Simulation (part 1)

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basic concepts

- simulation as a method
  - experiments using models

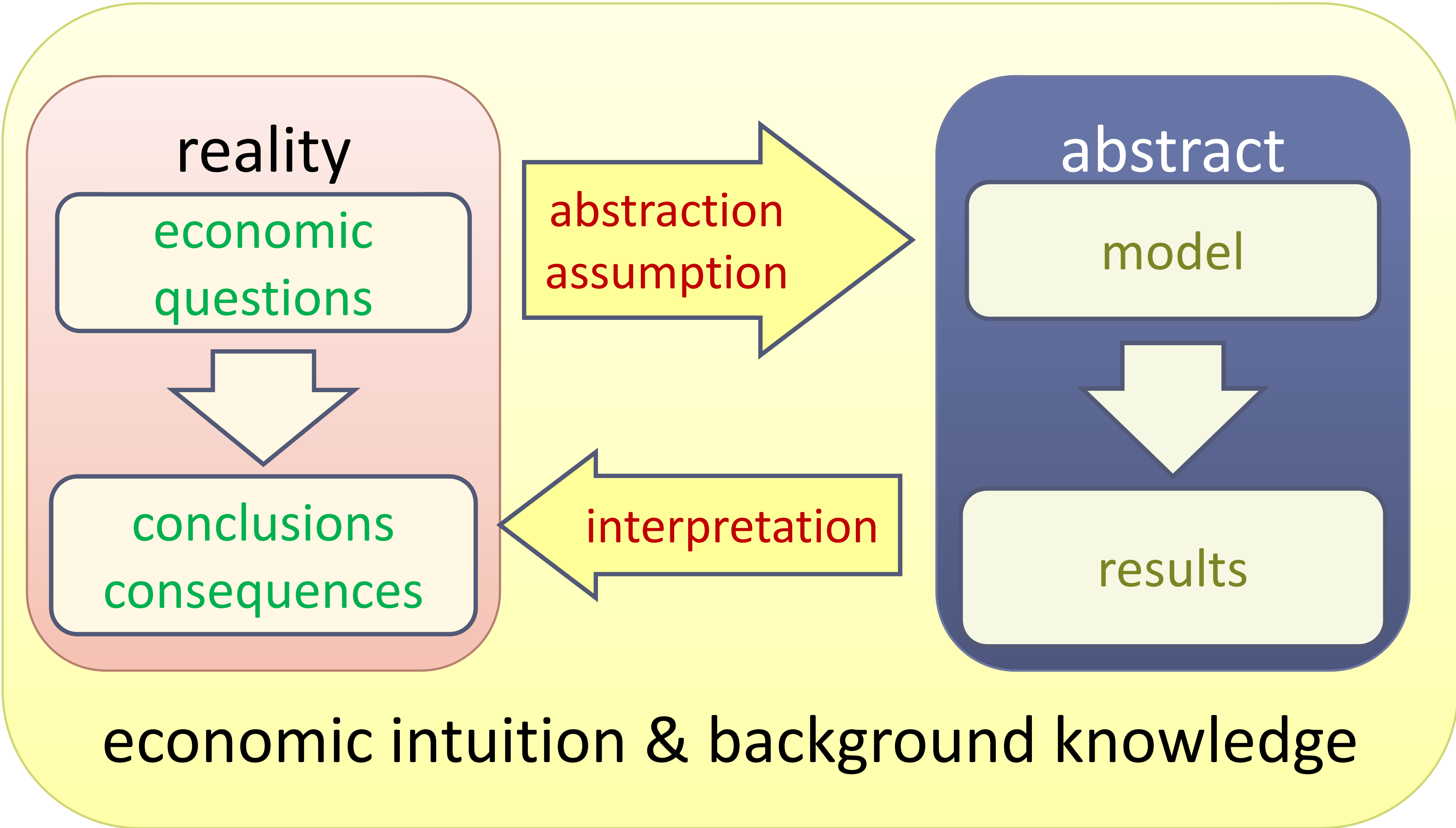
- some types of models
  - physical models
  - statistical models
  - micro-level / agent based models

