MATLAB Introduction for Finance and Economics

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Content

Getting Started

- Short for MATrix LABoratory, developed by 'The Mathworks'
- Matrix-based programming language and software for academia and industry
- Typical uses:
  - Math and computation
  - Data analysis and visualisation
  - Numerical simulations and modelling
  - Algorithm development
- Prepackaged functions and Toolboxes for statistics, econometrics, finance, optimisation,...

MATLAB Layout

- The GUI:
  - Current Folder
  - **Command Window**
  - **Editor** *(click on New Script button first)*
  - **Workspace**
  - File Details or Command History
- Home toolbar
  - New Script, New Function, Import Data, Preferences (Fonts), Set Path, Help
- Editor toolbar
  - New, Open, Save, Insert Section/Comment, Indent
  - **Run**, Run and Advance, Run Section, Advance

Getting Help

- When confronted with an unfamiliar function or command, type `help` or `doc` followed by the function name

```
help clear

clear Clear variables and functions from memory.
clear removes all variables from the workspace.

...

help clc

clc Clear command window.
clc clears the command window and homes the cursor.
```

MATLAB File Formats

- **Scripts** (`.m`) are for writing your program/script
- **Functions** (`.m`) can receive input arguments and return output values, unlike Scripts
  - Call a function by enclosing input arguments in parentheses: `z = functionname(x,y)`
- **Data** files (`.mat`) are variables you can import to, or save from, the Workspace
- The folder in which you save your main script is treated as the root directory

Matrix Types

- MATLAB is a matrix/array-based programming language
- **Elements** stored in an array can be numbers, characters (strings), logical states (true or false)
- A **matrix** is a rectangular array with m rows and n columns
- The square matrix is most common, where $m = n$
- A **row vector** has one row covering n columns
- A **column vector** has one column covering m rows
- A **scalar** is like a 1×1 matrix with only one element

Matrix Examples

$a = 2$ is a scalar

$\mathbf{a} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \end{bmatrix}$ is a 1×5 row vector

$$\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$ is a 4×1 column vector

$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$ is a 3×2 matrix

Matrix Examples written in MATLAB

$$a = 2$$

```
a = 2
```

$$\mathbf{a} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \end{bmatrix}$$

```
a = [1 2 3 4 5]
```

$$\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

```
b = [1; 2; 3; 4]
```

$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

```
A = [1 4; 2 5; 3 6]
```

Creating Variables in MATLAB

- Use assignment operator '=' to assign values to a variable
- Naming conventions:
  - First character must be a letter (`a-z`, `A-Z`)
  - Any combination of letters, numbers or underscores can follow
  - Case sensitive (`x1` is different from `X1`)
- Apostrophize non-numerical elements in string variables and character arrays

Matrix Construction in MATLAB

- For matrix construction, use square brackets `[ ]`
- A comma ‘`,`’ or a space ‘ ’ separates row elements
- A semicolon ‘`;`’ starts a new row in an array
- All rows should have the same number of elements

Variable Types in MATLAB

Variable type	Example
Scalar	`x = 5`
String	`y = 'oranges'`
Row Vector	`a = [0, 2, 3, 4]`
Column Vector	`b = [0; 2; 3; 4]`
Matrix	`A = [0, 2; 3, 4]`
Character array	`y = ['oranges'  'lemons']`
Logical arrays	`z = [0 1 1 0 1 0]`

Variable Assignment

- When MATLAB encounters a new variable name, it automatically creates the variable and allocates the appropriate amount of storage. If the variable already exists, MATLAB changes/overwrites its contents, allocating more storage if necessary.

Example

```
x = [2; 3];
y = x;
x = x*3
```

Sequencing Numbers

- The colon operator ':' is useful for creating vectors quickly.
- Each of the following commands create the same array:

Examples

```
a = [1,2,3,4,5]
a = 1:5
a = 1:1:5
a = linspace(1,5,5)
```

- Sequence `0:2:10` would run from 0 to 10 in increments of 2
- Reverse sequence with negative increment, `10:-2:0`. (Default increment is 1.)

