Non-renewables, pollution and creative destruction: Combining vertical innovations, resource economics and growth.

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Plan of the lecture

- The model of growth with non-renewables, environment and vertical innovations,
- Solution for social planner,
- Solution for the market economy and the comparison,
- Pollution and tradeable permits in this model.

Overview

- The endogenous growth model with vertical innovations;
- Technical change comes through **creative destruction**
- Only one intermediary - no range of technologies
- Production depends on (non-renewable) resource extraction (as in Barbier (1999))
- No feedback from environment to social welfare (yet)
- Comparison of decentralised and socially optimal solutions.

Setup I

- Single final output is produced **competitively**:

$$Y_t = A_t x_t^{\alpha} R_t^{1-\alpha}$$

  where R_t is the resource, x_t - an intermediate product and A_t is level of technology
- Labour is used for intermediate good production and for research:

$$L = 1 = x_t + n_t$$

Setup II

- Technology advances in **leaps** as in Aghion&Howitt:

$$A_\tau = \gamma A_{\tau-1},\ \gamma > 1$$

- There is finite resource stock and no extraction costs as in Barbier:

$$S_t = S_0 - \int_0^t R_v dv.$$

- Finally we have representative household utility:

$$\int_0^\infty \frac{c^{1-\epsilon}}{1-\epsilon} e^{-\rho t}.$$

Technology is the driver of growth

formulas

- The growth rate of the economy is the linear combination of technology and resource growth rates
- In the steady state the extraction of the resource must be limited
- This naturally requires $g_A > g_Y$ to compensate for finite resource stock

Social planner

formulas

- Social planner in this economy is:
  1. Maximizing lifetime consumption utility, (16)
  2. Given technology dynamics,(17)
  3. Given resource stock dynamics, (18)
- Optimal control over resource extraction R and research labor n
- This constitutes two-states two-controls infinite horizon optimal control problem.

Remark: Economy-wide technical change is described by

$$\dot{A}_t = (\gamma - 1)\lambda n_t A_t,$$

resembling Romer-type models

Steps to derive growth rates

formulas 1, formulas 2

- Construct Hamiltonian for the social planner, (19)
- Derive first-order conditions on R, n (20)
- Derive co-state equations associated with technology and resource dynamics, (21)
- Assume constant growth rates along BGP

This allows to express optimal growth rates for all variables.

Socially optimal growth rates

formulas

- The social optimum is given by:
  1. Quantities n^o, x^o
  2. Growth rates (constant) g_Y^o, g_R^o
- The research labor n^o defines all quantities
- The intensity of research λ and size of innovation γ define growth rates

The output growth is positive only if

$$\lambda(\gamma - 1) - \rho \geq 0.$$

Properties of the optimal path

formulas

- Growth may be *negative*, if technological efficiency is lower than discount rate
- Increase in λ or γ leads to:
  - Research labor n^o increases, decreasing manufacturing labor,
  - Rates of growth of technology increase,
  - Output grows faster, and resources are exploited to a lesser extent.
- Increase in ρ leads to the opposite
- Increase in ϵ increases elasticity of marginal utility, shifting preference towards uniform consumption paths.

Market economy

- Is defined by the set of prices, w_t, p_t, p_t^R, r_t with output price normalized to 1
- Interest rate is defined on a perfect financial market
- There are two market failures: monopolistic manufacturing sector and intertemporal spillover
- To eliminate those, we use:
  - Research subsidy, σ,
  - Demand subsidy for intermediate good θ.

