A General Equilibrium Analysis of Inflation and Microfinance in Developing Countries
Corresponding author:

Daniel Müller, Dr. rer. pol.
Department of Economic Theory
University of Basel
Peter Merian-Weg 6
Postfach
CH-4002 Basel
Mail: dani.mueller@unibas.ch
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Daniel Mueller†

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Abstract

This paper analyses the welfare effects of microfinance and inflation in developing countries. Therefore, we introduce a moral hazard problem into a monetary search model with money and credit. We show how access to basic financial services affects households’ decisions to borrow, to save and to hold money balances. The group lending mechanism of the microfinance institution induces peer monitoring, which in turn enables entrepreneurship. Our main result is that there exists an inflation threshold beyond which entrepreneurship collapses. We show that inflation affects the impact of microfinance on social welfare in a nonlinear way. The positive effect of microfinance is largest for moderate rates of inflation and drops substantially for inflation rates above the threshold.

Keywords: Microfinance, Moral Hazard, Group Lending, Peer Monitoring and Monetary Policy.

JEL Classification: D82, E44, G21, O16.

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†University of Basel, Faculty of Business and Economics, Peter Merian-Weg 6, 4002 Basel, Switzerland. E-mail address: dani.mueller@unibas.ch.
1 Introduction

It is generally accepted that better access to finance reduces poverty. Firstly, credit allows poor households to start small businesses, invest in new production machines, buy livestock, or simply to consume. Access to basic financial services facilitates consumption smoothing, payments for their children’s education and wealth accumulation. Secondly, savings accounts pay interest and, thus, mitigate the negative effects of high inflation rates prevalent in developing countries. Finally, well functioning financial institutions have a positive effect on growth (see for instance Levine (2005) for a comprehensive literature survey on the relationship between finance and growth). However, in developing countries, the majority of households have no access to financial institutions. Empirical studies have shown that women, rural populations and, in particular, poor households are most concerned by this issue. In low income countries, 76 percent of adults have no account at a formal financial institution (Demirguc-Kunt and Klapper, 2012), while the access rate lies at 89 percent in high income countries.¹ Poor people have difficulty in general in gaining access to financial services. The World Bank reports that 77 percent of adults earning less than $2 a day are unbanked (World Bank, 2012). Comparing access to finance across countries and regions shows that large differences exist. To make things worse, inflation rates are on average much higher in developing countries than in industrialized countries.²

While governments in developing countries are well aware of the benefits of an efficient financial system, the question naturally arises as to why so little effort is made towards improving access to financial services. The reason is that basic banking services are complicated by a number of issues in developing countries. First, poor households have no valuable belongings, nor wealth that they could use as collateral for a loan. Second, transaction costs are especially high for small loans, and enforcement of repayments is difficult in countries with weak legal institutions. Third, asymmetric information between lenders and borrowers leads to principal-agent problems which may result in a dwindling of the already weak credit market. Microfinance—the provision of financial services on a small scale—has shown that there are ways to overcome these problems and that lending to the poor is not a one-way street. By adopting new approaches such as peer-monitoring schemes or the village banking model, microfinance institutes can overcome the asymmetric information problem. These lending mechanisms show high repayment rates without requesting any collateral (Armendáriz and Morduch, 2010).

¹The terminology formal financial institutions in Demirguc-Kunt and Klapper (2012) includes banks, credit unions, cooperatives, post offices, and microfinance institutions.
²Easterly and Fischer (2001) analyze the effects of inflation on the poor.
Microfinance started in the late 1970s and has expanded quickly over the last three decades. The original idea was to give credit to the poor. Over the last two decades, there has been a paradigm shift from highly subsidized microfinance institutions with limited outreach to a large-scale and financially sustainable microfinance industry (Robinson, 2001). Furthermore, since the early 2000s, many microfinance institutions have broadened their financial services and now also offer clients the possibility to open saving accounts (Matin et al., 2002). A leading example of a microfinance institute is the Grameen Bank in Bangladesh. The Grameen Bank and its charismatic founder Professor Muhammad Yunus were rewarded in 2006 with the Nobel peace price in appreciation of their achievements in poverty reduction and economic development in Bangladesh by providing the poor with access to finance. Today, over 2000 microfinance institutions exist all around the world and serve roughly one billion customers. They are mostly situated in developing countries, but are also to be found in high income countries.

The success of microfinance has not gone unnoticed and is considered today as an important tool for generating access to finance and reducing poverty. A large body of theoretical and applied literature exists on microfinance. Theoretical contributions have thoroughly analyzed the mechanism utilized in microfinance to reduce transaction costs and to mitigate the asymmetric information problem. Numerous field studies have investigated to what extent access to microfinance institutes increases the wealth of poor households. However, most empirical studies that have analyzed the impact of microfinance neglect the monetary policy dimension in developing countries which is often characterized by high inflation rates. Moreover, general equilibrium effects on prices, caused by the financial intermediation of microfinance institutions, are often neglected. In this paper, we intend to fill this gap by analyzing the effects of inflation on microfinance in a general equilibrium model. Using a model where money and credit are essential, allows us to derive the total welfare of an economy depending on the government’s respective monetary policy and the outreach and efficiency of its microfinance institutes. Monetary policy plays an important role, as it directly determines the inflation rate and indirectly determines the market rates of borrowing and lending. Therefore, we use a monetary

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3Stiglitz (1990) pioneered the work on group lending. Ghatak and Guinnane (1999) provide an extensive analysis of group lending extend the model to study four different agency problems and also discuss practical issues. Armendáriz (1999) analyses the problem of ex-post moral hazard.

4Two studies of particular interest are Kaboski and Townsend (2012) and Banerjee and Duflo (2010). The first study evaluates the impact of the Million Baht Village Fund program in Thailand, and the second study runs a random field experiment, conducted in collaboration with an Indian microfinance institution.

5An exception is the paper by Kaboski and Townsend (2011), they develop a structural model to evaluate the impact of large-scale microcredit policy interventions.
search model similar to Berentsen et al. (2007) to study the welfare effects of establishing a large-scale and sustainable microfinance institution in developing countries. To represent the agency problem between borrower and lender, we introduce a moral hazard problem in the style of Holmstrom and Tirole (1997). Moreover, we analyze the welfare implications for individual households.

We show that establishing sustainable microfinance institutions in developing countries allows poor households to increase their standards of living above the subsistence level. The reason is that former credit-constrained households are afterwards able to take out consumer loans or to invest in small businesses. Moreover, we show that the actual magnitude of the welfare impact of microfinance crucially depends on the prevailing monetary policy regime.