Matrix Concatenation

- Connect the following arrays together:

```
a = 1:5;
b = 6:10;
```

Examples

Horizontal concatenation (side-by-side columns):

`[a,b]` $\rightarrow \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{bmatrix}$

Vertical concatenation (stack rows):

`[a;b]` $\rightarrow \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \end{bmatrix}$

Indexing Elements of a Matrix

- In MATLAB, indexing starts with 1, not 0
- To reference a particular element in a matrix, either:
  - Indicate the element's row and column position: `A(1,2)`
  - Or index of the actual storage sequence: `A(4)`

Examples

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

`A(0)`	$\rightarrow$	`error`
`A(1)`	$\rightarrow$	`1`
`A(1,2)`	$\rightarrow$	`4`
`A(4)`	$\rightarrow$	`4`
`A(end)`	$\rightarrow$	`6`

Indexing Vectors of a Matrix

- The ‘`:`’ operator can also be used as an index argument
- Useful for extracting rows, columns or subsets of a matrix

Examples

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

`A(:)`	$\rightarrow$	6×1 vector
`A(:,2)`	$\rightarrow$	`[4; 5; 6]`
`A(1:end, end)`	$\rightarrow$	`[4; 5; 6]`
`A(1:2:end, 1)`	$\rightarrow$	`[1; 3]`
`A([1,3], :)`	$=$	$\begin{bmatrix} 1 & 4 \\ 3 & 6 \end{bmatrix}$

Deleting Array Elements and Variables

- The command `who` returns a list of all existing variables, while `whos` gives more information about them
- `a(3)=[]` deletes third element in vector `a`
- `A(1,:)=[]` deletes the first row of an array
- `clear A` deletes the variable called `A`
- `clear A*` deletes all variables whose names begin with ‘A’

Special Matrices

$$\texttt{ones(2)} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\texttt{zeros(2)} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\texttt{eye(2)} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\texttt{diag(x)} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} \quad \text{, where } \texttt{x} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Special Matrices

$$\texttt{rand(2)} = \begin{bmatrix} 0.6463 & 0.7547 \\ 0.7094 & 0.2760 \end{bmatrix}$$

$$\texttt{randn(2)} = \begin{bmatrix} 0.0774 & -1.1135 \\ -1.2141 & -0.0068 \end{bmatrix}$$

$$\texttt{randi(2)} = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$$

$$\texttt{magic(2)} = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$$

Useful Built-in Functions

- For inspecting array dimensions:
  `length(), numel(), size()`
- For sorting and indexing:
  `find(), sort(), sortrows(), flipud(), fliplr()`
- For rounding:
  `ceil(), round(), floor()`
- Special math operations:
  `sum(), cumsum(), prod(), cumprod()`

Operators

1. **Arithmetic Operators** (for numeric computations)
   $+, -, *, /, ^\wedge$
2. **Relational Operators** (for quantitative comparisons)
   $>, <, <=, ...$
3. **Logical Operators** (for logical operations)
   $\&, |, \sim$

- Relational operators compare two arrays element-by-element, returns a same-sized logical array containing `1` (true) or `0` (false).
- Logical operators and functions also return a logical array but by examining the elements of one or two arrays altogether.

Arithmetic Operators

Symbol	Operation	Example
+	Addition	2+3
−	Substraction	2−3
*	Multiplication	2*3
/	Division	2/3
^	Exponentiation	2^3

- The usual algebraic precedence ordering holds for these operators

Matrix and Array Arithmetics

- 2 approaches:
  1. Matrix arithmetic - defined by the rules of **linear algebra**
  2. Array arithmetic - carried out **element-wise**
- Putting a '.' in front of operator instructs MATLAB to use 2nd approach, i.e. `.*`, `./`, `.^`
- To add/subtract two arrays with '+' and '−', array sizes must match unless one is a scalar because done element-wise for both approaches