Final goods sector

Profit of the final sector is:

$$\pi_t^Y = A_t x_t^{\alpha} R_t^{1-\alpha} - p_t(1-\theta)x_t - p_t^R R_t$$

F.O.C.s of profit maximization give:

$$x_t = \left(\frac{\alpha A_t}{p_t(1-\theta)}\right)^{\frac{1}{1-\alpha}} R_t, \qquad p_t^R = (1-\alpha)A_t\left(\frac{x_t}{R_t}\right)^{\alpha}. \qquad (1)$$

Intermediate goods sector

Profit function of the monopoly is:

$$\pi_t^m = p_t x_t - w_t x_t,$$

Maximization with demand given by eq. (1) yields:

$$x_t = \left(\frac{\alpha^2 A_t}{w_t (1-\theta)} \right)^{\frac{1}{1-\alpha}} R_t$$

and

$$p_t = w_t / \alpha.$$

R&D sector

formulas

- Profit of R&D is a random variable because of randomness of innovations arrival rates
- **Expected profit of R&D** is the full profit of the subsequent intermediate monopolist
- The **value of R&D** is the sum of present values of expected profits onwards
- **Arbitrage condition** equalizes labor costs and expected payoffs:

$$w_t(1 - \sigma_t) = \lambda V_t.$$

Resource market

formulas

- The maximization of the profit function given by **resource rent** over time
- Subject to constraint of finite resource stock
- Leads to the usual Hotelling's rule:

$$\forall t : \; \frac{\dot{p}_t^R}{p_t^R} = r_t$$

and asymptotic ($t \to \infty$) exhaustion of the resource stock

Government

Government's budget constraint is intertemporal (so borrowing is possible)

$$\int_0^{\infty} (\theta_t p_t x_t + \sigma_t w_t n_t - T_t) e^{-\int_t^s r_u du} dt = 0, \qquad (2)$$

with T_t being total tax revenues.

- Government sets subsidies σ_t and θ_t to maximize welfare
- The choice of taxes profile is made such that government borrowings are paid for in the end and budget is balanced.

Households

formulas

- Households maximize utility from consumption, (34)
- Subject to the **dynamic budget constraint** (35), which includes:
  1. Wages w_t, resource rent $p_t^R R_t$, profits of intermediate monopolist π_t^m as incomes
  2. Taxes T_t and consumption c_t as expenditures
  3. Savings are financial assets changes $\dot{B}_t$
- Optimal consumption path is given by difference is interest rate r_t and subjective discount rate ρ, (36)

Market equilibrium

Defined as:

- Set of **quantities**, $n^c, x^c, A_t^c, Y_t^c, R_t^c$ formulas 1;
- Set of **prices**, r, w_t, p_t, p_t^R, formulas 2;
- Set of resulting constant **growth rates**, formulas 3.

In these, only n^c depends on σ and only wage rate w_t on θ.
However:

- The research labor defines the growth rate of output, consumption, wages, technology, resource;
- Equilibrium wage defines all other prices

Thus subsidies regime define the growth rate of the economy

Remarks on equilibrium

formulas

- Market equilibrium makes it possible for the output to have negative growth (in the absence of R&D):
  1. For relatively small discount rate, ρ only positive growth rate equilibrium exists, $g_Y^c > 0$;
  2. For high discount rate only equilibrium with decaying output exists: $g_Y^c > 0$;
- Changes in R&D efficiency, discount rates, elasticity ϵ give the same results as for command optimum.

Comparison of equilibrium and optimum

formulas 1, formulas 2

We have *three* different cases on relation of command optimum and equilibrium, depending on where the discount rate lies:

1. Case of low discount rate:
$$\rho < \rho_+^c < \rho_+^o;$$
2. Case of medium discount rate:
$$\rho_+^c < \rho < \rho_+^o;$$
3. Case of high discount rate
$$\rho > \rho_+^o > \rho_+^c;$$

with ρ_+^c given by (9) and ρ_+^o by (51).

Discussion

- In the cases of low and medium discount rates command optimum yields higher growth rates than the market economy;
- In the case of high discount rate the opposite holds;
- The reason is the underinvestments of market economy into the R&D;
- As a result market overexploits exhaustible resource and slows down the technical change;
- The way out is **market regulation** through implementation of command optimum.