simulation

- basic concepts

observation based	model based (data driven ⇔ by construction)	
historical	statistical / econometric	agent based
assumption • history repeating **example** • back testing	**assumption** • formal model **example** • mean / variance	**assumption** • micro-structure ⇔ aggregation **example** • artificial stock market

process


reality
economic questions
conclusions consequences
abstraction assumption
interpretation
abstract
model
results
economic intuition & background knowledge

stages in the simulation process

1. designing a model
   - level of abstraction & details
     - assumptions
     - precision of relationships, estimations, etc.
     - sufficient level of details, but as simple as possible
   - rule of the thumb: go for …
     - … accuracy ➔ prediction
     - … simplicity ➔ coneptual understanding

2. building the model
   - formal description
   - implementation
     - own code
     - software packages

stages in the simulation process

3. verification & validation
   - verification
     - quality of implementation
     - check for implementation errors
   - validation
     - is simulation good enough for target?
     - model behaviour ⇔ simulation errors
     - limitations of model
       - level of abstraction
       - underlying assumptions & simplifications
       - precision / reliability of target
4. publication

Monte Carlo approaches

- generating random numbers
  - uniform
    - linear congruential RNG: $u_i = (a\, u_{i-1} + c) \mod m$
      - try: *a* = 65539, *c* = 0, *m*= 2^{31}
      - try: *a* = 1‘664‘525, *c* = 1‘013‘904‘223, *m*= 2^{32}
    - Mersenne twister
  - specialised methods
    - e.g. Box Muller for standard normal $z_1, z_2 \sim \mathrm{N}(0,1)$

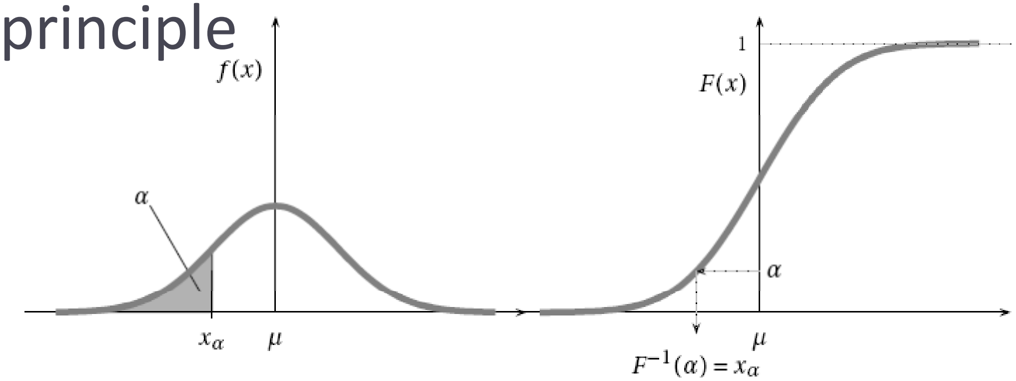
$$u_1, u_2 \sim U(0,1)$$

$$z_1 = \underbrace{\sqrt{-2\log(u_1)}}_{r}\ \underbrace{\cos(2\pi\, u_2)}_{c_x}$$

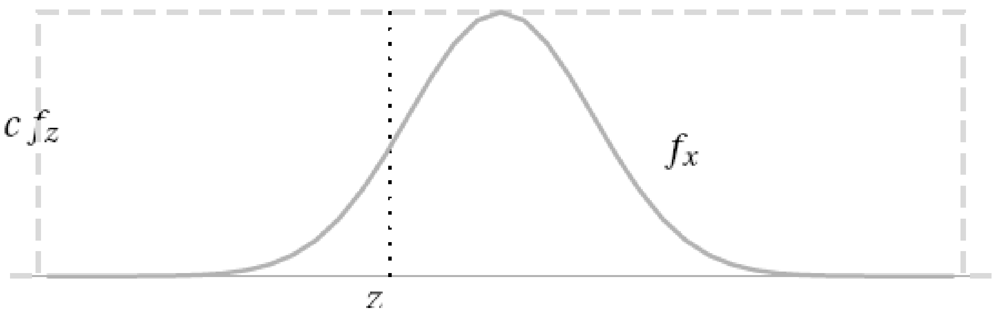
$$z_2 = \underbrace{\sqrt{-2\log(u_1)}}_{r}\ \underbrace{\sin(2\pi\, u_2)}_{c_y}.$$