Matrix and Array Arithmetics

$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

Examples

`A = [1 4; 2 5; 3 6]` → 3×2 matrix
`B = [1 2 3; 4 5 6]` → 2×3 matrix

`A+B`	→	Invalid operation	`A/B`	→	Invalid
`A+B'`	→	Valid operation	`A./B'`	→	Valid
`A.*B`	→	Invalid	`B^2`	→	Invalid
`A*B`	→	Valid	`B.^2`	→	Valid

Rules for Matrix Operations

- To multiply matrices with '$*$', number of columns in A must equal # rows in B, unless one is scalar
- To multiply/divide matrices with '`.*`' or '`./`', both matrices must have same size, unless one is scalar
- Use the transpose operator `'` or the `reshape` and `repmat` functions to make sizes compatible
- Only square matrices and scalars can be raised '^' to a power
- Exponent can be another matrix for '`.^`' operation as long as both matrices are the same size, unless one is a scalar

Relational Operators

Operation	MATLAB
equal	==
not equal	∼=
greater than	>
less than	<
greater of equal	>=
less or equal	<=

Example 1

```
a = 1;  b = 4;
a < b       →  1 (true)
a == b      →  0 (false)
```

Logical Arrays

Example 2

Given two datasets:

```
x = [2 4 6 8];
y = [9 7 3 1];
```

Which elements are greater than 5?

```
a = x > 5          →  logical([0 0 1 1])
b = y > 5          →  logical([1 1 0 0])
```

Logical arrays can be used to index other arrays, and single out elements that meet our condition:

```
x(a)     x(x > 5)  →  [6 8]
y(b)     y(y > 5)  →  [9 7]
```

Logical Operators

Example 2 (cont'd)

Which elements are true in both $\mathbf{a}$ and $\mathbf{b}$?

`a&b`   `and(a,b)` $\rightarrow$ `logical([0 0 0 0])`

Which elements are true in either $\mathbf{a}$ or $\mathbf{b}$?

`a|b`   `or(a,b)` $\rightarrow$ `logical([1 1 1 1])`

What is the opposite of $\mathbf{b}$ ($\neg\mathbf{b}$)?

`~b`   `not(b)` $\rightarrow$ `logical([0 0 1 1])`

Are all/any elements in $\mathbf{a}$ true?

`all(a)` $\rightarrow$ `0`
`any(a)` $\rightarrow$ `1`

Built-in Functions and Constants

- Many standard mathematical functions such as `sin`, `cos`, `log`, `sqrt`, `exp` are built-in
- Built-in math constants include: `pi`, `nan`, `inf`, `i`

Example 3

```
x = [0; pi/2; pi; 3*pi/2; 2*pi]
y = sin(x)
```

Statistics Toolbox

- Built-in statistical functions in MATLAB:
  `mean`, `median`, `min`, `max`, `mode`, `std`, `var`, `cov`, `corrcoef`
- The **Statistics Toolbox** is an expansion pack containing a wider range of statistical tools
- Higher moments: `skewness`, `kurtosis`
- Gaussian distribution functions:
  `normrnd`, `normcdf`, `norminv`, ...
- Hypothesis tests:
  `jbtest`, `kstest`, `ttest`, `ttest2`, `ztest`, `anova1`, `anova2`, ...

Running Scripts

- A script is a saved MATLAB file (`.m`) that contains lines of commands or code
- Press 'New → Script' in the Home toolbar. This opens the MATLAB Editor
- Pressing **Run** will execute all lines of code in sequence. The script must be saved first

Background for Script Example

- Firms may issue bonds to investors to raise money without giving them ownership. Bond investors are essentially lenders
- During the life of the bond, the investor might collect periodic payments called coupons
- At maturity, the investor receives the face value of the bond plus the final coupon
- What is the present value of a $\$100$ bond that pays an $\$8$ coupon annually for 5 years if the interest rate is 7%?

$$PV = \sum_{t=0}^{T-1} \frac{C_t}{(1+r_f)^t} + \frac{FV_T + C_T}{(1+r_f)^T}$$

Script Example

Example: Present Value of a Bond

```
clear; clc; clf;

CF=[0 8 8 8 8 108];
r=0.07;
T=5;

DCF=CF./(1+r).^(0:T)
PV=sum(CF./(1+r).^(0:T))
```