Implementation of the optimum path

- To realise the optimal path in market economy one has to choose **particular** subsidies rates;
- Observe, that producers' subsidy, θ, influences only prices;
- Thus to obtain optimum only research subsidy has to be set optimally;
- This level is

$$\sigma : n^c = n^o, \sigma > 0.$$

- At the same time n^c, g_Y^c, r are increasing in σ, and resource extraction rate g_R^c decreases for high elasticity ϵ;
- Thus optimal positive subsidy to R&D **maximizes** growth, utility and slows down resource exploitation.

Overview

- We add environment to the mix;
- There is a continuum of sectors, $i \in [0:1]$;
- Each sector generates its own innovations, B_{it};
- However, there exists the common level of knowledge in the economy: $B_t = \int_0^1 B_{it} di$;
- For every sector innovations have the same Poisson arrival rates as before (with sector-specific labor).

Setup

- Labour is used for final output and research:

$$L = 1 = L_{Y_t} + \int_0^1 L_{RD_{it}} di;$$

- Output depends on labor, total R&D productivity and resource exploitation:

$$Y_t = B_t^{\nu} L_{Y_t}^{\alpha} R_t^{1-\alpha};$$

- Household consumes **total output** and cares about **pollution**:

$$u(C_t, E_t) = \int_0^{+\infty} \frac{(C_t(-E_t)^{\omega})^{1-\epsilon}}{1-\epsilon} \mathrm{e}^{-\rho t} dt,$$

$$C_t = Y_t.$$

Environmental module

- **Pollution**, P_t is generated by the usage of resource in production:
$$P_t = \gamma R_t;$$
- **Resource stock** , S_t is limited and decreasing:
$$\dot{S} = -R_t;$$
- **Environmental quality**, E_t decreases from pollution:
$$\dot{E}_t = -P_t - \phi E_t.$$

the stock is bounded from below and **negative**

Final producers

Equilibrium is obtained in a similar manner:

- Representative firm in final sector maximizes profits:

$$\pi_t^Y = B_t^{\nu} L_{Y_t}^{\alpha} R_t^{1-\alpha} - w_t L_{Y_t} - p_{R_t} R_t - p_{R_t} q_t P_t + p_{R_t} q_t Q_t$$

- Yielding wage and resource price:

$$w_t = \alpha B_t^{\nu} L_{Y_t}^{\alpha-1} R_t^{1-\alpha},$$
$$p_{R_t} \theta_t = (1-\alpha) B_t^{\nu} L_{Y_t}^{\alpha} R_t^{-\alpha}$$

R&D sector

- In this version there is no intermediate product;
- The profit of R&D is no longer the profit from using technology;
- Rather, it is the *willingness to pay* of users of this knowledge;
- B_t is used in production Y_t and research:

$$B_{\tau+1(i)} = B_{\tau(i)} + \sigma B_{t(\tau(i))} \rightarrow \dot{B}_t = \lambda \sigma L_{RD_t} B_t. \tag{3}$$

- From expected R&D profits we get *free entry condition*:

$$w_t = \lambda \sigma B_t V_t. \tag{4}$$

Resource sector and households

- The competitive natural resource market sets the price as of Hotelling's rule:

$$\frac{\dot{p}_{R_t}}{p_{R_t}} = r_t; \tag{5}$$

- Households maximize utility subject to flow budget constraint

$$\dot{D}_t = w_t = rD_t + p_{R_t} R_t - T_t - C_t, \tag{6}$$

- Which yields intertemporal consumption rule as a function of the degradation of environment:

$$\frac{\dot{C}}{C} = \frac{r - \rho - \omega(1-\epsilon)\dot{E}/E}{\epsilon}. \tag{7}$$

Government

- In this economy government has 2 activities:
  1. It subsidizes research by $s_t^{RD} = \nu_t^{RD} B_t$,
  2. It distributes pollution permits in the output sector and subsidizes research acquisition by the final sector:

$$s_t^Y = \nu_t^Y B_t - p_{R_t} q_t Q_t. \tag{8}$$

- The budget is balanced at any point in time by charging lump-sum taxes from households:

$$T_t = s_t^Y + s_t^{RD}. \tag{9}$$

Equilibrium

- Equilibrium has the same properties as before;
- Existence is different:
  1. Equilibrium exists only for low discount rate;
  2. There is an additional condition on marginal disutility of pollution, ω.
- The evolution path (in case of existence) is defined by the growth rate of permits and subsidies: there is a **continuum** of possible growth rates!.

