Creative destruction and quality ladders

Anton Bondarev

Department of Businness and Economics,
Basel University

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Outline

Creative Destruction

- The term coined by Schumpeter (1942)
- He claimed, that innovation is
  1. Destructive
  2. Creative
- It destroys old products, which are no longer desired
- It creates new products **which replace older ones**
- "This process of creative destruction is essential fact about capitalism".

Growth through creative destruction

- **Vertical innovations** (quality ladders): Industrial innovations which improve the quality of products
  - Different from *horizontal* innovations as of Romer (1990)
  - Range of products is fixed, while qualities are not
- **Creative destruction**: better products render previous obsolete
  - Innovation process is alike the patent race literature
  - The successful innovator acquires patent that will grant monopoly for the new product
- **Uncertain** process of innovations: random arrival time.

Setup

- Two products:
  - Final product y, being consumed;
  - Intermediate product x, being used in production of final product.
- Three sectors:
  - Final good productio
  - Intermediate good production;
  - Research sector: increases the productivity of intermediate good.

Assumptions

- There is no capital accumulation in the model
- The population is continuous mass of individuals, L, constant
- These individuals have *linear intertemporal preferences*:

$$u(y) = \int_0^\infty y_\tau \mathbf{e}^{-rt} d\tau$$

- There is no disutility from labour
- Risk-neutrality introduced to safely eliminate intertemporal choices of capital (absent)
- Only one consumption good with unitary price.

Labour

- Each individual has one unit of labour (flow)
- Thus total labour supply is constant and equals L
- There are three possible types of labour:
    1. Unskilled labour which can be used only for final good production, M
    2. Skilled labour used in intermediaries production and R&D, N
    3. Specialized labour for R&D only, R
- The total skilled labour is distributed among research and intermediary production:

$$N = l + n. \tag{1}$$

Production

- There is single final good, y
- This good is produced with the help of an *intermediate good*, x through dynamic production function:

$$y_t = A_t F(x_t)$$

- There is only **one** intermediate product with linear technology, $x = l$
- Index t denotes **generation** of intermediate product, NOT time
- Labour is not used in final goods production at all.

Innovations

- The **innovation** is the creation of new intermediary, x
- Such an innovation raises the productivity parameter, A, by some factor γ:
$$A_{t+1} = \gamma A_t$$
- Research is the function of labour being used, n
- The research is a random process (Poisson process)
- The (expected) arrival rate of new intermediary is proportional to labour used:

$$\mathbb{E}(x_{t+1}) = \lambda n_t || \tau_{t+1} - \tau_t ||$$

Final producer problem

formulas

- Final producers are perfectly competitive
- They employ intermediary x at the level where its price equals marginal productivity
- Price of final product is numeraire $P_y = 1$
- Price for intermediary equals it's marginal product at t.

Intermediate monopolist

formulas 1 Production of x_t is a **finite-time monopoly**:

- For each generation t there is exactly **one** producer, which is replaced after new innovation arrives
- This firm faces p_t and chooses optimally the production(supply) level x_t by solving profit maximization
- This is the same as to choose the **wage rate** w_t
- This implies that **monopolist's profit is a decreasing function of real wage** formulas 2

Wages and employment effects

- Wage is determined by the demand for skilled labour in both research and intermediate production sectors
- Consider the situation after the next innovation:
  - Higher demand for future research labour will increase future wages, ω_{t+1}
  - This will decrease future profits, $\pi(\omega)_{t+1}$
- Motivation for current research is the prospective innovator profit
- A lower expected future profit will discourage current research
- Thus **higher** future research labour, n_{t+1} implies **lower** current research labour, n_t.

Problem of the R&D sector

- In R&D there is a **free-entry**
- A firm in research sector employs two factors:
  - The research-specific labour (scientists), R, which is exogenously given
  - The fraction of skilled labour, n, which is defined by the demand.
- It experiences innovations at the Poisson arrival rate $\lambda\phi(n, R)$
- This is the **rate** of expected innovations
- Since the specific labor can be employed only in research, in equilibrium it is fully employed:

$$\phi(n, R) = \varphi(n).$$

Problem of the R&D sector II

formulas

- The typical research firm thus sets **labor demand** n_t
- This is profit maximization problem in perfectly competitive sense, but:
  1. Profit of R&D is in the **future**
  2. Firm has to invest labor now
  3. Implying intertemporal optimization
- The labor demand n_t is a function of $t+1$ generation of innovations, $V_{t+1}(0)$!