Our model discloses the relationship between the lending terms of microfinance and the monetary policy of the government. Higher money growth rates increase inflation and this in turn affects deposit and lending terms of the microfinance institution: On the one hand, depositors have to be compensated by a higher interest rate to encourage saving. The higher refinancing costs of the microfinance institution lead, in turn, to an increase in the lending rate. Moreover, higher inflation rates decrease real prices and output, which reduce the gains from trade (real balance effect). Entrepreneurs, who rely on external funding, are more affected by inflation than subsistence producers. Above a specific inflation threshold, entrepreneurship collapses and is displaced by subsistence production. Our numerical example shows that the positive impact of microfinance on social welfare is largest for moderate inflation rates, where entrepreneurship exists. However, for inflation rates above the threshold, the positive impact of microfinance drops substantially.

The structure of the article is as follows. Section 2 introduces the agents and describes the framework of the general equilibrium model with moral hazard and group lending. Section 3 presents the maximization problem of households in the two markets. In Section 4, the market outcome of the equilibrium and the optimal group lending contract are presented. In Section 5, we give a numerical example to present the impact and the welfare effect of microfinance in developing countries. Section 6 concludes.

2 The Basic Model

The model is based on Berentsen et al. (2007). It uses the standard Lagos-Wright structure, where time is discrete and every period consists of two subperiods. There exists
a continuum $[0,1]$ of infinitely living households and a single microfinance institution (MFI). In each period, households trade their produced goods at two sequentially opening markets. In subperiod A, households produce and trade the production good, and in subperiod B, the general good. Both goods are perishable and cannot be stored. We assume that the two markets are competitive and that no trading frictions exist. We will proceed by illustrating the structure of the economy and the characteristics of households. Then, we introduce fiat money and show how households can deposit money balances, as well as take out consumption loans from the microfinance institution. Subsequently, we illustrate in Sections 2.1 how entrepreneurs start a business and address the moral hazard problem with external funding. Section 2.2 shows the social planner problem.

Figure 1 displays the timeline of the model for a representative period $t$. At the beginning of subperiod A, households are hit by a temporary preference and technology shock. With probability $1 - n$, the household is a buyer; with probability $n\theta$, he is an entrepreneur; and with probability $n(1 - \theta)$, he is a producer. Buyers can consume but cannot produce in the first market. In contrast, producers can produce but cannot consume in the first market. Producers may either produce in home production or work for entrepreneurs. Finally, entrepreneurs have the possibility to start a small enterprise with one employee. In contrast to producers, entrepreneurs cannot produce in home production. We assume that the production technology of the enterprise is superior to the home production technology. From now on, we will use the term subsistence production for the inferior home production technology. Buyers, entrepreneurs and subsistence producers trade in the first market, and subsequently the market closes. In subperiod B, all households can consume, produce and trade the general good in the second market.
Households have quasi-linear preferences, where $q_b (q_s)$ is the amount of the production good consumed (produced) in subperiod $A$.$^6$ In subperiod $B$, $x (h)$ is the amount of the general good consumed (produced), and all households have the same productivity. Equation (1) displays the utility function of a household. To account for the preference shock, the utility function of households is modeled with an indicator function. When a household is hit by the preference shock, then the indicator is one, otherwise it is zero. The utility function of a representative household is

$$U(q_b, q_s, x, h) = \mathbb{1} u(q_b) - (1-\mathbb{1})c(q_s) + U(x) - h,$$

where $\mathbb{1}$ is the indicator, and $c(q_s)$ are the utility costs of producing the amount $q_s$ of good $q$ in subperiod $A$. The cost function $c(\cdot)$ is a convex function with respect to $q$, where $c(0) = 0$, $c'(\cdot) > 0$ and $c''(\cdot) > 0$. The utility function $u(\cdot)$ is a concave function with respect to $q$, where $u(0) = 0$, $u'(0) = \infty$, and $u''(\cdot) < 0$. Good $x$ can be consumed and produced in home production in subperiod $B$ by every household, where $h$ are the utility costs. To ease the calculation, we assume that the production of the general good is linear.$^7$ The utility function $U(x)$ of the general good is a convex function.

We assume that households trade in anonymous goods markets. Households are not able to recognize former trading partners in future meetings. Hence, a role for a medium of exchange emerges. As the medium of exchange, we introduce fiat money. Money is essential in the markets of the production good and the general good, because there is no commitment and no record-keeping in the two markets. Access to financial services is solely feasible over the MFI. Households deposit money balances at the end of subperiod $B$ and receive interest in the next subperiod $B$, if they do not withdraw their balances. Buyers use their deposits and can additionally take out small loans for consumption in subperiod $A$.$^8$ Entrepreneurs can issue risky debt with a special group lending contract. We will describe the group lending contract below when we show how entrepreneurs start businesses.

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$^6$The subscript $b$ stands for buyer and the subscript $s$ for subsistence producers. The production of an enterprise will be denoted by subscript $e$.

$^7$This is the standard assumption in Lagos-Wright models, that makes the model tractable. Actually, we could also assume that the cost function is nonlinear and that $U(x)$ is linear to find a solution. For further discussions, see e.g., Lagos and Wright (2005).

$^8$For the saving accounts of the Grameen Bank, it was initially only possible to withdraw savings at an assigned time. In 2004, Grameen allowed customers to withdraw money at will. Thus, today, saving accounts are utilized like current accounts. This has led to large increases of Grameen’s deposit portfolio. Since the end of 2004, the deposits of the Grameen Bank exceed their outstanding loans (Rutherford, 2006).
The central bank directly influences the amount of fiat currency by means of lump-sum transfers at the beginning of subperiod B. We assume that money grows at a specific but constant rate. The stock of money is indicated by $M$, and the money growth rate is $\gamma$, where $\gamma \geq 1$ ($M = \gamma M_{-1}$). Variables referring to the previous (subsequent) period are indexed by $-1 (+1)$. Households receive lump-sum transfers of $\tau M_{-1}$ from the central bank. To meet the targeted growth rate, it has to be the case that $\tau = \gamma - 1$. The real price of money in subperiod B is indicated by $\phi$. The assumption of a constant money growth rate implies that real money balances are time-invariant. Therefore, it is the case that $\phi/\phi_{t+1} = M_{t+1}/M = \gamma$. This is the standard way to model monetary policy in the Lagos-Wright framework.