- Save file as 'bondprice.m'
- To run script, enter '`bondprice`' at Command Prompt, or press **Run**

Clean Slate

- After running a script, variables, files and functions are retained in the Workspace and background
- Can either leave these artifacts there, or start from clean slate
- The `clear`, `clc` and `clf` functions automatically cleared the Workspace at the start of the script
- To manually start from a clean slate after each Run:
  1. Right-click Workspace, select **Clear Workspace**
  2. Right-click Command Window, **Clear Command Window**

Comments

- Use comment symbol '`%`' to add descriptions to code or instructions on how to use it
- Typing '`%`' at front of a line of code tells MATLAB to skip that line when running the script
- Comment-out multiple lines of code by selecting them and pressing **Comment** in the Editor toolbar
- Double comment symbols '`%%`' specify a new section in the script. Press **Run Section** to execute section by itself
- To write a multi-line comment, type '`%{`' for first line then end last line with '`%}`'

Debugging

- After running a script, Command Window prints any errors with hints to fix them
- Commenting out lines of code can help narrow in on problem
- Debugging is methodical process of finding, resolving errors
- Examine line-by-line execution by setting **Breakpoints** (red ball) next to line numbers
- When breakpoint reached, unpause by pressing **Continue** or **Quit Debugging**
- Search your error/question online, i.e. Stackoverflow

Control Structures

- For the implementation of algorithms, a programming language requires control structures for the tasks listed below
- MATLAB offers the following constructs for each task
  1. Conditional Execution
     - `if ... end`
     - `if ... else ... end`
     - `if ... elseif ... else ... end`
     - (`switch` can be used instead of the above)
  2. Repetition, looping and iteration
     - `for ... end`
     - `while ... end`
     - `continue`, `break`, `return`
  3. Comparison

if ... end

- Syntax:

  if expr
  cmd
  end

- If the evaluation of expr yields logical 1 (true) or a non-zero result, MATLAB executes one or more commands denoted by cmd. Otherwise, MATLAB skips cmd.

Example Script

```
x = rand()-0.5;
if x<0
   disp('x is negative')
end
```

if ... else ... end

- Syntax:

  if expr
  cmd1
  else
  cmd2
  end

Example Script

```
x = rand()-0.5;
if x < 0
   disp('x is negative')
else
   disp('x is positive or zero')
end
```

if ... elseif ... else ... end

- Use this construct if there are over 2 possibilities

Example Script

```
m = 90;
if m >= 70
   disp('First Class')
elseif m >= 60
   disp('Second Class')
elseif m >= 50
   disp('Third Class)
else
   disp('Fail')
end
```

Repetition, Looping and Iteration

- A sequence of calculations is repeated until either
  - All elements in a vector or matrix have been processed
  - The calculations have produced a result that meets a predetermined termination criterion
- Loop constructs in MATLAB:
  - **for ... end**
  - **while ... end**

for loop

- The statements in the **for** loop repeat continuously for a specific number of times
- Syntax:

  for index = first:step:last

  cmd

  end
- Loop variable
  - defined as a vector
  - is a scalar within the command block
  - does not need to have consecutive values

1. Save the following as any name other than 'ou.m':

Example: Ornstein-Uhlenbeck Stochastic Process

$$dX_t = \kappa(\theta - X_t)dt + \sigma dW_t$$

```
kappa = 0.04; theta = 0; v = 0.2; ou = 0;

for t=2:100
   ou(1,t) = ou(1,t-1) + kappa.*(theta-ou(1,t-1)) ...
      + v.*randn(1,1);
end
plot(ou')
```

2. Run the code. Next, put a breakpoint by `end` and iterate by repeatedly pressing **Run** and **Continue** to watch variable `ou` fill

while loop

- The **while** loop executes a statement or a group of statements repeatedly as long as the controlling expression is true
- The number of iterations required for the termination criterion/criteria to be met is not necessarily known in advance
- Beware of infinite loops! Pressing **CTRL+C** stops code execution
- Syntax:

```
while expr
    cmd
end
```

