Directed technical change in endogenous growth theory

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Outline

Origins

- The idea of **direction** of technical change has been discussed already in 1960-s
- Technical change may be **biased** towards one or another factor of production
- These concepts has been formulated in papers on induced innovations
- This framework relies on the concepts of:
  - **Innovations possibility frontier** (Kennedy (1962))
  - **Bias of technical change** vs rate of technical change.

Empirical foundations

- The technical change in the 19-th century has been capital-biased
- However, in 1970-s this bias has changed
- The new, **skill-biased**, technical change required another approach
- This skill-biased technical change has **delayed** structure
- Explanation is needed, why skill-biased technical change leads to higher inequality
- **What is the nature of technical change today?**

New Growth Theory era

- These ideas of the biased technical change has been long unused
- With NGT the interest to these models re-emerged
- Models of Acemoglu and others tried to include the notion of bias into quality ladders and variety expansion type models
- These models usually distinguish between **two** possible directions of technical change
- At the same time technical change is modelled along the path of NGT
- Combination of new ideas of endogenous technology and old ideas of the biased technology.

Shortcomings

This most recent framework of technical change has shortcomings:

- The number of sectors is usually limited to 2
- The range of intermediaries is allowed to change in both sectors, but quality of intermediaries is constant
- No new sectors are allowed to appear.

These shortcomings are being dealt with (currently) by so-called **2-nd generation R&D-based models**:

- Allows for variety expansion and quality ladders simultaneously: Peretto, Connolly (2007)
- Allows for dynamic number of sectors: Chu (2011)
- Allows for both dynamic sectors and 2-dim. R&D: Bondarev&Greiner (2017).

Overview

- Is considered to be classical as of today
- Combines ideas of technological bias with Grossman-Helpman economy
- There are two possible directions of the bias:
  - Labour-augmenting
  - Capital-augmenting.
- There exists **equilibrium bias** of technology
- This bias is subject to 2 effects:
  - Price effect, favouring bias towards scarce factor;
  - Market size effect, favouring abundant factor.
- Relative scarcity of production factors is the main drive of the **direction of technical change**!

Production technology

Consider the aggregate production function:

$$Y = F(L, Z, A),$$

where L is labour, Z is anything except labour (capital, human capital, land, Martians..) and A is technology.

- Technical progress A is *L-augmenting*, if:

$$\frac{\partial F}{\partial A} = \frac{L}{A}\frac{\partial F}{\partial L}$$

and *Z-augmenting* defined similarly through Z.

Technological bias

Details On the other hand, **technical change** is *L-biased*, if:

$$\frac{\partial \frac{\partial F/\partial L}{\partial F/\partial Z}}{\partial A} > 0,$$

That is, technology increases productivity of L to a greater extent, than that of Z.

- Whether technical change is L or Z -augmenting, depends on the shape of production function
- The *bias* of technical change depends, in contrast, on the ratio of **marginal products**
- This is (partial) intuition, why only 2 factors may be considered.

Representative consumer

Representative consumer has standard life-time utility:

$$\max_{C} \int_{0}^{\infty} \mathrm{e}^{-\rho t} \frac{C^{1-\theta} - 1}{1 - \theta} dt$$

subject to the budget constraint:

$$C + I + R \leq Y \equiv [\gamma Y_L^{\alpha} + (1 - \gamma) Y_Z^{1-\alpha}]^{1/\alpha}$$

where I are investments into capital and R are R&D expenditures, and Y_L, Y_Z are two intermediate products.

Intermediate products

These are produced from the range of technologies (machines) being used in both sectors (sector-specific):

$$Y_L = \frac{1}{1-\beta}\Big(\int_0^{N_L} x_L(j)^{1-\beta} dj\Big) L^{\beta},$$

$$Y_Z = \frac{1}{1-\beta}\Big(\int_0^{N_Z} x_Z(j)^{1-\beta} dj\Big) Z^{\beta}.$$

where β is the productivity parameter (identical across sectors) and L, Z are total supplies of both factors in the economy (inelastical).