### 2.1 Starting a Business and External Funding

Entrepreneurs have a business idea and start a small business with one employee. Every entrepreneur is matched with one producer and makes a take-it-or-leave-it wage offer that the producer accepts or declines. If the producer declines the wage offer, he produces in home production at the subsistence level. We assume that entrepreneurs have to pay wages in advance of the production (cash-in-advance). This assumption can be motivated through a lack of commitment. Thus, households are only willing to work if they are compensated for their utility costs beforehand. The production technology of the enterprise is superior to the home production technology. We assume that the employee’s utility costs of producing the amount $q_e$ are $K + \tilde{c}(q_e)$. The cost function consists of a fixed setup cost term and a variable cost term. The variable cost term $\tilde{c}(\cdot)$ has the same features as $c(\cdot)$.

Entrepreneurship is subject to risk. Production is successful with a probability $\{\mu_h, \mu_l\} < 1$, depending on the behavior of the entrepreneur. If he shirks, the production is successful with the lower probability $\mu_l$, but the entrepreneur receives real, private benefit $B$. The difference between the two probabilities of success is denoted by $\Delta \mu$.\footnote{We assume that the expected profit of a shirking entrepreneur, who could finance the production internally, is smaller than zero. The expected profit consists of sales if the production is successful (with probability $\mu_l$) plus the private benefit minus the wage costs for the employee. Moreover, we assume that $\tilde{c}(q)/\mu_h < c(q)$, for all $q > 0$.} Agents can verify whether production was successful, but the behavior of entrepreneurs is private information and can only be revealed by monitoring. If the investment is financed externally, then the entrepreneur has an incentive to shirk. This is the standard moral hazard problem similar to (Holmstrom and Tirole, 1997). We assume that monitoring costs are proportional to the loan size and are denoted by $\delta_m$. For simplicity, we assume...
that monitoring entrepreneurs will always detect a shirking peer and that strong enough means exist to sanction shirking peers. Hence, if entrepreneurs monitor, shirking can be ruled out.

Let us shortly recapitulate the three obstacles that entrepreneurship faces: First, the already discussed cash-in-advance constraint for enterprises. Second, entrepreneurs are capital-constrained and need external funding. More precisely, their savings are insufficient for self-financing the business and they have no collateral. Finally, asymmetric information leads to a moral hazard problem. The MFI solves the agency problem through group-lending contracts with joint liability and enables entrepreneurship.

**Group Lending Contract**

We suppose that commercial loans are only available as group-lending contracts. After the shock has been revealed, all entrepreneurs meet with the local branch of the MFI to contract for loans. They are divided into small groups of two. A representative group consists of entrepreneurs $i$ and $j$. We assume that they are protected by limited liability. Thus, the MFI can only claim the returns of the project. Moreover, we assume joint liability, which means that entrepreneurs have to take responsibility for the repayment if the peer defaults. In our group lending contract, this means that borrower $i$ pays interest rate $i_s$ if borrower $j$ repays his loan, and $i_f$ if the peer defaults (The subscript $s$ stands for success and $f$ for failure). Typically, in this kind of contract the interest rate $i_f$ is greater than $i_s$.

### 2.2 The Social Planner Allocation

The mission of the planner is to choose the quantities buyers consume and producers produce of the production good and the general good. We assume that the planner sees whether entrepreneurs behave or shirk and is able to force them to behave. The social planner maximizes the aggregated lifetime utility of the households subject to the feasibility constraints. It is obvious that the planner decides that all entrepreneurs start a business, since the returns are higher than under subsistence production. The

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10 MFI s offer larger loans that households can use to finance a marriage, a funeral and, especially, to start small enterprises. These loans can only be used for the stated purpose and the requirements are higher. For example, the Grameen Bank offers different loan contracts, ranging from housing loans to special investment loans intended for entrepreneurs (Rutherford et al., 2004).

11 For instance, the Grameen Bank lends money to groups of 5 (called kendras).

12 In practice, institutions not only use group lending mechanisms with joint liability. Armendáriz and Morduch (2000) describe other mechanisms such as regular repayment schedules or non-refinancing threats.
The optimization problem of the social planner is
\[
W = \frac{1}{1-\beta} \{(1-n)u(q_b) - n\theta K - n\theta \tilde{c}(q_e) - n(1-2\theta)c(q_s) + U(x) - h\}, \tag{2}
\]
where welfare consists of net utility of subperiod A plus net utility of subperiod B. The planner chooses the quantities produced and consumed in the two subperiods. Equation (3) displays the optimal amount consumed \((q^*_b, q^*_e, q^*_s)\) in subperiod A.
\[
u'(q^*_b) = \tilde{c}'(q^*_e) = c'(q^*_s). \tag{3}
\]
In subperiod B, every household produces and consumes the amount \(x^*\) of the general good such that \(U'(x^*) = 1\). In the optimal allocation, aggregate production has to equal aggregate demand for both goods \((q, x)\). Furthermore, the social planner forces entrepreneurs to behave. Thus, the production is successful with probability \(\mu_h\). The production market in subperiod A clears if Equation (4) holds.
\[
(1-n)q^*_b = n\theta \mu_h q^*_e + n(1-2\theta)q^*_s. \tag{4}
\]

## 3 The Goods Markets

The expected lifetime utility of a household can be specified in a recursive way by value functions. In particular, \(V(\cdot)\) denotes the value function at the beginning of subperiod A, and \(W(\cdot)\) denotes the value function at the beginning of subperiod B. The ex-ante value function \(V(d)\) of a representative household at the beginning of period \(t\) with deposits \(d\) is given by
\[
V(d) = (1-n)\left\{u(q_b) + W(0, l_b, d + l_b - pq_b)\right\} + n\theta \left\{-\phi \delta m_l_j + E\{W\}\right\}
+ n\theta \left\{-K - \tilde{c}(q_e) + W(d, 0, w)\right\} + n(1-2\theta)\left\{-c(q_s) + W(d, 0, pq_s)\right\}. \tag{5}
\]

To fully understand the value function \(V(d)\), we give a short description of the four states. First, with probability \(1-n\), the household is a buyer. The term in the curly brackets indicates the utility of a buyer consuming the amount \(q_b\) of the production good plus the continuation value of a buyer in subperiod B. Second, with probability \(n\theta\), the household is an entrepreneur. The term in curly brackets indicates the monitoring costs (which depend on the peer’s loan size) plus the ex-ante expected continuation value of
an entrepreneur. The continuation value depends on the outcome of his own production and the peer’s production. Third, with probability \( n\theta \), the household is an employee. The term in curly brackets indicates the disutility of producing the amount \( q_e \) plus the continuation value of an employee in subperiod B. Finally, with probability \( n(1-2\theta) \), the household is a subsistence producer. The term in curly brackets indicates the disutility of producing the amount \( q_s \) plus the value function of a producer with deposits \( d \) and income \( pq_s \) in subperiod B.