Environmental policy

- The governmental policy has two competitive uses;
- Setting high enough pollution permits, Q, decreases the need for governmental R&D subsidy;
- This happens because final producers have *free* permits, generating additional profit, spent on R&D;
- If there is no permits at all, the whole R&D activity is financed by the government;
- At the same time increase in permits may slow down R&D through L_{RD} channel;
- This happens if disutility from pollution is **low**:

$$\Delta g^{Q} > 0 \rightarrow \Delta g^{\theta} < 0 \rightarrow \delta L_{RD} < 0, \tag{10}$$

iff

$$\omega < 1 - \alpha. \tag{11}$$

Conclusions

- Both models illustrate the importance of environmental and R&D policy;
- Environmental policy may be partially substituted by the R&D policy;
- Still market failures of two types may lead to degradation of the economy;
- This cannot be offset by the smart policy choices (depends on parameters);
- If consumers are myopic (ρ is high), nothing can help;
- We need both smart R&D policy and better far-seeing people!

References

- Griamud A., Rougé L., (2003) Non-renewable resources and growth with vertical innovations: optimum, equilibrium and economic policies. *Journal of Environmental Economics and Management 45*, pp. 433-453;
- Griamud A., Rougé L., (2004) Polluting non-renewable resources, tradeable permits and endogenous growth. *Int. J. Global Environmental Issues, Vol. 4, Nos. 1/2/3*.

Next time

- Concept of **directed** technical change;
- Corresponds to the old idea of **induced** technical change from 1960's;
- In multi-sectoral setup, **which** sector has to be stimulated?
- Seems to be way forward to "green growth"?
- Paper: Why do new technologies complement skills? Directed technical change and wage inequality. Acemoglu D. (1998)

Preliminaries

Back

- The growth rate of the economy is

$$g_Y = g_A + (1-\alpha)g_R; \tag{12}$$

- The extraction of the resource must be limited;

$$\int_0^{\infty} R_0 \mathrm{e}^{(g_Y - g_A)t/(1-\alpha)} \leq S_0; \tag{13}$$

- This integral converges only if

$$g_A > g_Y, \tag{14}$$

requiring technocentric society.

Social planner

Back **Remark:** Economy-wide technical change is described by

$$\dot{A}_t = (\gamma - 1)\lambda n_t A_t, \tag{15}$$

resembling Romer-type models

- Social planner problem is thus:

$$\max_{n,R} \int_0^\infty \frac{1}{1-\epsilon}\left(A_t(1-n_t)^\alpha R_t^{1-\alpha}\right)^{1-\epsilon} \mathrm{e}^{-\rho t} dt \tag{16}$$

$$s.t.$$

$$\dot{A}_t = (\gamma - 1)\lambda n_t A_t, \tag{17}$$

$$\dot{S}_t = -R_t. \tag{18}$$

Hamiltonian and Co.

Back

- Hamiltonian function of the social planner:

$$\mathcal{H}^{SW} = \frac{1}{1-\epsilon} A_t^{1-\epsilon} (1-n_t)^{\alpha(1-\epsilon)} R_t^{(1-\epsilon)(1-\alpha)} + \mu_t (\gamma - 1) \lambda n_t A_t - \nu_t R_t, \tag{19}$$

- F.O.C.s:

$$\frac{\partial \mathcal{H}^{SW}}{\partial n} = 0 \rightarrow \mu_t = \frac{\alpha A_t^{-\epsilon} (1-n_t)^{\alpha(1-\epsilon)-1} R_t^{(1-\alpha)(1-\epsilon)}}{\lambda(\gamma-1)},$$
$$\frac{\partial \mathcal{H}^{SW}}{\partial R} = 0 \rightarrow \nu_t = (1-\alpha) A_t^{1-\epsilon} (1-n_t)^{\alpha(1-\epsilon)} R_t^{(1-\alpha)(1-\epsilon)-1}, \tag{20}$$

