

Innovations heterogeneity, 2-dimensional r&d and fertility choices

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Plan of the lecture

Overview of previous models

Q&A's

Innovations by entrants and incumbents

Growth on a finite planet: resources, technology and population in the long run

Concluding remarks

Technology

- ▶ Neoclassical theory: Factor-augmenting technical change;
- ▶ Semi-endogenous growth: Induced innovations, inventions possibility frontier;
- ▶ Endogenous growth: Horizontal innovations, vertical innovations, directed technical change

Environment

- ▶ Environment in neoclassical theory: Resource-dependant production, pollution;
- ▶ Endogenous growth: ways to combine technical change and environment;
- ▶ Taste-for-variety: technology gradually replaces resource;
- ▶ Quality ladders: optimal regulation, pollution permits;
- ▶ Directed technical change: technology lock-in and green growth.

- ▶ Exam questions
- ▶ Mathematical background
- ▶ State of the theory today

Overview

- ▶ Paper of Acemoglu& Cao (2015), JET;
- ▶ Essentially based on quality ladders setup, but with two types of innovations;
- ▶ An attempt to depart from symmetry assumption
- ▶ Still, no horizontal innovations at all.

Setup

- ▶ Usual representative consumer with life-time utility;
- ▶ Budget includes spending on machines and on R&D;
- ▶ Single final output is produced via labor and machines $x(v, t)$.
- ▶ Range of machines is normalized to one, but **quality changes over time**
- ▶ This quality is the main driver of growth

Innovations

There are two types of innovations:

1. By incumbents, which increase their quality via incremental innovations:

$$q(v, t) = \lambda^n q(v, s) \quad (1)$$

with n number of incremental innovations from s to t ;

2. By entrants, who may invent new machine (drastic innovation), leading to the quality

$$\kappa q(v, t), \quad \kappa > \lambda \quad (2)$$

Equilibrium

An **allocation** of time paths of:

- ▶ Consumption, aggregate spending on machines and r&d, $[C(t), X(t), Z(t)]$
- ▶ Distribution of r&d spending across entrants and incumbents, $[z(v, t), \hat{z}(v, t)]$
- ▶ Prices and quantities for each machine type and net present value of profits for each machine type
- ▶ Paths of interest rates and wage rates $[r(t), w(t)]$

All firms maximize profits, households maximize utility and labor and final goods markets clear.

Results

- ▶ Under **some** assumptions only the highest quality machine is sold for each v ;
- ▶ The total quantity sold is proportional to labor, $x(v, t|q) = qL$
- ▶ Final output is proportional to labor and **average quality**:

$$Y(t) = \frac{1}{1-\beta} Q(t)L, \quad Q(t) = \int_0^1 q(v, t)dv \quad (3)$$

- ▶ Everything else is defined by the evolution of this $Q(t)$ - growth driver as in old Aghion&Howitt (1998).

Definition of BGP

In this economy not any equilibrium grants a balanced growth path (BGP). BGP is defined as:

- ▶ Equilibrium, where r&d, output and consumption all grow at a **constant** rate;
- ▶ Still there is firms turnover (entry & exit)
- ▶ **Stationary** BGP is such that firms' sizes distribution is stationary (does not depend on time)

Once intertemporal elasticity of substitution is high enough, the BGP is unique.

GDP growth

Under assumptions on existence and uniqueness of (linear, stationary) BGP:

- ▶ The GDP growth is given by the sum of both types of innovations:

$$g^* = \phi(z^*)(\lambda - 1) + \hat{z}^* \eta(\hat{z}^*)(\kappa - 1) \quad (4)$$

- ▶ The interest rate is constant:

$$r^* = \rho + \theta g^* \quad (5)$$

Stationary distribution of firms

- ▶ In general, does not exist, since at infinity some firms **accumulate** infinite assets
- ▶ With addition of **imitating** firms the stationary distribution exists
- ▶ Role of **knowledge spillovers**: imitator firm may enter with technology $q^e(v, t) = \omega Q(t)$ with costs $\mu_e \omega Q(t)$;
- ▶ Entry by imitation is profitable if exists ϵ , such that $q(v, t) \leq \epsilon Q(t)$
- ▶ Then stationary distribution is Pareto and exists and
- ▶ growth rate is average from **three** components:
 1. Incumbent innovation
 2. Entrants drastic innovation
 3. Imitators