Equation (6) shows the expected value of entering subperiod B of entrepreneur \( i \), given that \( i \) and \( j \) behave. The expected continuation value \( W(\cdot) \) is the weighted sum of three possible outcomes: In the first outcome, the production is a failure. In this case, entrepreneur \( i \) defaults and enters subperiod B with no income. In the second outcome, the production of entrepreneur \( i \) is successful and at the same time entrepreneur \( j \) repays his loan. In this case, entrepreneur \( i \) only pays for his own obligations and makes a large profit. In the third outcome, entrepreneur \( i \) is successful, but entrepreneur \( j \) defaults. In this case, entrepreneur \( i \) not only has to come up for his own obligation, he has also to repay part of borrower \( j \)’s loan. Entrepreneur \( i \) enters subperiod B with a lower net profit than in the second case, which is indicated by \( \Pi_l < \Pi_h \). The business of entrepreneur \( i \) \((j)\) is successful with probability \( \mu_{h,i} \) \((\mu_{h,j})\).

\[
E\{W\} = \mu_{h,i}\mu_{h,j}W(0, l^p, \Pi_h) + \mu_{h,i}(1 - \mu_{h,j})W(0, l^p, \Pi_l) + (1 - \mu_{h,i})W(0, 0, 0). \tag{6}
\]

To find the equilibrium, we start with the equilibrium conditions of the general goods market and solve backwards to find the equilibrium in the production market.

### 3.1 The General Goods Market (Subperiod B)

In subperiod B, households consume and produce the general good at home. The amount consumed is denoted by \( x \), and \( h \) denotes the produced amount. Households discount time with \( \beta \in (0, 1) \). Households enter subperiod B with heterogeneous portfolios of deposits \( (d) \), loans \( (l) \) and cash \( (m) \) and maximize the value function with respect to \( x \), \( h \) and \( d_{+1} \). The maximization problem of a household entering subperiod B with the portfolio \((d, l, m)\) is

\[
W(d, l, m) = \max_{x, h, d_{+1}} \{U(x) - h + \beta V_{+1}(d_{+1})\} \tag{7}
\]

\[
s.t. \quad x - h = \phi[(1 + i_d)d + m + \tau M_{-1} - d_{+1} - (1 + i)l],
\]
where \( d_{t+1} \) are deposits households place on the MFI for the subsequent period. The interest rate \( i \) on loans depends on whether the household was a buyer \((i_d)\) or a successful entrepreneur \((i_s, i_f)\) in subperiod A. All values are stated in real terms. Households have to choose \( x, h \) and \( d_{t+1} \), thereby satisfying the intertemporal budget constraint. The left-hand side of the budget constraint is consumption \( x \) less the amount \( h \) produced of the general good. The right-hand side consists of deposits of period \( t \) charged with interest, the lump-sum transfer of the central bank less the deposits for period \( t + 1 \), and loans charged with the respective interest. Substituting the budget constraint for \( h \) gives

\[
W(d, l, m) = \max_{x, d_{t+1}} \{ U(x) - x + \beta V_{t+1}(d_{t+1}) \\
+ \phi[(1 + i_d)h + m + \tau M_{t-1} - d_{t+1} - (1 + i)l] \}.
\]

The optimal quantities of good \( x \) and deposits \( d_{t+1} \) for the subsequent period follow from the first order conditions:

\[
U'(x) = 1, \tag{8}
\]

\[
\beta V'_{t+1}(d_{t+1}) = \phi. \tag{9}
\]

The marginal value of deposits has to be equal to \( \phi/\beta \) in equilibrium. The envelope condition for private saving is: \( W_d = \phi(1 + i_d) \). The envelope condition of borrowing money for consumption is: \( W_l = -\phi(1 + i_d) \). And lastly, the envelope condition of holding money is: \( W_m = \phi \). All households will enter the next period with the same amount of deposits. This implies that at the beginning of the subsequent period, the money holdings are degenerate, and the liability side of the MFI’s balance sheet is equal to the aggregate of all households’ deposits \( d_{t+1} \). The general goods market serves to simplify calculations, since we do not have to keep track of the history of households’ deposits.

### 3.2 The Production Goods Market (Subperiod A)

At the beginning of subperiod A, the preference shock determines whether households are buyers, producers or entrepreneurs. In the following, we will present the optimization problem for each group.
Buyers

Buyers choose how much to demand of good $q$, taking prices as given. For their expenses, they use their deposits and in addition have the possibility to take out consumption loans ($l_b$) from the MFI. The optimization problem of a representative buyer is:

$$\begin{align*}
\max_{q_b, l_b} & \{u(q_b) + W(0, l_b, d + l_b - pq_b)\}, \\
\text{s.t.} & pq_b \leq d + l_b, \\
& l_b \leq \bar{l},
\end{align*}$$

where the budget constraint states that households can dispense up to the sum of deposits $d$ and the loan $l_b$. The loan constraint states that the buyer can borrow up to the limit $\bar{l}$.

We will assume that buyers have no possibility to default and that the MFI can reclaim consumption loans without costs; thus, the (LC) is not binding. If this is the case, buyers optimally choose $q_b$ such that the following equation is satisfied:

$$u'(q_b) = \phi p (1 + i_d).$$

For the detailed derivation with the first-order conditions, see Appendix A.1.