Value of R&D components

formulas 1

- Denote by δ some small time interval in between two generations t and $t+1$
- At this time δ the value of the patent for the innovation t is:
  - Discounted flow of profits during this period δ (green)
  - Plus the expected value (zero) of being replaced by the next generation with some probability (red)
  - Plus the value of NOT being replaced after this period expires with probability (blue)
- In general the value of R&D at time τ is thus profit+continuation value

This forms the backbone of the **Hamilton-Jacobi-Bellman** approach. formulas 2

Creative destruction effect

- Consider the expected value of the **next** innovation:

$$V_{t+1}(\tau) = \frac{A_{t+1}\pi_{t+1}(\tau)}{r + \lambda\varphi(n_{t+1}(\tau))}; \tag{2}$$

- The denominator is the *obsolescence-adjusted* interest rate
- It shows the effect of creative destruction:
  - The more research is expected, the higher is the arrival rate $\lambda\varphi(n_{t+1}(s))$
  - This shortens the duration of the monopoly on technology (since new generation would appear faster)
  - This decreases the expected stream of monopoly profits.

Arrow replacement effect

- The holder of the patent for the innovation of generation t does not do research
- If the current monopolist would do research, this will cancel out its current patent
- Hence, the value of such a research for this monopolist is:

$$V_{t+1} - V_t$$

- Which is less than the value for the outside firm, V_{t+1}
- Hence, every monopolist is replaced by the new firm when new generation of innovation arrives.

Intertemporal spillover effect

- Any innovation raises productivity permanently by factor γ
- Some portion of this productivity gain is captured by the innovator
- However, monopoly lasts only one generation, $t+1-t$
- When replacement occurs, this productivity gain is no longer captured by the inventor
- New innovators build up on the basis of this achieved productivity, but do not compensate the previous innovator
- Thus, **lack of compensation discourages research**.

Equlibrium

- In this basic model equilibrium is fully characterized by:
  - Labour market clearing condition,

$$N = x_t + n_t$$

    which defines the residual supply of labor for manufacturing (given n_t)
  - And the demand for skilled labor in research sector, defined from the problem of R&D firm,

$$\omega_t = \lambda \varphi'(n_t) V_{t+1}. \tag{3}$$

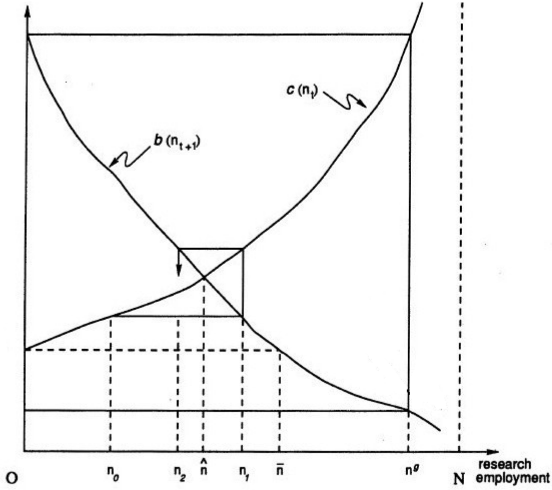
- This last condition actually depends on intertemporal spillovers of knowledge.

Determinants of the equilibrium

formulas From (3) with $n_t > 0$ we obtain **arbitrage condition**

1. Costs of research $c(n)$ depend on **current** labor demand, n_t
2. Profits from research $b(n)$ depend on **future** labor demand n_{t+1}
3. Since free entry, R&D profit is zero
4. Thus the two curves have to intersect defining equilibrium
5. Since arrival time is uncertain, these are NOT smooth time-dependent curves

Research labour market clearance via tâtonnement



Commments

- The arbitrage condition is about intergenerational allocation of labor
- The left-hand side is the costs of research, w_t, normalized by the speed of research
- The right-hand side is the discounted stream of profits of the next-generation innovator
- They together define the evolution path of the research labor as the recurrent relation:

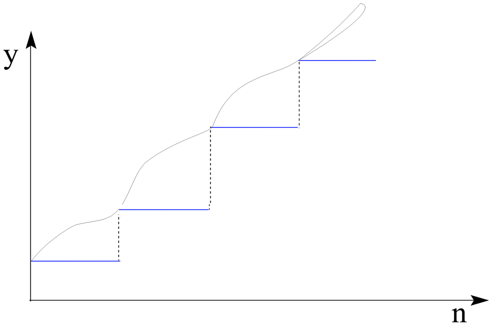
$$n_t = \psi(n_{t+1})$$

- **No growth trap** occurs with $c(n_t) = 0, b(n_t) = n^g$
- **Two-cycles** are possible as an equilibrium.

Stationary equilibrium (aka steady state)

formulas

- In the steady state, the $n_t = \psi(n_{t+1})$ relation is simply identity:
- Stable research labor implies stable growth by factor γ
- The output has a form of a **random step function** with step size γ.



Uniqueness of equilibrium

- From now on, assume for simplicity

$$\varphi(n) = n; F(x) = x^{\alpha}.$$

- In the steady state the labour market is cleared according to:

$$\bar{\omega} = \lambda \frac{\gamma \pi(\bar{\omega})}{r + \lambda \bar{n}}; \qquad (4)$$
$$\bar{n} + x(\bar{\omega}) = N.$$

- First schedule has a negative slope in $\omega - n$ space:

$$\frac{\partial \omega}{\partial n} < 0;$$

- The second schedule is increasing function of n:

$$\frac{\partial N}{\partial n} > 0$$

Comparative statics: research

- Equilibrium level of research labour (and research) is **raised** by:
  - Lower interest rate, r
  - Higher size of the skilled labour force, N
  - Higher productivity of R&D, λ
  - Higher size of innovation, γ
- Real wage is that higher, the higher is the research labor
- Research is **decreasing** in intermediate demand elasticity, α
- This means, **product market competition is bad for growth**!

Comparative statics: expected output

- The output is a random step function
- Time interval between each step is exponentially distributed with parameter $\lambda \bar{n}$
- In unit time interval :

$$\ln y(\tau + 1) = \ln y(\tau) + \ln \gamma \epsilon(\tau)$$

where $\epsilon(\tau)$ is the number of innovations occurring between τ and $\tau + 1$.

- The **expected** growth rate of log output is:

$$\mathbb{E}(\ln y(\tau + 1) - \ln y(\tau)) = \lambda \bar{n} \ln \gamma = g. \tag{5}$$

- Which is the average growth rate of the economy,

Social welfare

formulas

- We compare the market equilibrium with social planner problem;
- Social welfare is given by maximizing utility of consumers:

$$\max_{\bullet} U = \int_{0}^{\infty} \mathbf{e}^{-rt} y(\tau) d\tau$$

- But given **total** output evolution over time
- This is probabilistic, so social planner takes into account future r&d
- Which makes it different from the decentralised system

Social optimum

formulas

- The social welfare may be integrated to yield the static optimization problem over n
- As such, we obtain **constant** optimal research labor n^*
- Growth rate is obtained similarly to decentralised economy
- The average growth rate of planned economy would be then:

$$g^* = \lambda n^* \ln \gamma.$$

Comparison of research

- Whether the socially optimal growth rate is higher or lower than the market one, depends solely on the R&D speed
- The R&D in market economy is given by:

$$1 = \frac{\lambda\gamma(\frac{1-\alpha}{\alpha})(N - \bar{n})}{r + \lambda\bar{n}}; \tag{6}$$

- While the socially optimal research is given by Eq. (38):

$$1 = \frac{\lambda(\gamma - 1)(\frac{1}{\alpha})(N - n^*)}{r - \lambda n^*(\gamma - 1)}.$$

- They differ by spillover effect, appropriability effect and business-stealing effect.

Intertemporal spillover effect again

- The private discount rate is
$$r + \lambda n;$$
- The social planner discount rate is
$$r - \lambda n(\gamma - 1).$$
- Social planner takes into account the fact, that new innovation will continue to have effect forever
- Private innovator is not interested in the value of innovation after the new generation innovation arrives
- Social planner values innovations **more** than the private investor

Appropriability effect

- The Eq. (6) contains the term $\frac{1-\alpha}{\alpha}$ instead of $\frac{1}{\alpha}$
- This reflects the fact, that monopolist may capture only the fraction of total productivity growth
- The social planner would capture the whole productivity growth γ
- Thus, the monopolist receives less profits, than the innovation generates
- This further discourages private innovations in comparison with the socially optimal one.