Example 1

```
maxIt = 100; sumX = 0; it = 1;
while it <= maxIt
    sumX = sumX + it;
    it = it+1;
end
```

Example 2

```
min_error = 0.1; x_true = 0.5;
it = 1;
while error > min_error
    x_hat = rand();
    error = abs(x_true-x_hat);
end
```

break, continue and return

- The **continue** statement passes control to the next iteration of the loop in which it appears, skipping any remaining statements in the body of the loop.
- The **break** statement terminates the execution of a **for** loop or **while** loop. When a break statement is encountered, execution continues with the next statement outside of the loop.
- The **return** command forces early termination allowing us to exit a function/script prior to the point of normal completion

continue vs. break

- What will be the value of sum_x?

Example

```
sum_x = 0;
for k = 1:10
   if sum_x == 5
      continue;
   end
   sum_x = sum_x + x(k);
end
```

- Now try replacing continue with break

Importing Data into MATLAB

- Common data formats are `.csv, .txt, .xlsx, .mat`
- Alternative ways to import data:
  - The **Open** button with 'All Files (`*.*`)' chosen
  - Double-clicking the data file in **Current Folder** panel
  - Import Wizard, by pressing **Import Data** button
  - In Excel, import and delimit file through Data tab, copy all cells, then in MATLAB create **New Variable** and paste data into this array

Importing Data into MATLAB

- MATLAB functions that import data:
  - `csvread()` replaced with `readmatrix()` in version 2019a
  - `load()` if `.mat` data file already prepared and saved
- We'll use Import Wizard and `load()` to import `.csv` stock market data from Yahoo Finance

Importing Data with `load()`

1. In `http://finance.yahoo.com` **Quote Lookup** search box, type and select ‘AAPL’ on the NASDAQ exchange
2. Click on **Historical Data** tab in Yahoo Finance
3. For time period, change start date to 01/01/2000, end date to 01/30/2020, then press **Apply** button
4. Press **Download Data** and save the `.csv` file to your MATLAB current folder
5. (Could assign `price = readmatrix('AAPL.csv')`)

Importing Data with `load()`

6. In MATLAB, press **Import Data** button to open the `.csv` file
7. In Import Wizard, select 'Close' column only (E), then adjust Range to exclude non-numerical headers (e.g. E2:E5051)
8. Change **Output Type** to 'Numeric Matrix'
9. Press **Import Selection**
10. Double-click AAPL variable in **Workspace** to view
11. Right-click AAPL variable in Workspace, save as 'AAPL.mat'
12. Can now assign `price = load('AAPL.mat');`

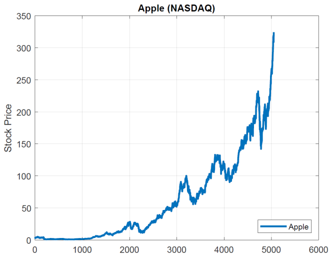
plot Function

- `plot(x)`, where x is some vector of length N, draws the data points contained according to their index $\{(1, x(1)), (2, x(2)), \ldots, (N, x(N))\}$
- `plot(x,y)` where x and y are vectors of the same length, draws the points $\{(x(1), y(1)), (x(2), y(2)), \ldots, (x(N), y(N))\}$
- Data points are connected with a straight line

Plot Data

Example Script: Stock Price Chart

```
figure
plot(AAPL,'linewidth',2);
grid on
ylabel('Stock Price')
title('Apple (NASDAQ)')
legend('Apple','Location','southeast');
```



Customize Plot

- Third argument `S` in **`plot(x,y,S)`** is a string that combines line style, color, marker preferences. See all options with `help plot`

`k`	black	`.`	dots	`-`	solid line	`-.`	dash-dot
`g`	green	`o`	circles	`:`	dotted line	`v`	triangle (down)
`r`	red	`x`	x-mark	`--`	dashed line	`*`	star

Examples

`plot(x, 'k.-')` plots a black line marked with dots
`plot(x, y, '.')` plots different colored dots with no line
`plot(x, y, 'c+--')` plots a cyan dashed line marked with +'s
`plot(x, y, 'bd')` plots blue diamonds with no line