Producers

Subsistence producers choose the amount $q_s$ that maximizes profit, thereby taking as given the price $p$. Producers incur utility costs $c(q_s)$ for producing the amount $q_s$. The optimization problem of a representative subsistence producer is:

$$\begin{align*}
\max_{q_s} & \{-c(q_s) + W(d, 0, pq_s)\},
\end{align*}$$

Because real balances enter the value function of submarket B in a linear fashion, the optimization problem of subsistence producers can be stated as

$$\phi \Pi_s = \max_{q_s} \{\phi pq_s - c(q_s)\},$$

where real profit $\phi \Pi_s$ depends on the produced amount $q_s$ and the real price $\phi p$. Assuming a convex cost function gives the standard solution where producers set their marginal costs equal to the real price. Equation (14) displays the first order condition that maximizes the profit of a household at the subsistence level for a given real price.

$$c'(q_s) = \phi p.$$
Entrepreneurs
Entrepreneurs take out loans from the MFI and start small enterprises that produce with superior technology. To mitigate the agency problem between lender and borrower, the MFI offers group lending contracts with joint liability. In the following, we will present the optimization problem of a representative group with entrepreneurs $i$ and $j$ that takes as given the lending mechanism of the MFI. We assume that both entrepreneurs monitor each other and show afterwards how the MFI’s group lending contract has to be designed to be incentive compatible. The optimization problem of entrepreneur $i$ when he monitors $j$ is:

$$\max_{l_i,w,q_e} E(\Pi) = \mu_{h,i} [pq_e - \mu_{h,j} (1 + i_s)l_i - (1 - \mu_{h,j})(1 + i_f)l_i] - d - \delta ml_j, \quad (15)$$

$$\text{s.t.} \quad d + l_i \geq w, \quad (FC)$$

$$w \geq K/\phi + \tilde{c}(q_e)/\phi + \Pi_s. \quad (PC)$$

With probability $\mu_{h,i}$, the project is successful and $i$ sells the amount $q_e$ at the production market. The actual interest rate that entrepreneur $i$ has to pay depends on whether $j$ repays his loan or not. With probability $\mu_{h,j}$, entrepreneur $j$ repays his loan and $i$ has to pay interest rate $i_s$. With probability $1 - \mu_{h,j}$, entrepreneur $j$ defaults and $i$ has to pay interest rate $i_f$. The last term indicates the monitoring costs. Entrepreneur $i$ has to satisfy two constraints: First, the sum of deposits and the loan has to be greater or equal to the wage. Second, the wage has to be greater or equal to the disutility costs of producing $q_e$ plus the outside option of the employee. The outside option is production at the subsistence level, which achieves a profit of $\Pi_s$. See Section A.3 for the optimization problem of a subsistence producer. The participation constraint is satisfied if the wage is greater than the disutility of producing $q_e$ plus the foregone profit $\Pi_s$. Finally, the entrepreneur will only start the business if the expected profit is greater than his own outside option–principal and interest on deposits. The Lagrangian of the profit maximization problem with the two Lagrange multiplier $\lambda_l$ and $\lambda_w$ is

$$L(q_e, l_i, w) = \mu_{h,i} pq_e - \mu_{h,i} \mu_{h,j} (1 + i_s)l_i - \mu_{h,i} (1 - \mu_{h,j})(1 + i_f)l_i - d - \delta ml_j$$

$$- \lambda_l [w - d - l_i] - \lambda_w [\frac{K}{\phi} + \frac{\tilde{c}(q_e)}{\phi} + \Pi_s - w].$$
The first-order conditions are:

\[ q_e : \quad \phi \mu_{h,i} p = \lambda_w \tilde{c}'(q_e), \]
\[ l_i : \quad \lambda_l = \mu_{h,i} \left[ \mu_{h,j} (1 + i_s) + (1 - \mu_{h,j})(1 + i_f) \right], \]
\[ w : \quad \lambda_l = \lambda_w, \]
\[ \lambda_l : \quad l_i = w - d, \]
\[ \lambda_w : \quad w = \frac{K}{\phi} + \tilde{c}'(q_e)/\phi + \Pi_s. \]  

(16)

Combining the second and the third FOC gives \( \lambda_w = \mu_{h,i} \left[ \mu_{h,j} (1 + i_s) - (1 - \mu_{h,j})(1 + i_f) \right] \).

The fourth and the fifth FOC are the standard loan and wage constraints which have to hold with equality if the interest rate is greater than zero and the entrepreneur maximizes his profit. Substituting the Lagrange multiplier \( \lambda_w \) in the first FOC and canceling \( \mu_{h,i} \) on both sides gives the following condition which has to hold if entrepreneurs maximize profit:

\[ \left[ \mu_{h,j} (1 + i_s) + (1 - \mu_{h,j})(1 + i_f) \right] \tilde{c}'(q_e) = (1 + \tilde{i}) \tilde{c}'(q_e) = \phi p. \]  

(17)

The term in the squared brackets is the expected interest rate an entrepreneur has to pay if production is successful. It depends on the two interest rates and also on the behavior of entrepreneur \( j \).

4 Equilibrium

In this section, we assume that the microfinance institution maximizes social benefit and is not profit-oriented. But in contrast to a social planner, the institution cannot force households to behave. Moreover, outstanding credits have to be fully backed by deposits (no external sourcing). Furthermore, financial operations have to be sustainable, as we suppose that the MFI receives no subsidies. Therefore, the MFI offers group lending contracts with expected returns that are equal to the deposit rate. Even though the returns from specific groups are stochastic, aggregated returns of the MFI are fully predictable. The reason is that production failures are uncorrelated, and the law of large numbers applies.

The Optimal Group Lending Contract

The MFI has to design the group lending contract with joint liability such that entrepreneurs have incentives to monitor their peers.\(^{13}\) Remember that an entrepreneur

---

\(^{13}\) In reality, collusion between entrepreneurs can be a serious threat for the success of group lending. However, the consideration of collusion is beyond the scope of our analysis and we therefore assume that
will behave if the peer monitors, since a detected entrepreneur would be punished by severe social sanctions. Hence, monitoring induces good behavior, and production of the peer is successful with probability $\mu_h$. Assume that entrepreneur $i$ is monitored by the peer. It is optimal for entrepreneur $i$ to monitor if

$$
\mu_{h,i}[pq_e - \mu_{h,j}(1 + i_s)l_i - (1 - \mu_{h,j})(1 + i_f)l_i] - d - \delta_ml_j \geq \\
\mu_{h,i}[pq_e - \mu_{l,j}(1 + i_s)l_i - (1 - \mu_{l,j})(1 + i_f)l_i] - d.
$$