Equilibrium

In this economy, equilibrium is given by:

- Set of **prices of machines** for both sectors,maximizing profits of technology monopolists, $\chi_L(j), \chi_Z(j)$
- **Machine demands** from two intermediate sectors,maximizing intermediate goods producers profits, $x_L(j), x_Z(j)$
- **Factor prices**, that clear factor markets, ω_L, ω_Z
- **Product prices**, that clear product markets, p_L, p_Z.

This equilibrium is unique **given evolution of technologies**, N_L and N_Z.

Products markets clearing

Two intermediate products markets are competitive, and thus their prices ratio is proportional to marginal costs of production:

$$p \equiv \frac{p_Z}{p_L} = \frac{1-\gamma}{\gamma}\left(\frac{Y_Z}{Y_L}\right)^{-1/\epsilon} \tag{1}$$

where $\epsilon = \frac{1}{1-\alpha}$ is the elasticity of factor substitution.
Taking price of the final good as a numeraire, we have another relation between prices:

$$\gamma p_L^{1-\epsilon} + (1-\gamma) p_Z^{1-\epsilon} = 1,$$

which together with (1) defines both prices.

Demand: Labour-intensive sector

formulas Demand in labour-intensive is the result of profit maximization of producers:

- Usual first-order conditions yield
  1. Demand for labour-augmenting intermediaries, $x_L(j)$ as function of prices $p_L, \chi_L(j)$,
  2. Price for labour (as inverse of demand) ω_L as function of $x_L(j), p_L$.
- The major role is played by the **intermediate demand**

Demand: Capital-intensive sector

formulas Demand in capital-intensive sector is defined in a similar way:

- Profit maximization yields:
  1. Demand for Z-augmenting intermediaries, $x_k(j)$ as function of prices $p_Z, \chi_Z(j)$,
  2. Price for Z, ω_Z, as an inverse demand for this factor being function of $x_Z(j), p_Z$.
- Again **intermediate demand** is of importance.

Prices of machines

formulas

- Each monopolist in each sector faces constant marginal costs of producing machines, ψ
- Then the profit of the monopolist may be written in per unit terms
- Since the demand for machines in both sectors is isoelastic, producers are monopolists, the prices $\chi_{L,Z}$ are similar and are the constant mark-up over costs ψ
- Normalizing $\psi = 1 - \beta$, prices are equal:

$$\chi_L(j) = \chi_Z(j) = 1, \forall j. \tag{2}$$

Value of technology

- Using Eqs. (2), (35), (38) we rewrite profits:

$$\pi_L = \beta p_L^{1/\beta} L, \pi_Z = \beta p_Z^{1/\beta} Z \tag{3}$$

- Technology monopolist is interested in the total discounted stream of profits, rather than in the instantaneous profit
- This value is obtained via the standard HJB dynamic equations:

$$rV_L - \dot{V}_L = \pi_L, \; rV_Z - \dot{V}_Z = \pi_Z$$

- In the steady state $\dot{V}_{L,Z} = 0$, giving expressions for value functions:

$$V_L = \frac{\beta p_L^{1/\beta} L}{r}, V_Z = \frac{\beta p_Z^{1/\beta} Z}{r}.$$

Price effect and market size effect

Two forces determine the direction of technical change (relative N_L/N_Z change):

$$V_L = \frac{\beta {p_L}^{1/\beta} L}{r}, V_Z = \frac{\beta {p_Z}^{1/\beta} Z}{r}. \tag{4}$$

1. Price effect: targets more expensive goods (influence of p_L, p_Z)
2. Market size effect: targets more abundant factor (influence of L, Z).

- Innovations are directed at the more profitable factors (as of early induced innovations literature);
- The equilibrium **bias** is defined by relative change in factors supply, Z/L and associated price changes, p_Z/p_L.