Discussion

- ▶ One of the first attempts to make r&d non-symmetric
- ▶ Still, no new technologies appear;
- ▶ Growth is positive and constant, but
- ▶ Imitators play a crucial role, stabilizing the system
- ▶ **Positive role of imitations**

Overview

- ▶ Paper of Peretto&Valente (2015), JEG;
- ▶ Essentially based on older Peretto&Connolly (1998) setup;
- ▶ Main idea: combine horizontal and vertical innovations, but:
 1. Horizontal innovations eventually stop
 2. Long-run growth is due to vertical innovations only
- ▶ Current paper adds resources and population into the mix

Setup: Households

- ▶ Population growth:

$$\dot{L}(t) = B(t) - dL(t) \quad (6)$$

with $B(t)$ **mass of children**

- ▶ Utility:

$$U = \int_0^{\infty} \log u(t) e^{-\rho t} dt \quad (7)$$

- ▶ Instantaneous utility depends on mass of adults and children and their consumption, $L(t)$, $B(t)$, $C_L(t)$, $C_B(t)$
- ▶ The flow budget constraint is

$$\begin{aligned} \dot{A}(t) = & r(t)A(t) + w(t)L(t) + p(t)\Omega - \\ & P_C(t)(C_L(t)L(t) + C_B(t)B(t)) \end{aligned} \quad (8)$$

Setup: Producers

- ▶ Final producers are perfectly competitive and produce a **single homogeneous** output, which is totally consumed using

$$C(t) = \left(\int_0^{N(t)} X_i(t)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \quad (9)$$

- ▶ This results in demand for every intermediate good, $P_{X_i}(t)$;
- ▶ Intermediate producers use **labor** $L_{X_i}(t)$ and resource $R_i(t)$ with some efficiency $Z_i(t)$ (firm-specific knowledge):

$$X_i(t) = Z_i^\theta(t) F(L_{X_i}(t) - \psi, R_i(t)) \quad (10)$$

Setup: R&D

- ▶ There are two types of R&D:
 1. **Vertical** is given by firm-specific knowledge $Z_i(t)$:

$$\dot{Z}_i(t) = \alpha K(t) L_{Z_i}(t) \quad (11)$$

proportional to spillover from **public knowledge** $K(t)$

2. **Horizontal** is given as the entry of new firms given labor sunk costs L_{N_i} , implying the free-entry condition:

$$V_i(t) = \beta P_{X_i}(t) X_i(t) = w(t) L_{N_i}(t) \quad (12)$$

- ▶ Each new entrant operates as a normal firm increasing its productivity $X_N(t)$ after entry and no-arbitrage condition ensures equilibrium.

General equilibrium

- ▶ Households choose optimally their (total) consumption profile:

$$\frac{\dot{P}_C(t)}{P_C(t)} + \frac{\dot{C}(t)}{C(t)} = r(t) - \rho \quad (13)$$

- ▶ Plus the **fertility choice**
- ▶ In the intermediate sector we have usual **symmetric equilibrium**: each firm charges the same price $P_{X_i} = P_X$ and produces the same quantity $X_i = X$
- ▶ Then the final output is alá Romer:

$$C(t) = N(t)^{\frac{\epsilon}{\epsilon-1}} X(t) \quad (14)$$

R&D

- ▶ Intermediate firms are symmetric and thus their knowledge is also symmetric, evolving via

$$\frac{\dot{Z}(t)}{Z(t)} = \alpha \frac{L_Z(t)}{N(t)} \quad (15)$$

- ▶ The total mass of intermediate firms grows via new entries which is simultaneously the productivity in (14):

$$\frac{\dot{N}(t)}{N(t)} = \frac{w(t)}{\beta P_C(t) C(t)} L_N(t) - \delta \quad (16)$$

- ▶ No-arbitrage ensures associated returns are equal and rates of both types of innovations are **independent!**

General equilibrium dynamics

In the general equilibrium there are **two** components of the system:

1. The resource-population (environment) part is given by dynamical system

$$\begin{aligned}\dot{\omega}(t) &= \omega(t)(d - b(t)) \\ \dot{b}(t) &= \left(\frac{1}{(1 - \eta)y^*(\omega(t))} - 1 \right) \frac{1}{1 - \mu} - \rho b(t)\end{aligned}\quad (17)$$

with $\omega(t)$ resource use per capita and $y(\omega)$ equilibrium consumption expenditures per capita;