Business stealing effect

- In market economy new innovation forces the current monopolist to close business
- The private firm does not internalize losses of this previous monopolist
- The social planner takes into account the fact, that new innovation x_{t+1} replaces the old one, x_t
- This creates some welfare losses from the social viewpoint
- This effect leads to **more** research under market economy than under social optimum.

Overall picture

- The market driven growth may be **higher** or **lower** than socially optimal
- This depends on the relative size of above effects
- When size of innovation is large, socially optimal research is larger, $n^* > \bar{n}$
- When there is much monopoly power (low α) *and* small innovations (γ small), private research is higher, $n^* < \bar{n}$
- This is intuitive: big research projects are rather financed by the government, than private firms
- Private firms will research more actively. when there is monopoly power and not much research have to be done!

Literature

- Aghion P., Howitt P.: *Endogenous Growth Theory*, The MIT Press, 1998;
- Aghion P., Howitt P. (1992) A Model of Growth through Creative Destruction. *Econometrica 60*, pp. 323-51;
- Aghion P., Howitt P. (1994) Growth and Unemployment. *Economic Studies 61*, pp. 477-494;

Next lecture

- Combination of vertical R&D and environment;
- What policy tools are the most adequate one?
- Can environmental policy be substituted by R&D policy?
- Paper: Griamud A., Rougé L., (2003) Non-renewable resources and growth with vertical innovations: optimum, equilibrium and economic policies.

Final producer problem

Back

- Representative firm solves profit maximization the problem

$$\max_{x_t} \pi_y = A_t F(x_t) - p_t x_t \tag{7}$$

- Giving usual profit condition

$$\frac{\partial \pi_y}{\partial x_t} = A_t \frac{\partial F}{\partial x_t} - p_t = 0 \tag{8}$$

- The marginal rule of demand:

$$p_t = A_t F'(x_t) \tag{9}$$

Intermediate producer problem

Back

- Monopolist in intermediate sector chooses the profit-maximizing output (given demand):

$$\max_{x_t} p_t x_t - w_t l_t; \tag{10}$$

- With given linear production technology:

$$x_t = l_t. \tag{11}$$

- And with demand for x_t given by Eq. (9) transforms into:

$$\max_{x_t} A_t F'(x_t) x_t - w_t x_t; \tag{12}$$

- Then the F.O.C. for x_t

$$\frac{\partial \pi}{\partial x_t} = A_t(F'(x) + F''(x)) - w = 0 \tag{13}$$

Intermediate monopolist's profit

Back

- Denote the supply of intermediary as $X(\omega)$ with $\omega = \frac{w}{A}$;
- Then the profit of intermediate monopolist may be rewritten as:

$$\pi(\omega) = [X(\omega)F'(X(\omega)) - \omega X(\omega)] \quad (14)$$

- The effect of real wage on profit of the monopolist:

$$\pi'(\omega)_\omega = -X(\omega)X'(\omega)[2F''(X(\omega)) + XF^3(X(\omega))] < 0; \quad (15)$$

given $X'(\omega) < 0$ and $2F''(X(\omega)) + XF^3(X(\omega)) < 0$

Profit of R&D

Back

- The research firm maximizes its expected profits:

$$\max_{n_t} \lambda\varphi(n_t)V_{t+1}(0) - w_t n_t - w_t^R R \tag{16}$$

where $V_{t+1}(0)$ is the value of $t+1$ generation of innovations.

- The first-order condition for this problem is

$$\lambda\varphi'(n_t)V_{t+1}(0) - w_t = 0. \tag{17}$$

implying the demand for skilled labour from R&D sector.

- Now we have to define $V_t(\tau)$!