Elasticity of substitution

Define ϵ, σ to be elasticity of substitution in consumption and production

- With symmetric prices (and thus demands) for both types of machines outputs are functions of **ranges**, N_L, N_Z
- Giving relative output price as a function of the **ratio of technical change**:

$$p = \left(\frac{1-\gamma}{\gamma}\right)^{\beta\epsilon/\sigma} \left(\frac{N_Z Z}{N_L L}\right)^{-\beta/\sigma} \tag{5}$$

Details

- Relative profitability of research is function of technical change and scale:

$$\frac{V_Z}{V_L} = \left(\frac{1-\gamma}{\gamma}\right)^{\epsilon/\sigma} \left(\frac{N_Z}{N_L}\right)^{-1/\sigma} \left(\frac{Z}{L}\right)^{\frac{\sigma-1}{\sigma}}$$

Details

Overview of substitution role

- The value of σ defines, whether factors are gross substitutes or complements:
  - With $\sigma > 1$ factors are gross substitutes and market size effect dominates
  - With $\sigma < 1$ they are gross complements and price effect dominates
- The gross substitutability of factors also plays a role in the way, technical progress bias influences factor prices:
  - With $\sigma > 1$ the increase in N_Z/N_L increases ω_Z/ω_L, as is predicted by the old induced innovations literature
  - However, with $\sigma > 1$ the relation is reversed
- This effect appears because factor prices depend on the marginal product increase of factors, not on their physical productivity increase.

Innovations possibility frontier

- The form of **innovations possibility frontier** defines the form of stead-state dynamics
- Two different forms are considered:
  - **Lab equipment model**, where technical change depends only on the final good spending:

$$\dot{N}_L = \eta_L R_L, \dot{N}_Z = \eta_Z R_Z \tag{6}$$

  - **Productive knowledge model**, where previous knowledge has positive spillovers on current technology:

$$\dot{N}_L = \eta_L N_L^{1+\delta} N_Z^{1-\delta} S_L, \dot{N}_Z = \eta_Z N_L^{1-\delta} N_Z^{1+\delta} S_Z. \tag{7}$$

- They result in different degrees of *state dependence* of R&D.

Lab equipment model

$$\dot{N}_L = \eta_L R_L, \dot{N}_Z = \eta_Z R_Z. \tag{8}$$

- There is no state dependence of ratio of technical change in this model:
$$\frac{\partial \dot{N}_Z / \partial R_Z}{\partial \dot{N}_L / \partial R_L} = \frac{\eta_Z}{\eta_L} = const. \tag{9}$$
- Varieties of both types grow proportionally to the resources (in terms of final output) being used
- Coefficients η_Z, η_L allow the productivity in two kinds of knowledge to be different.

BGP in lab equipment model

- BGP is defined as a path, where:
  - Prices of the output of both sectors are constant,
  - N_Z and N_L grow at the same rate.
- These implies technology market clearing condition:

$$\eta_L \pi_L = \eta_Z \pi_Z; \tag{10}$$

- Using expressions for prices and profits, Eqs. (5) and (3) this gives ratio of technical changes as functions of factors:

$$\frac{N_Z}{N_L} = \eta^{\sigma} \left(\frac{1-\gamma}{\gamma} \right)^{\epsilon} \left(\frac{Z}{L} \right)^{\sigma - 1}, \tag{11}$$

where $\eta = \frac{\eta_Z}{\eta_L}$.

Bias of technical change in lab equipment model

- The case $\sigma > 1$:
  - The higher Z/L ratio *increases* N_Z/N_L and physical productivity of abundant factor will be *higher*;
  - Because of gross substitutability the higher N_Z/N_L will correspond to *Z-biased* technical change overall (marginal productivity ratio).
- The case $\sigma < 1$:
  - The higher Z/L ratio *decreases* N_Z/N_L and physical productivity of abundant factor will be *lower*;
  - However, since factors are gross complements, lower physical productivity translates into *higher* value of marginal product and technical progress is still *Z-biased*
- The case $\sigma = 1$:
  - The production function is a Cobb-Douglas one and technical progress is neither Z- nor L-biased.