2. The R&D system, (15), (16)

In total, the **growth rate** of the economy is composed of long-run part, $F_1(\dot{Z}(t), \dot{N}(t))$ and transitory part, $F_2(\dot{y}(t), \dot{p}(t))$

The long-run growth

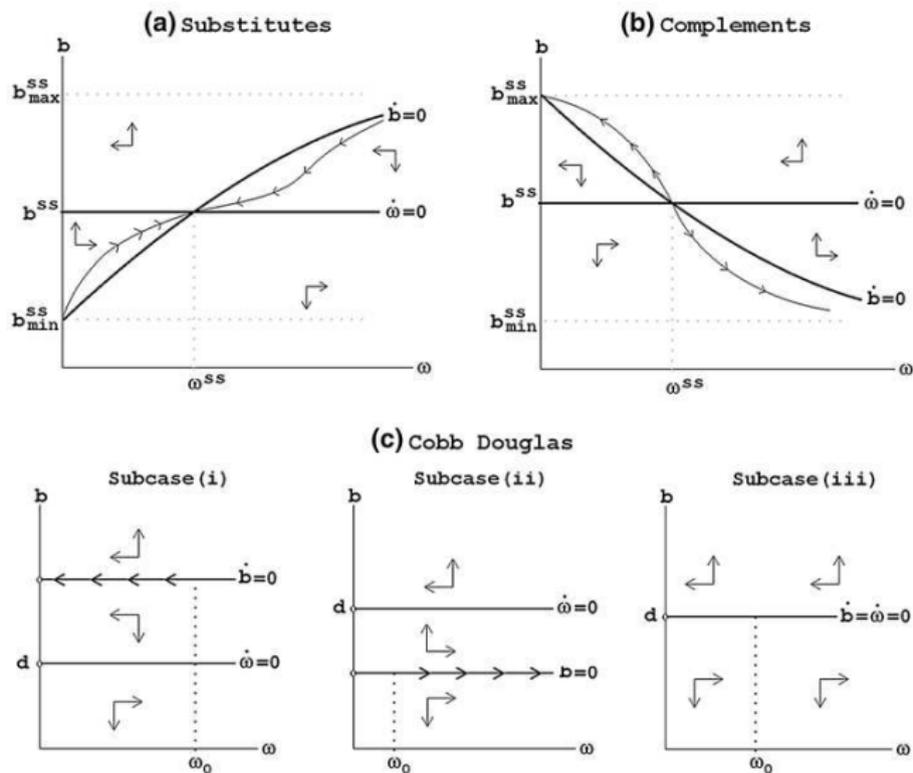
- ▶ In this economy the long-run growth happens only through the $\frac{\dot{Z}(t)}{Z(t)}$ channel;
- ▶ The reason is **fixed entry costs**
- ▶ growth may be positive or negative, depending on parameters
- ▶ Resource influences only the transitory dynamics (as it is finite)
- ▶ Stability of the BGP depends on steady-state resource and expenditures

Main result

- ▶ Economy converges to the steady state (in b, y) with constant population if labor and resource are substitutes;
- ▶ Economy under complementarity of the resource and labor diverges off the equilibrium implying
 1. Collapse of population in finite time, if labor is more scarce;
 2. Exponential explosion of population if resource is more scarce
- ▶ Under Cobb-Douglas population again may collapse, grow exponentially or stay constant;

There is no equilibrium sustainable population growth in this model! This is the replication of *knife-edge* argument.

Illustration



Discussion

- ▶ Model tries to unify 2-dim R&D with population growth and resource scarcity;
- ▶ Still it generates rather pessimistic outcomes
- ▶ Reasons are:
 1. R&D is essentially 1-dimensional in the long-run (**spillover role**)
 2. It does not help to fight off resource scarcity
 3. Population growth is costly: children consume
 4. Symmetric innovations!

- ▶ There are **three** main types of endogenous growth models;
- ▶ Recent activity is focused on DTC models
- ▶ Still some new research is done in:
 1. Heterogeneity of r&d
 2. Combining vertical and horizontal innovations
- ▶ Dynamic structural change is an issue;
- ▶ It is important how we specify environment-technology interrelationships
- ▶ Predictions of recent models are more and more pessimistic
- ▶ The discipline has went in 30 years from optimism to careful consideration of details
- ▶ More and more empirical testing is discussed

**THANKS FOR ATTENTION AND
GOOD LUCK NEXT WEEK!**