In a symmetric equilibrium $l_i = l_j = l$. Then, it follows that the incentive constraint to monitor is satisfied if

$$
\mu_h \Delta \mu (i_f - i_s) \geq \delta_m. \tag{18}
$$

**Zero Profit Condition of the MFI**

The MFI has to pay interest rate $i_d$ on deposits. Thus, the expected return from the group lending contract has to be large enough. To break even, the two interest rates $i_s$ and $i_f$ have to satisfy the following condition:

$$
2\mu_h^2(1 + i_s) + 2\mu_h(1 - \mu_h)(1 + i_f) = 2(1 + i_d), \tag{19}
$$

where the first term of the left-hand side is gross repayment of the group if both households are successful (which occurs with probability $\mu_h^2$), and the second term is gross repayment if only one household is successful (which occurs with probability $2\mu_h(1 - \mu_h)$). The right-hand side are the gross deposit costs the MFI has to pay. Entrepreneurs’ loans and deposits have been normalized. To derive the interest rate $i_s$, we assume that Equation (18) is satisfied with equality and substitute for $i_f$. We obtain

$$
\mu_h^2(1 + i_s) + \mu_h(1 - \mu_h)(1 + i_s + \frac{\delta_m \mu_h}{\mu_h \Delta \mu}) = 1 + i_d. \tag{20}
$$

**Business Funding**

Entrepreneurship exists if two constraints are satisfied: On the one hand, the MFI has to satisfy the incentive constraints of the entrepreneurs, and it has to respect the limited liability clause. The MFI will only give credit if profits are greater than the repayment obligation in the event that the peer defaults.

$$
pq_e^* - (1 + i_f)[K/\phi + \hat{c}(q^*_e)/\phi + \Pi_s - d] \geq 0. \tag{21}
$$

entrepreneurs do not collude. See e.g., Laffont and Rey (2003) on collusion and group lending.
On the other hand, entrepreneurs will only start the business if expected profits are greater than the principal and interest on deposits. Entrepreneurs only take out loans if the following condition is satisfied:

\[
\mu_h[pq_e^* - (1 + \tilde{i})(K/\phi + \tilde{c}(qv_e^*)/\phi + \Pi_s - d)] - \delta_m[K/\phi + \tilde{c}(qv_e^*)/\phi + \Pi_s - d] \geq (1 - i_d)d. \tag{22}
\]

Whether the former or the latter constraint is more restrictive depends on the parameterization of the monitoring costs and the difference between \(i_f\) and \(\tilde{i}\). Usually, Equation (22) is more restrictive. Furthermore, above the threshold exists a small range, where only part of the entrepreneurs are active. The rate of active entrepreneurs is determined through an indifference condition: Entrepreneurs enter production up to a rate where the expected profit is equal to the outside option.

**Equilibrium of the Financial and the Real Market**

In the equilibrium of the production market, supply has to equal demand. The demand side is the fraction of households hit by the preference shock \(1 - n\). The supply side consists of the aggregate output of successful entrepreneurs and of subsistence producers. The market clearing condition of the production market, assuming no search frictions, is

\[
n(1 - 2\theta)q_s + n\theta\mu_hq_e = (1 - n)q_b. \tag{23}
\]

Combining the FOCs of entrepreneurs and subsistence producers with the FOC of buyers yield the relationship between the equilibrium quantities produced and consumed and the interest rates.

\[
\frac{u'(q_b)}{c'(q_s)} = (1 + i_d), \tag{24}
\]

\[
\frac{u'(q_b)}{\tilde{c}'(q_e)} = (1 + \tilde{i})(1 + i_d), \tag{25}
\]

where \(\tilde{i} \equiv \mu_hi_s + (1 - \mu_h)i_f\).

Rate \(\tilde{i}\) is the expected interest rate a successful entrepreneur has to pay, given that the peer behaves. The interest rates on deposits and on loans drive a wedge between the marginal utility of consumption and the marginal cost of production. The higher the interest rates are, the further away is the economy from the first best allocation where the ratio of the marginal utility of consumption and the marginal cost of production are equal.
At the beginning of the section, we ruled out external sourcing possibilities for the MFI. More specifically, we assume that all loans have to be fully covered by deposits and that the MFI is not dependent on subsidies. Hence, aggregated deposits have to be greater or equal to demanded loans. In the optimal allocation, the two measures are equal.

\[ n(1 - \theta)d = n\theta l + (1 - n)b. \]  

(26)

Marginal Value of Deposits
To obtain the marginal value of deposits, we first take the derivative of Equation (5).

\[
\frac{\partial V(d)}{\partial d} = (1 - n)\frac{u'(q_b)}{p} + n(1 - \theta)(1 + i)\phi + n\theta \frac{\partial E\{W\}}{\partial d}.
\]  

(27)

If the household is a buyer, he receives marginal utility of \( u'(q_b)/p \). If he is a producer, he receives principal and interest in subperiod B and can consume the general good. If, instead, he is an entrepreneur, Lemma 1 below reveals that the marginal value of deposits is the same as for the producer.

**Lemma 1.** For \( l > 0 \), the marginal value of holding deposits for an entrepreneur is equal to \((1 + i_d)\phi\).

**Proof.** To verify Lemma 1, note that if entrepreneurs are not credit-constrained, then the equilibrium wage of the employee is independent of \( d \). This implies that, when the entrepreneur increases his deposits, he is able to decrease the loan by the same amount. This implies that the marginal value of deposits is equal to the negative value of the expected marginal value of loans of an entrepreneur.

The expected marginal value of loans depends on the outcome of production and the respective interest rate that the entrepreneur has to pay. There are three outcomes: In the first outcome, the entrepreneur defaults and pays zero. In the second outcome, both entrepreneurs are successful, and the gross interest rate is \( 1 + i_s \). Finally, the peer entrepreneur defaults, and the gross interest rate is \( 1 + i_f \). Thus, the negative value of the ex-ante expected marginal value of loans is: \( \phi\mu_h(1 + i) \). Using Equation (19) gives: \( \phi\mu_h(1 + i) = \phi(1 + i_d) \).