Factor rewards and shares

- The ratio of factor prices is given by:

$$\frac{\omega_Z}{\omega_L} = \eta^{\sigma-1}\left(\frac{1-\gamma}{\gamma}\right)^{\epsilon}\left(\frac{Z}{L}\right)^{\sigma-2}. \tag{12}$$

- With enough elastic substitutability ($\sigma > 2$) the relationship may be upward-sloping
- With fixed technology ratio the more abundant the factor is, the less is its price
- However, since technology is biased towards more abundant factor, the overall effect is ambiguous
- The relative *shares* of factors is given by:

$$\frac{s_Z}{s_L} \equiv \frac{\omega_Z Z}{\omega_L L} = \eta^{\sigma-1}\left(\frac{1-\gamma}{\gamma}\right)^{\epsilon}\left(\frac{Z}{L}\right)^{\sigma-1}. \tag{13}$$

Growth rates in lab equipment model

- The maximization of consumption yields

$$g_c = g = \theta^{-1}(r - \rho) \tag{14}$$

- The free-entry condition for both research sectors imply

$$\eta_L \beta p_L^{1/\beta} L/r = 1 = \eta_Z \beta p_Z^{1/\beta} Z/r \tag{15}$$

- With this plus expressions for prices and technology ratios we obtain the growth rate of the economy along BGP:

$$g = \theta^{-1}(\beta[(1-\gamma)(\eta_Z Z)^{\sigma-1} + \gamma(\eta_L L)^{\sigma-1}]^{\frac{1}{\sigma-1}} - \rho). \tag{16}$$

Productive knowledge model

The evolution of varieties in both sectors:

$$\frac{N_Z}{N_L} = \eta^\sigma \left(\frac{1-\gamma}{\gamma}\right)^\epsilon \left(\frac{Z}{L}\right)^{\sigma-1}, \tag{17}$$

- It is assumed that there is an exogenous supply of scientists in the economy, S;
- With only one research sector the growth of knowledge should be proportional to this S: $\dot{N}/N \sim S$;
- With two different sectors, however, the *distribution* of scientists is important;
- The parameter δ measures the degree of state-dependence: with $\delta = 0$ one obtains the previous version of the model (more or less)

BGP in productive knowledge model

- In the productive knowledge model the technology market clearing depends on the level of varieties being achieved:

$$\eta_L N_L^{\delta} \pi_L = \eta_Z N_Z^{\delta} \pi_Z \tag{18}$$

- The equilibrium level of relative technology is then:

$$\frac{N_Z}{N_L} = \eta^{\frac{\sigma}{(1-\delta)\sigma}} \left(\frac{1-\gamma}{\gamma} \right)^{\frac{\epsilon}{(1-\delta)\sigma}} \left(\frac{Z}{L} \right)^{\frac{\sigma-1}{(1-\delta)\sigma}} \tag{19}$$

- Now the ratio of technologies depends on the *degree of state-dependence*, δ.

Factor prices and shares

- In this version of the model ratio of factor prices is:

$$\frac{\omega_Z}{\omega_L} = \eta^{\frac{\sigma}{(1-\delta)\sigma}} \left(\frac{1-\gamma}{\gamma}\right)^{\frac{(1-\delta)\epsilon}{(1-\delta)\sigma}} \left(\frac{Z}{L}\right)^{\frac{\sigma-2+\delta}{(1-\delta)\sigma}}. \qquad (20)$$

- And relative factor shares:

$$\frac{s_Z}{s_L} \equiv \frac{\omega_Z Z}{\omega_L L} = \eta^{\frac{\sigma}{(1-\delta)\sigma}} \left(\frac{1-\gamma}{\gamma}\right)^{\frac{1-\delta\epsilon}{1-\delta\sigma}} \left(\frac{Z}{L}\right)^{\frac{\sigma-1+\delta-\delta\sigma}{(1-\delta)\sigma}}. \qquad (21)$$

- Both are reduced to the lab equipment model if $\delta = 0$.