Monte Carlo approaches

- generating random numbers (cont.'d)
  - inversion principle



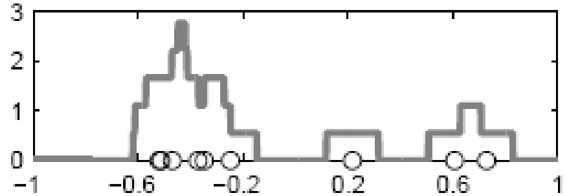
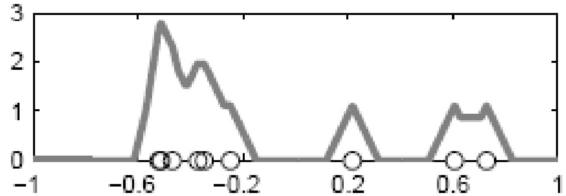
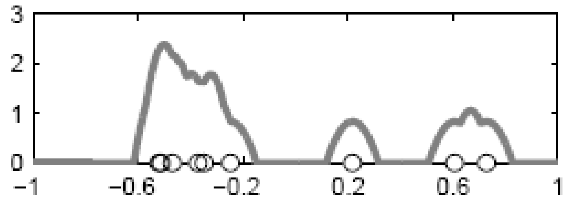
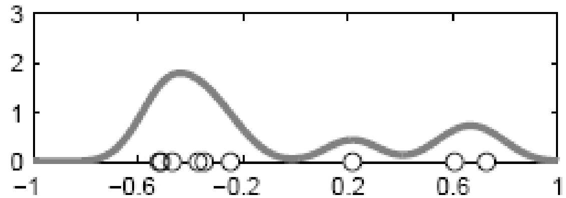
  - acceptance—rejection method



Monte Carlo approaches

- kernel density estimation

$$f_h(x) = \frac{1}{h\,n}\sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right)$$

Type	$K(y)$	Example
uniform	$\begin{cases} \frac{1}{2} & -1 \leqslant y \leqslant 1 \\ 0 & \text{otherwise} \end{cases}$	
triangular	$\begin{cases} 1 - \lvert y \rvert & -1 \leqslant y \leqslant 1 \\ 0 & \text{otherwise} \end{cases}$	
Epanechnikov	$\begin{cases} \frac{3}{4}(1 - y^2) & -1 \leqslant y \leqslant 1 \\ 0 & \text{otherwise} \end{cases}$	
Gaussian	$\frac{1}{\sqrt{2\pi}} e^{-y^2/2}$	

Monte Carlo approaches

- Markov chains
  - random walks
  - discrete Markov chains
  - Metropolis algorithm

```
1: initialize x_1, n, and s
2: for i = 1 : (n − 1) do
3:     while x_{i+1} not assigned do
4:         draw z ∈ [0,1] and u_i ∈ [−1,1]^d
5:         x_new = x_i + u_i s
6:         if f(x_new)/f(x_i) ≥ z then x_{i+1} = x_new
7:     end while
8: end for
```

Monte Carlo approaches

- specialities
  - discrete numbers
    - drawing from discrete set
    - roulette wheel selection
  - bootstrap
  - Quasi-Monte Carlo
  - stratified sampling
  - variance reduction
    - antithetic variables
    - importance sampling

literature

- background reading, additional material
  - J Gentle,
    Computational Statistics,
    Springer 2009.
  - M Gilli, D Maringer, E Schumann,
    Numerical Methods and Optimization in Finance,
    Academic Press 2011.
  - O Jones, R Maillardet, A Robinson,
    Scientific Programming and Simulation Using R,
    CRC Press 2009.
  - Kenneth Judd,
    Numerical Methods in Economics,
    MIT Press 1998.