In the next step, we substitute \( p \) by using the equilibrium condition of entrepreneurs (Equation (17)). Finally, we replace the left-hand side of Equation (27) by using the lagged intertemporal optimality condition (Equation (9)).\(^\text{14}\) This gives the equilibrium

\(^{14}\text{We use the long-term relation of the growth rate of money and the real value of money: } \phi = \phi_{-1}\gamma.\)
relationship between the marginal utility of consumption and the marginal cost of industrial production conditional on the growth rate of money and the interest rate.

\[
\gamma - \beta \beta = (1 - n) \left[ \frac{\mu h(q_b)}{(1 + \bar{i})c'(q_e)} - 1 \right] + ni_d. \tag{28}
\]

Using the equilibrium condition of industrial production (Equation (25)) shows that one plus the deposit rate is equal to \(\gamma/\beta\).

\[
\frac{\gamma}{\beta} = (1 + r)\gamma = 1 + i_d. \tag{29}
\]

**Real Value of Deposits**

The real value of deposits \(\phi_d\) can be derived by using the clearing condition of the MFI (Equation (26)). Recall from the optimization problem of entrepreneurs that \(l = K/\phi + \bar{c}(q_e)/\phi + \Pi_s - d\), and from the budget constraint of buyers that \(l_b = pq_b - d\). Using, once again, the optimality condition of the entrepreneur to replace \(p\) and the market clearing condition of the production market gives the real value of deposits, where the superscript \(\ast\) denotes equilibrium values:

\[
\phi_d = (1 - n)(1 + \bar{i})\bar{c}'(q_e^\ast)q_b^\ast + n\theta K + n\theta\bar{c}(q_e^\ast) + n\theta[(1 + \bar{i})\bar{c}'(q_e^\ast)q_s^\ast - c(q_s^\ast)]. \tag{30}
\]

Equation (30) shows that the real value of deposits depends on the expected interest rate that producing households have to pay in order to receive a loan. The real price of money \(\phi\) can be determined by substituting \(d\) by the money stock \(M_{t-1}\). All these conditions have to be satisfied in the symmetric, stationary equilibrium.

**Definition 1 (Equilibrium).** The symmetric, stationary equilibrium \(\{q_b, q_e, q_s, x, p, \phi, i_d\}\) satisfies the equilibrium equations (20), (21), (23), (24), (25), (26), (29) and (30). The interest rate \(i_d\), the real price of money \(\phi\), and the price \(p\) result from the monetary policy of the central bank, which decides over the parameter values \(\{M, \gamma\}\).

## 5 Discussion

In this section, we analyze the economic equilibrium of our model. We suppose that the microfinance institution receives no subsidies and is in our case fully independent of commercial banks or the financial market. It provides basic financial services to the households. In Section 5.1, we offer a numerical example to present the equilibrium outcome of our model and give the intuition to our results. In Section 5.2, we compute
the welfare costs for different money growth rates. Moreover, we compute the welfare
costs of having no access to financial services for given monetary policies.

5.1 Numerical Example

We suppose that a household is with probability 0.6 a buyer, with probability 0.3 a pro-
ducer, and with probability 0.1 an entrepreneur with a business idea. For our numerical
example, we use the same utility and disutility functions as in Lagos and Wright (2005).
We assume that the disutility function of the employee is identical to the function of
producers, but scaled down by the factor \( a < 1 \). Equation (31) displays the utility func-
tions and the disutility functions of the households. The disutility function in market 2
is linear for all households.

\[
\begin{align*}
  u(q) &= A \frac{(q + b)^{1-\eta} - b^{1-\eta}}{1 - \eta}, \\
  U(x) &= D \log(x), \\
  c(q) &= \varphi, \\
  \tilde{c}(q) &= a\varphi, \\
\end{align*}
\]

(31)

where \( D = 3 \) and \( \varphi = 2.5 \). For the cost function of entrepreneurs, we assume that
\( a = 0.25 \) and the setup costs are \( K = 0.2 \). The success probability is \( \mu_h = 0.9 \) if the
entrepreneur behaves, while it is \( \mu_h = 0.3 \) and he receives a private benefit of \( B = 0.3 \)
if the entrepreneur shirks. Monitoring costs of entrepreneurs are \( \delta_m \), with \( \delta_m = 0.05 \).
The remaining parameter values are \( A = 1, b \approx 0 \) and \( \eta = 0.3 \).\(^{15}\)

Figure 2 displays sales and expenses of a representative entrepreneur as a function of
the deposit rate \( i_d \). The dark blue line indicates the sales of the enterprise. Sales are
decreasing with the deposit rate. The other three lines display monitoring costs, labor
costs and interest costs. The cost components are added up in the figure. Thus, the blue
line marks the aggregated expenses of the enterprise. Increasing the inflation rate leads
to a rise of the equilibrium deposit rate. Higher inflation rates, therefore, increase the
funding costs of entrepreneurs. Funding costs are steadily increasing with the deposit
rate. Monitoring costs are low and increase only marginally with the deposit rate. Labor
costs account for the largest part of the expenses. Wages are slightly decreasing, since
real output declines if the inflation rate increases. For deposit rates above 20 percent, the
expected profits of entrepreneurs are smaller than their outside option–leaving deposits
at the MFI and instead receiving interest in subperiod B. Thus, above this threshold
rate, entrepreneurship collapses and entrepreneurs will be inactive.

\(^{15}\)Table 2 in the Appendix reports all underlying parameter values for the numerical example.
The funding costs for entrepreneurs are relatively small if the rate of inflation is low. In such an environment, enterprises can resort to their superior technology. Enterprises’ profits are greater than their outside option. However, if the inflation rate increases, subsistence production becomes more and more advantageous, for the reason that home production involves no external funding. Ultimately, very high inflation rates lead to a collapse of entrepreneurship.

Figure 3 illustrates how the threshold value of entrepreneurs’ production depends on the cost advantage and the inflation rate. In the area below the line, only subsistence production exists. For the case that the technology of enterprises is only slightly superior
to the subsistence technology, the threshold value is very low. The more advanced the production technology is, the larger is the array where entrepreneurship is profitable.

A further question of interest is how monetary policy affects the borrowing behavior of poor households. Figure 4 compares the credit volume of a consuming household to the credit volume of an entrepreneur as functions of the deposit rate. The commercial credit is nearly constant up to the threshold value, beyond which it collapses to zero. If the deposit rate is exactly 20 percent, only part of the entrepreneurs are active. At this rate entrepreneurs are indifferent between starting a business and their outside option. Contrary to commercial credits, the volume of the consumer credit increases regularly with the deposit rate. Above the threshold, the consumer credit jumps to a higher volume. If the deposit rate is 20 percent, the credit volume lies between the two values.