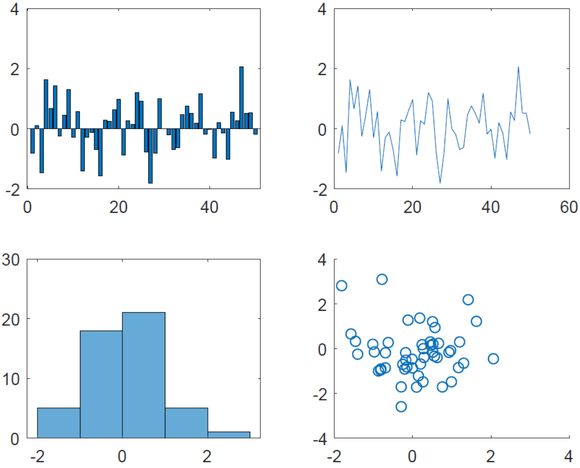
subplot Function

- `subplot(nrows, ncols, position)` is used to make multiple plots in the same figure

Example Script

```
y = normrnd(0, 1, 50, 1);
z = trnd(4, 50, 1);
subplot(2,2,1); bar(y)          % bar plot
subplot(2,2,2); plot(y)         % line plot
subplot(2,2,3); histogram(y)    % histogram
subplot(2,2,4); scatter(y,z)    % scatter plot
```

Subplot Output



MATLAB Tables

- The script below generates a table in the Command Window displaying data from Workspace variables

Example

```
Country = {'Austria'; 'Germany'; 'Liechtenstein';
'Switzerland'};
Imports = [14.1; 82.1; 0.47; 16.8];
Range = [14.5 0.06; 98.4 0.35; 0.9 0.15; 18 0.27];
inEUR = logical([1; 1; 0; 0]);

T = table(Country, Imports, Range, inEUR)
```

3D Plotting Functions

- `plot3(X,Y,Z)` displays a 3D line plot of a set of data points
- `surf(X,Y,Z)` creates a 3D surface plot colored for height

Examples

```
t = 0:pi/50:10*pi; st = sin(t); ct = cos(t);
plot3(st,ct,t)  % 3D Helix Line Plot

Z = peaks(30) % 30x30 data matrix
surf(Z) % Surface Plot of peaks(30)
```

fplot function

- `fplot(fname,lims)` plots the function `fname`, as long as it is univariate
- Default for axis limits is $[-5 \;\; 5]$, but can be customized with `lims = [xmin xmax]` argument
- `fplot(x,y,lims)` plots coordinates $x(t)$ and $y(t)$ for t between `[xmin xmax]`

Examples

```
fplot(@sin)
fplot(@(x) sin(1./x), [0.01 0.1])
fplot(@(t) cos(3*t), @(t) sin(2*t))
```

Functions

- A MATLAB function (`.m`) receives **input variables** as parameters for processing, then returns **output variables**
- Functions can be called from Command Window, a script, or other functions
- Offloading code to a function reduces length and clutter in the main script, making debugging easier

Declaring and Calling Functions

- Declare a function in MATLAB with:

  `function [y1, y2,...] = fname(x1, x2,...)`

  - This is just the first line. Below it would be the algorithm followed by `end`
  - The reserved word "**`function`**" must be at the beginning
  - The name of the function should match its file name, excluding `.m`

- Invoke the function with:

  `[y1, y2,...] = fname(x1, x2,...)`

  - The variable names called in as inputs and outputs can be different than how they are named within the function

Function Examples

1. Go to 'New', 'Function' and type the following in Editor:

Example 1: Statistical Moments of a Variable

```
function [mu, sigma, sk, ku] = moments(x)
    mu = mean(x);
    sigma = std(x);
    sk = skewness(x);
    ku = kurtosis(x);
end
```

2. Save function as 'moments.m' and try to run
3. Create data `x = randn(1000,1)` at Command Prompt
4. Must instead call function with
   `[mu, sigma, sk, ku] = moments(x)`