Costates and growth rates

Back

- Co-state equations yield:

$$\frac{\partial \mathcal{H}^{SW}}{\partial A} = \rho\mu_t - \dot{\mu}_t \rightarrow g_\mu = \rho - \frac{\lambda(\gamma-1)}{\alpha} + \frac{\lambda(\gamma-1)(1-\alpha)}{\alpha} n_t,$$
$$\frac{\partial \mathcal{H}^{SW}}{\partial S} = \rho\nu_t - \dot{\nu}_t \rightarrow g_\nu = \rho. \qquad (21)$$

- Last point is to assume constant growth rates along the steady state for all variables, thus

$$g_A^o = \lambda(\gamma-1)n_t^o = const \rightarrow n_t^o = const. \qquad (22)$$

- Then co-states and F.O.C.s may be recombined and used to acquire optimal growth rates and quantities of all variables.

Remarks

Back

- The transversality condition ensures, that

$$n^o < 1, \tag{23}$$

- Transversality condition is also equivalent to

$$g_R^o = g_S^o < 0, \tag{24}$$

- It ensures convergence of resource integral, eq. (5), and we have

$$R_0 = -g_A S_0 = (\rho - \lambda(\gamma - 1)(1 - \epsilon))S_0/\epsilon \tag{25}$$

- Since eq. (12) and (24) we have

$$g_Y^o < g_A^o, \tag{26}$$

Socially optimal growth rates

Back

$$
\begin{aligned}
n^o &= \frac{\alpha}{\epsilon}\left(1 - \frac{\rho}{\lambda(\gamma-1)}\right) = 1-\alpha, \\
x^o &= 1 - n^o, \\
g_A^o &= \frac{-\alpha\rho + \lambda(\gamma-1)(\epsilon+\alpha-\alpha\epsilon)}{\epsilon}, \\
g_Y^o = g_c^o &= \frac{\lambda(\gamma-1)-\rho}{\epsilon}, \\
g_R^o = g_S^o &= \frac{\lambda(\gamma-1)(1-\epsilon)-\rho}{\epsilon}.
\end{aligned}
\tag{27}
$$

R&D sector

Back

- *Expected* profit of R&D

$$\mathrm{E}(\tilde{\pi}_s) = \pi_s^m \mathrm{e}^{-\int_t^s \lambda n_u du}, \tag{28}$$

- The *value* of R&D:

$$V_t = \int_t^\infty \pi_s^m \mathrm{e}^{-\int_t^s (r_u + \lambda n_u) du} ds, \tag{29}$$

Resource market

Back

- The maximization of the profit function

$$\int_t^\infty p_s^R R_s \mathrm{e}^{-\int_t^s r_u) du} ds, \tag{30}$$

- Governed by the constraint

$$\dot{S}_s = R_s, \; R_s \geq 0, \; S_s \geq 0, s \geq t, \tag{31}$$

- Leads to the usual Hotelling's rule:

$$\frac{\dot{p}_t^R}{p_t^R} = r_t \, \forall t, \tag{32}$$

and asymptotic exhaustion of the resource stock under

$$\lim_{t \to \infty} S_t = 0. \tag{33}$$

Households

Back Households maximize utility

$$\int_0^\infty \frac{c^{1-\epsilon}}{1-\epsilon} \mathrm{e}^{-\rho t}. \tag{34}$$

subject to budget constraint:

$$\dot{B}_t = w_t + p_t^R R_t + \pi_t^m - T_t - c_t, \tag{35}$$

giving usual intertemporal condition

$$\frac{\dot{c}_t}{c_t} = \frac{r_t - \rho}{\epsilon}. \tag{36}$$

Equilibrium I

Back Market equilibrium path is the set of **quantities**:

$$n^c = f(\lambda, \epsilon, \gamma, \alpha, \rho, \sigma); \tag{37}$$

$$x^c = 1 - n^c; \tag{38}$$

$$A_t^c = A_0 \mathrm{e}^{g_A^c t}; \tag{39}$$

$$Y_t^c = c_t^c = A_t^c (x^c)^{\alpha} (R_t^c)^{1-\alpha}; \tag{40}$$

$$R_t^c = R_0 \mathrm{e}^{g_R^c t}. \tag{41}$$

Equilibrium II

Back **prices**:

$$r = \frac{\epsilon\lambda(\gamma - 1)n^c + \alpha\rho}{\alpha + \epsilon(1-\alpha)}; \tag{42}$$

$$w_t = \frac{\alpha^2 A_t^c}{(1-\theta)(x_t^c)^{1-\alpha}}; \tag{43}$$

$$p_t = w_t/\alpha; \tag{44}$$

$$p_t^R = (1-\alpha)A_t^c \left(\frac{x^c}{p_t}\right)^\alpha . \tag{45}$$

Equilibrium III

Back and **growth rates**:

$$g_n^c = g_x^c = g_r = 0; \tag{46}$$

$$g_Y^c = g_c^c = g_w = g_\rho = \frac{r-\rho}{\epsilon} = \frac{\lambda(\gamma-1)n^c - \rho(1-\alpha)}{\alpha+\epsilon(1-\alpha)}; \tag{47}$$

$$g_A^c = (\gamma-1)\lambda n^c; \tag{48}$$

$$g_S^c = g_R^c = g_Y^c - g_{p^R} = \frac{\lambda(\gamma-1)(1-\epsilon)n^c - \rho}{\alpha+\epsilon(1-\alpha)}; \tag{49}$$

$$g_{p^R} = r. \tag{50}$$

Remarks on equilibrium

Back

Positive growth rate equilibrium exists if:

$$\rho < \frac{\lambda\gamma(\gamma-1)(1-\alpha)}{\gamma-\alpha}, \epsilon > 1 - \frac{\rho}{\lambda\gamma(\gamma-1)(1-\alpha)} \rightarrow \exists! g_Y^c > 0. \tag{51}$$

Decaying output exists if:

$$\rho > \frac{\lambda\gamma(\gamma-1)(1-\alpha)}{\gamma-\alpha}, \epsilon < 1 - \frac{\rho}{\lambda\gamma(\gamma-1)(1-\alpha)} \rightarrow \exists! g_Y^c < 0. \tag{52}$$

Comparison of equilibrium and optimum

Back

1. Case of low discount rate:

$$\rho < \frac{\lambda\gamma(\gamma-1)(1-\alpha)}{\gamma-\alpha}, \epsilon > 1 - \frac{\rho}{\lambda(\gamma-1)}, \tag{53}$$

2. Case of medium discount rate:

$$\frac{\lambda\gamma(\gamma-1)(1-\alpha)}{\gamma-\alpha} < \rho < \lambda(\gamma-1), \tag{54}$$

$$\textit{and} \tag{55}$$

$$\gamma < 1/\alpha, \epsilon > 1 - \frac{\rho}{\lambda(\gamma-1)}, \tag{56}$$

$$\textit{or} \tag{57}$$

$$\gamma > 1/\alpha, \epsilon > \frac{\alpha}{(1-\alpha)}\left(\frac{\rho-\lambda\gamma(1-\alpha)}{\lambda\gamma(1-\alpha)}\right). \tag{58}$$

Case of high discount rate

Back

$$\rho > \lambda(\gamma - 1), \tag{59}$$

$$\textit{and} \tag{60}$$

$$\gamma < 1/\alpha, \epsilon > \frac{\alpha}{(1-\alpha)} \left(\frac{\rho}{\lambda(\gamma - 1)} - 1 \right), \tag{61}$$

$$\textit{or} \tag{62}$$

$$\gamma > 1/\alpha, \epsilon > \frac{\alpha}{(1-\alpha)} \left(\frac{\rho}{\lambda\gamma(\gamma - 1)} - 1 \right). \tag{63}$$

in which case equilibrium growth may be higher than in the optimum.