Value of R&D components

Back Thus, the value of R&D firm generation t at time τ is:

$$V_t(\tau) = \frac{1}{1 + r\delta}\Big\{A_t\pi_t(\tau)\delta + \lambda\varphi(n_t(s))\delta \bullet 0 + (1 - \lambda\varphi(n_t(s))\delta)V_t(s + \delta)\Big\} \tag{18}$$

Hamilton-Jacobi-Bellman

Back

- As $\delta \to 0$, the Eq. 18 is transformed into the usual HJB equation:

$$rV_t(\tau) - \frac{\partial V_t(\tau)}{\partial \tau} = A_t \pi_t(\tau) - \lambda \varphi(n_t(s)) V_t(\tau); \tag{19}$$

- Since the Poisson arrival rate of the new innovation is independent of time:

$$\frac{\partial V_t(\tau)}{\partial \tau} = 0; \tag{20}$$

- And the value of R&D firm is:

$$V_t(\tau) = \frac{A_t \pi_t(\tau)}{r + \lambda \varphi(n_t(s))}. \tag{21}$$

Equilibrium research labour

Back

- First, with condition $n_t > 0$ we have from Eq. 17:

$$\lambda\varphi'(n_t)V_{t+1}(0) = A_t\omega(x_t). \tag{22}$$

- Now recall, that manufacturing labour is a residual:

$$x_t = N - n_t \rightarrow w(x_t) = w(N - n_t); \tag{23}$$

- And the expected value of t+1st innovation at time zero is:

$$V_{t+1}(0) = \frac{A_{t+1}\pi_{t+1}(0)}{r + \lambda\varphi(n_{t+1}(0))}; \tag{24}$$

- Then the **arbitrage condition** relates future and current research labour through innovation costs $c(n_t)$ and profits $b(n_{t+1})$:

$$c(n_t) \overset{def}{=} \frac{w(N - n_t)}{\lambda\varphi'(n_t)} = \frac{\gamma\pi(w(N - n_{t+1}))}{r + \lambda\varphi(n_{t+1})} \overset{def}{=} b(n_{t+1}) \tag{25}$$

BGP

Back

$$n_t \equiv \bar{n} : \bar{n} = \psi(\bar{n}); \tag{26}$$

$$w_t \equiv \bar{w}. \tag{27}$$

This implies the balanced growth path:

$$y_t = A_t F(N - \bar{n}); \tag{28}$$

$$y_{t+1} = \gamma y_t. \tag{29}$$

Random step function for the output

Back

- Output starts at the level

$$\ln y_0 = \ln A_0 + \ln F(N - \bar{n}) \tag{30}$$

- It grows in jumps size γ but in a non-deterministic way;
- The time to elapse between consecutive jumps is defined **by a random Poisson process**:

$$\ln y_{t+1} = \ln y_t + \ln \gamma|_{\Delta_i} \tag{31}$$

but t denotes generation, not time!
- Time path is defined by the sequence $\Delta_1, \Delta_2, ..$ of time intervals which separate one jump from the other;
- This time depends (randomly) on intensity of research:

$$\Delta_i \overset{iid}{\sim} Exp(\lambda\psi(\bar{n})) \tag{32}$$

Social welfare

Back

$$\max_{\bullet} U = \int_0^\infty \mathbf{e}^{-rt} y(\tau) d\tau = \int_0^\infty \mathbf{e}^{-rt} \left(\Pi(t,\tau) A_t x^\alpha \right) d\tau \tag{33}$$

where $\Pi(t,\tau)$ is the probability that exactly t innovations will occur up to time τ.
Given the same Poisson process for innovations, we have:

$$\Pi(t,\tau) = \frac{(\lambda n \tau)^t}{t!} \mathbf{e}^{-\lambda n \tau}. \tag{34}$$

Social optimum

Back

- The social welfare may be integrated to yield the optimization problem:

$$\max_n U(n) = \frac{A_0(N-n)^{\alpha}}{r - \lambda n(\gamma - 1)} \tag{35}$$

$$s.t. \tag{36}$$

$$N = x + n. \tag{37}$$

- The first-order condition yields:

$$U'(n^*) = 0 \rightarrow 1 = \frac{\lambda(\gamma - 1)(\frac{1}{\alpha})(N - n^*)}{r - \lambda n^*(\gamma - 1)}. \tag{38}$$

- The average growth rate of planned economy would be then:

$$g^* = \lambda n^* \ln \gamma. \tag{39}$$