Function Examples

1. Go to 'New', 'Function' and type the following in Editor:

Example 2: Return on Equity

```
function ROE = return_on_equity(NI, sales, TA, TE)
    % NI is net income
    % TA is total assets
    % TE is total equity
    ROE = (NI/sales)*(sales/TA)*(TA/TE);
end
```

2. Save as 'return_on_equity.m'
3. Set `NI=5e6; sales=20e6; TA=30e6; TE=8e6;`
4. Now call the function with

   `ROE = return_on_equity(NI, sales, TA, TE)`

Types of Functions

1. **Local functions**
   - Contained within the main script/same file, but not tabbed
   - In a function file, they can appear in any order after the main function in the file
   - In a script file, they must be at the end of the file
2. **Nested functions**
   - Contained (tabbed) completely within a parent function
   - Can use variables defined in parent function without explicitly passing those variables in as input arguments
3. **Anonymous functions**
   - Called with a defined handle without saving to a function file
   - Handy for defining and evaluating a mathematical expression over a range of values

Example: Local Functions

```
u=randn(20,1);
[avg, med] = newstats(u)

function[avg, med] = newstats(u) % Parent function
    n = length(u);
    avg = mean(u, n);
    med = median(u, n);
end

function a = mean(v,n) % Local function
    a = sum(v)/n;
end

function m = median(v,n) % Local function
    w = sort(v);
        if rem(n,2) == 1
            m = w((n+1)/2);
        else
            m = (w(n/2)+w(n/2+1))/2;
        end
end
```

Background for Nested Function Example

- The **Black-Scholes-Merton model** computes the value of an option based on underlying stock price information

Valuing a Call Option

$$C_0 = Se^{-\delta\tau} \times N(d_1) - Xe^{-r_f\tau} \times N(d_2)$$

where $d_1 = \frac{ln(S/X)+(r_f-\delta+\sigma^2/2)\tau}{\sigma\sqrt{\tau}}$ and $d_2 = d_1 - \sigma\sqrt{\tau}$

S is the current stock price	r_f is the risk-free rate
X is the exercise or strike price	δ is the dividend yield
τ is the time to maturity, $T-t$	σ^2 is the stock variance

Example: Nested Function

```
S = 75; X = 70; tau = 0.75; rf = 0.01; delta = 0.05; v = 0.35;
call = callBSM(S,X,tau,rf,delta,v)

function C0 = callBSM(S,X,tau,rf,delta,v) % Parent function
    [d1, Nd1] = N_d1;
    d2 = d1 - sqrt(v*tau);
    Nd2 = normcdf(d2, 0, 1);
    C0 = S.*exp(-delta.*tau).*Nd1 - X.*exp(-rf.*tau).*Nd2;

    function [d1, Nd1] = N_d1() % Nested function
        d1 = (log(S./X) + (rf - delta + v / 2) .* tau) ...
            ./ (sqrt(v * tau));
        Nd1 = normcdf(d1, 0, 1);
    end
end
```

- Save as 'callBSM_nested.m' then **Run**

Anonymous functions

- Handy for defining and evaluating a math formula over a range of values
- Can be constructed at command prompt, in a script, or in a function
- Syntax:

  fhandle = @(arglist) expr

  - expr: expression/formula to be evaluated containing variables
  - arglist: list of input arguments to be passed to the function
  - @ sign: operator that assigns function handle used to invoke the function

Example: Parabola as Anonymous Function

```
a=1; b=0; c=0;
y = @(x) a*x.^2 + b*x + c
fplot(y)
```

Example: Range of Parabolas

```
figure
hold on
b=0; c=0;
for a = 1:5
    fplot(@(x) a*x.^2 + b*x + c,'b--')
end
hold off
```

Optimization Background

- *What are the optimal inputs for a function that minimizes/maximizes its output?*
- Unconstrained optimization problem:

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} f(\mathbf{x})$$

  - $f(\mathbf{x})$ is objective function that generates output
  - $\mathbf{x} = [x_1, x_2, \ldots x_n]$ is decision variable, or input
  - Global optimum $\mathbf{x}^*$ is the stationary point where $f(\mathbf{x}^*) \leq f(\mathbf{x}) \;\; \forall \mathbf{x} \in S \subseteq \mathbb{R}^N$ because $f'(\mathbf{x}^*) = 0$
  - Local optima are also stationary, but sub-optimal to $\mathbf{x}^*$
  - Unconstrained problem puts no constraints on the range of $\mathbf{x}$

Optimization Background

- Constrained optimization problem:

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} f(\mathbf{x}) \text{ s.t. } \mathbf{x} \geq \mathbf{0}$$

  - Objective function is now subject to a non-negativity constraint on $\mathbf{x}$
  - Equality constraints: $g(\mathbf{x}) = \mathbf{0}$
  - Inequality constraints: $g(\mathbf{x}) \geq \mathbf{0}$
  - Bound constraints: $\texttt{lb} \leq g(\mathbf{x}) \leq \texttt{ub}$

Optimization Background

1. **Gradient-based methods**
   - Iterate through sequence of candidate solutions $\mathbf{x}^{(k)}$ until convergence towards stationary point $\mathbf{x}^*$
   - Approximate $f(\mathbf{x})$ based on its partial derivative(s) $f'(\mathbf{x})$
   - Reliable if $f(\mathbf{x})$ is smooth
   - Doesn't work if $f(\mathbf{x})$ is non-differentiable
2. **Derivative-free methods**
   - Direct local search evaluates trial solutions in a local area until no improvement can be made
   - Use when $f(\mathbf{x})$ is non-smooth or noisy
   - Includes Nelder-Mead simplex, genetic algorithms, metaheuristics (simulated annealing)

Optimization Toolbox

- The following functions and routines can be implemented as an M-file, function handle, or inline function
  - Linear programming: `linprog`
  - Quadratic programming: `quadprog`
  - Unconstrained optimization: `fminunc`
  - Constrained optimization: `fmincon`
  - Derivative-free optimization: `fminsearch`
- Search Matlab documentation to see each's objective function
- Matlab optimizers minimize. To maximize, solve $\min\left[-f(\mathbf{x})\right]$

- Convex problem ($f''(\mathbf{x}) > 0$) ensures local minimum is a global minimum

Example: Convex Optimization problem

```
tc = @(q) q+q.^2

figure
fplot(tc)

min = fminunc(tc,0) % find the minima of tc
tc(min) % value of function at min
```

- Also try `fminsearch(tc,0)` at Command Prompt

- Non-convex problem ($f''(\mathbf{x}) < 0$) might trap optimizer at a local minima

Example: Non-convex Optimization problem

```
tc = @(q) (1/5)*q.^5 + (1/2)*q.^4 - 4*q.^3
... - q.^2 + 6*q;
mc = @(q) q.^4 + 2*q.^3 - 12*q.^2 - 2*q + 6;

figure
fplot(tc)
figure
fplot(mc)

min1 = fminunc(mc,0)   % local minima 1
min2 = fminunc(mc,-4) % local minima 2 is lower
tc(min2) % value of tc when mc at min2
```

Third-party Functions and Packages

- Researchers sometimes share their code on their homepage
  - Gilli, Maringer, Schumann - NMOF
    `http://www.nmof.info`
  - Kevin Sheppard (Oxford) - MFE toolbox
    `http://www.kevinsheppard.com/MFE_MATLAB`
  - Jondeau and Rockinger (Lausanne) - Financial applications
    `http://www.hec.unil.ch/MatlabCodes`
  - Kendrick, Mercado, Amman (Texas) - Comp. economics
    `http://www.laits.utexas.edu/compeco`
- MATLAB's File Exchange website has many user-created packages

References

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- C. Moler (2004) *Numerical Computing with MATLAB*
- K. Nyholm (2008) *Strategic asset allocation in fixed-income markets*. Chapter 2: Essential Elements of MATLAB
- P. Getreuer, *Writing Fast MATLAB Code*
- D.F. Griffithis, *An Introduction to MATLAB*
- Mathworks website, MATLAB Help (printable documentation)