Growth rate of the economy

- In productive knowledge model, growth rate is determined by the number of scientists, S
- In BGP both sectors should grow at the same rate:

$$\dot{N}_L/N_L = \dot{N}_Z/N_Z; \tag{22}$$

- Taking into account the technology market clearing condition, Eq. (18), this is equivalent to

$$\eta_L N_L^{\delta-1} S_L = \eta_Z N_Z^{\delta-1} S_Z \tag{23}$$

- Which results in equilibrium allocation of scientists and growth rates:

$$S_L = \eta_Z S/(\eta_Z + \eta_L), g = \eta_L \eta_Z S/(\eta_Z + \eta_L). \tag{24}$$

Stability of BGP

- For the case of productive knowledge, the steady state is not always stable (as in the lab equipment model)
- Unstable here means that in the end only one type of R&D is undertaken
- BGP is the one, where **both** types of R&D have non-zero growth rates
- The condition for existence of such a path is:

$$\sigma < 1/\delta \tag{25}$$

- Additionally, for downward sloping factor demands it is necessary, that

$$\sigma > 2 - \delta \tag{26}$$

- These conditions may be simultaneously satisfied only if $\delta < 1$!

Full state-dependence with $\delta = 1$

- In this case stability of BGP requires

$$\sigma < 1 \tag{27}$$

so factors are gross complements

- To have research in both sectors along BGP factor shares must be constant:

$$\frac{s_Z}{s_L} \equiv \frac{\omega_Z Z}{\omega_L L} = \eta^{-1} \tag{28}$$

- This confirms the result of Kennedy (1964) with constant shares along BGP, but only with full state-dependence!

Market size effect and population growth

- So far we abstracted from population growth;
- Positive population growth leads to the so-called "scale effect": growth rate positively depends on population growth
- Modify the knowledge production equations as to:

$$\dot{N}_L = \eta_L N_L^\lambda S_L, \dot{N}_Z = \eta_Z N_Z^\lambda S_Z \tag{29}$$

- In the absence of population growth such a model will not have a stable BGP
- However with population growth BGP is

$$g = \frac{n}{1-\lambda} \tag{30}$$

- The market size effect will still be present in this model:

$$\eta_L N_L^\lambda \pi_L = \eta_Z N_Z^\lambda \pi_Z \tag{31}$$

- Which will give the same market effect as before.

Applications

The model of endogenous skill-biased technical change

- In the previous model substitute Z for H (human capital)
- In such a model the increase in the supply of human capital (skills) creates a bias of technical progress towards this skilled labour.

Model of international trade

- Assume the factors are H and L - skilled and unskilled labour
- Assume all the technologies are developed in the "North" and all other countries are less-developed
- These countries may copy the technologies of the North
- Depending on whether the factors are gross substitutes or complements, the increase in relative technology N_H/N_L increases or decreases the income gap between countries.

Conclusions

- The idea of induced direction of technical change originates back to Fellner (1962)
- Acemoglu (1998) adapted this idea to endogenous growth theory with varieties
- This adaptation amounts to inclusion into the analysis two (instead of one) R&D sectors
- Wide variety of economic applications may be found of this simple idea
- However it is essential that only two sectors exist
- Currently we are trying to extend this to continuum of sectors.

Literature

- Fellner W. (1961) Two Propositions in the Theory of Induced Innovations. *The Economic Journal, Vol. 71, No. 282*, pp. 305-308
- Kennedy C. (1964) Induced Bias in Innovation and the Theory of Distribution. *The Economic Journal, Vol. 74, No. 295*, pp. 541-547
- Acemoglu D. (1998) Why do new technologies complement skills? Directed Technical Change and Wage Inequality. *The Quaterly Journal of Economics, Vol. 113(4)*, pp. 1055-1089.

Next time

- The Green growth concept
- Application of directed technical change to environment: technology lock-in
- Paper: Acemoglu, Aghion, Bursztyn, Hemous (2012) *The Environment and Directed Technical Change*.

Details

Back Consider production function as of *CES* type:

$$Y = [\gamma(A_L L)^{\frac{\sigma-1}{\sigma}} + (1-\gamma)(A_Z Z)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} \tag{32}$$

where A_L is labour-augmenting technical progress and A_Z is capital-augmenting technical progress.
At the same time, *direction* of technical change is governed by:

$$\frac{MP_Z}{MP_L} = \frac{1-\gamma}{\gamma}\left(\frac{A_Z}{A_L}\right)^{\frac{\sigma-1}{\sigma}} \cdot \left(\frac{Z}{L}\right)^{-\frac{1}{\sigma}}, \tag{33}$$

Now we incorporate this notion of direction into the standard variety expansion type model.

