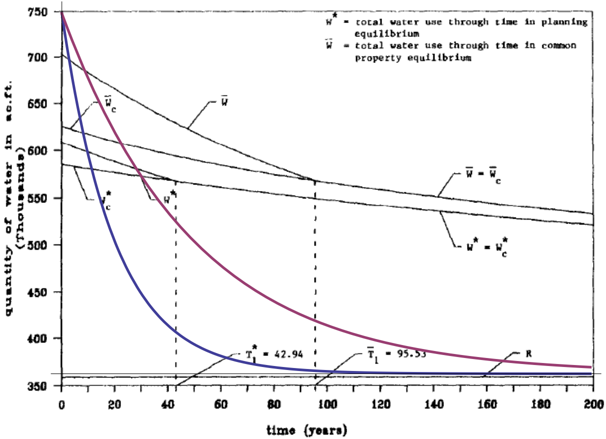


FIG. 1. Groundwater mining paths and water allocation pattern between crops for planning equilibrium and common property equilibrium.

Conclusions

- Optimal control methods at micro-level resource management;
- Concept of multi-stage optimal control and sustainable resource usage;
- Role of discount factor: dynamic efficiency;
- Common property equilibrium and social optimum differences.

Next lecture

- Role of strategic interactions in open access resource usage;
- Introduction to differential games theory;
- Model of shallow lakes: interactions, irreversibility, multiple equilibria;
- Paper: Mäler K-G., Xepapadeas A., De Zeeuw A. (2003) The Economics of Shallow Lakes. *Environmental and resource Economics, 26*: pp. 603-624.