Demand: Labour-intensive sector

Back Demand in labour-intensive is the result of profit maximization of producers:

$$\max_{L,\{x_L(j)\}} p_L Y_L - \omega_L L - \int_0^{N_L} \chi_L(j) x_L(j) dj; \tag{34}$$

taking $p_L, N_L, \chi_L(j)$ as given, this problem yields the demand for machines and labour in labour-intensive sector:

$$x_L(j) = \left(\frac{p_L}{\chi_L(j)}\right)^{1/\beta} L; \tag{35}$$

$$\omega_L = \frac{\beta}{1-\beta} p_L \left(\int_0^{N_L} x_L^{1-\beta}(j) dj\right) L^{\beta-1}. \tag{36}$$

Demand: Capital-intensive sector

Back Demand in capital-intensive sector is defined in a similar way:

$$\max_{Z,\{x_Z(j)\}} p_Z Y_Z - \omega_Z Z - \int_0^{N_Z} \chi_Z(j) x_Z(j) dj; \tag{37}$$

$$x_Z(j) = \left(\frac{p_Z}{\chi_Z(j)}\right)^{1/\beta} Z; \tag{38}$$

$$\omega_Z = \frac{\beta}{1-\beta} p_Z \left(\int_0^{N_Z} x_Z^{1-\beta}(j) dj\right) Z^{\beta-1}. \tag{39}$$

- Since Z and L are used only in respective sectors, all of the supply of factors is used in production;
- It is crucial, that there are only 2 sectors - this enables to define relative prices.

Prices of machines

Back

- Each monopolist faces constant marginal costs of producing machines:

$$MC_{x_{L,Z}(j)} = \psi; \tag{40}$$

- Then the profit of the monopolist may be written in per unit terms:

$$\pi_{L,Z} = (\chi_{L,Z}(j) - \psi)x_{L,Z}(j); \tag{41}$$

- Since the demand for machines in both sectors is isoelastic, Eqs. (35), (38), price is a constant mark-up:

$$\chi_{L,Z} = \frac{\psi}{1-\beta}; \tag{42}$$

- Normalizing $\psi = 1 - \beta$, prices are equal:

$$\chi_L(j) = \chi_Z(j) = 1, \forall j. \tag{43}$$

The role of elasticity of substitution I

Back Define

$$\epsilon \equiv 1/(1-\alpha), \sigma \equiv \frac{1-\alpha(1-\beta)}{1-\alpha}; \tag{44}$$

elasticity of substitution in consumption and production respectively.

- Making use of machine demands, one may rewrite outputs as functions of ranges of machines:

$$Y_L = \frac{1}{1-\beta} p_L^{(1-\beta)/\beta} N_L L, Y_Z = \frac{1}{1-\beta} p_Z^{(1-\beta)/\beta} N_Z Z; \tag{45}$$

- Now the relative price is a function of relative technology, N_Z/N_L:

$$p = \left(\frac{1-\gamma}{\gamma}\right)^{\beta\epsilon/\sigma} \left(\frac{N_Z Z}{N_L L}\right)^{-\beta/\sigma}; \tag{46}$$

The role of elasticity of substitution II

Back

- The relative profitability of both kinds of innovations is defined by the ratio of values (from Eqs. (4)):

$$\frac{V_Z}{V_L} = \left(\frac{p_Z}{p_L}\right)^{1/\beta} \frac{Z}{L}; \tag{47}$$

- Using the new expression for the relative price, Eq. (5), this turns out to be the function of ratio of technologies and factor supplies:

$$\frac{V_Z}{V_L} = \left(\frac{1-\gamma}{\gamma}\right)^{\epsilon/\sigma} \left(\frac{N_Z}{N_L}\right)^{-1/\sigma} \left(\frac{Z}{L}\right)^{\frac{\sigma-1}{\sigma}} \tag{48}$$

- The relative factor supply will increase values ratio as long as $\sigma > 1$ and vice versa.