![Figure 4: Consumer credit and commercial credit](image)

We will show in Section 5.2 that the impact on social welfare is largest if households take out commercial and consumer loans simultaneously. The discovered link between monetary policy and the credit composition (commercial versus consumer credits) in our model is also important for another reason: The empirical finance literature finds a positive relationship between enterprise credits and economic growth, whereas the relationship between consumer credits and growth is insignificant (Beck et al., 2012).
5.2 Welfare Implications

Welfare of households depends on the amount of production and consumption. The equilibrium outcome is affected by the inflation rate, which in turn is induced by the central bank through the money growth rate. The aggregated steady state lifetime utility of households is

\[ W = \frac{1}{1-\beta} \{ (1-n)u(q_b) - n\theta K - n\theta\bar{c}(q_e) - n\theta\phi\delta_m l - n(1-2\theta)c(q_s) + U(x) - x \}. \quad (32) \]

The expected utility of one household at the beginning of the period consists of the expected net utility of the two subperiods. In subperiod A, it is the probability of being a consuming household times the utility of consumption minus the probability of being an entrepreneur or a producer times the disutility of production. In subperiod B, it is the utility of consumption minus the production of the general good, since all households consume the general good.\(^{16}\)

To derive the welfare costs of inflation, we apply the standard approach used in search-theoretic monetary models (see e.g., Lagos and Wright (2005) or Craig and Rocheteau (2008)). We ask what fraction of consumption would households be willing to give up in order to change inflation from \(\pi_0\) to \(\pi_1\). We denote the fraction of consumption by \(\Delta\). To obtain the welfare costs of increasing inflation from \(\pi_0\) to \(\pi_1\), we compute the \(\Delta\), which sets \(W_\Delta(\pi_0) = W(\pi_1)\). The aggregated steady state lifetime utility of a household that decreases its overall consumption by the fraction \(\Delta\) is:

\[ W_\Delta = \frac{1}{1-\beta} \{ (1-n)u((1-\Delta)q_b) - n\theta(K + \bar{c}(q_e) + \phi\delta_m l) - n(1-2\theta)c(q_s) + U((1-\Delta)x) - x \}. \quad (33) \]

Table 1 compares the real price, consumption and production quantities in equilibrium for inflation rates of -5%, 10%, 14%, 20% and 30%. Entrepreneurship collapses at an inflation rate of 14 percent—which is the inflation threshold in our numerical example. For the two cases with higher inflation rates, the output of enterprises is zero. In contrast, the output of subsistence producers decreases only slightly with higher inflation rates.

\(^{16}\)The formula for aggregated steady state welfare is similar to the optimization problem of the social planner. However, in contrast to Section 2.1, the quantities produced and consumed are now chosen by the households.
In the lower part of the table, we display the welfare costs of inflation. The first row depicts the costs of inflation, given that households have access to the microfinance institution, in comparison to the Friedman rule (our benchmark). For low rates, the welfare costs of inflation are small. However, welfare costs are much higher if inflation rates are high. The reason is that if inflation lies above the threshold value, entrepreneurship collapses. Hence, at moderate inflation rates welfare costs are negligible, but, above a certain threshold, welfare costs increase substantially.

Another important result of our model is that monetary policy affects the impact of microfinance. The last row of the table depicts the costs of inflation given that households have no access to the microfinance institution. The comparison of welfare costs with and without a microfinance institution discloses that establishing microfinance institutions generally increases social welfare. However, the magnitude of the positive impact depends crucially on the inflation rate. The welfare impact of microfinance is largest for moderate inflation rates, where external funding enables entrepreneurship and consumption smoothing. For inflation rates above the threshold, the welfare impact of microfinance is smaller, since households use loans solely for consumption smoothing.

Lagos and Wright (2005) show that the costs of inflation can be significantly larger when trading frictions are taken into account or other pricing mechanisms are applied.

Welfare costs without access to financial services are again measured relative to the benchmark case, where every household has access to financial services. To calculate social welfare without access to the MFI, we closely follow Berentsen et al. (2007): We first set \( i_d = 0 \) in Equation (28) to derive the equilibrium amounts of the production good and afterwards calculate social welfare.
6 Conclusion

In this paper, we analyzed the effects of microfinance and inflation on social welfare in developing countries. In our theoretical model, group lending mitigates the agency problem between entrepreneurs and the microfinance institution, which in turn allows entrepreneurship to emerge. At the same time, access to financial services affects households’ decision to hold money balances, to save and to borrow. Under moderate inflation rates, entrepreneurship emerges, which in turn increases aggregate production and reduces real prices. Households receive interest payments on their deposits and benefit from the possibility to take out loans. Yet, subsistence producers, without profitable business ideas, are negatively affected by the general equilibrium effect on prices.

Our main result is that entrepreneurship collapses above an inflation threshold. A higher rate of inflation negatively affects entrepreneurship through two mechanisms. First, the standard real balance effect of inflation lowers output and expected real profits of entrepreneurs. Second, inflation increases the funding costs of entrepreneurs. The real balance effect affects subsistence producers and entrepreneurs alike, whereas the effect on funding costs applies only to entrepreneurs. There exists an inflation threshold beyond which entrepreneurship collapses, because entrepreneurs who rely on external funding are more affected by inflation than subsistence producers.

Our welfare analysis shows that the magnitude of the impact of microfinance on households’ welfare crucially depends on the prevailing monetary policy regime. Microfinance has the largest impact on social welfare if inflation is moderate and households use loans to start small enterprises as well as for consumption smoothing. If inflation is high, the impact of microfinance on welfare decreases substantially, as entrepreneurship collapses and households use loans solely for consumption smoothing. Our findings imply that a reasonable monetary policy is especially important in developing countries, where households face high transaction costs and information problems when applying for small loans. Better knowledge of the relationship between monetary policy and economic development and growth is of great interest for central banks, the World Bank and development agencies, but primarily to improve the quality of life of the people in developing countries.
References


A Appendix

A.1 Optimization Problem of Buyers

The optimization problem of the buyer with access to the MFI is:

\[
\max_{q_b, l_b} \{ u(q_b) + W(0, l_b, d + l_b - pq_b) \},
\]

\[ \text{s.t.} \quad pq_b \leq d + l_b, \quad \text{(BC)} \]
\[ l_b \leq \bar{l}. \quad \text{(LC)} \]

The Lagrangian of the maximization problem is:

\[
L(q_b, l_b) = u(q_b) + W(0, l_b, d + l_b - pq_b) - \lambda (pq_b - d - l_b) - \lambda_l (l_b - \bar{l}).
\]

The first-order conditions are:

\[
q_b : \quad u'(q_b) = pW_m + \lambda_l \phi p,
\]
\[ l_b : \quad (\lambda - \lambda_l)\phi = -W_l - W_m, \]
\[ \lambda : \quad l_b = pq_b - d,
\]
\[ \lambda_l : \quad 0 \geq l_b - \bar{l}. \]

Using the envelope conditions of holding money and borrowing money from Section 3.1 and substituting these into the second FOC gives \( \lambda - \lambda_l = id \). The difference between the multiplier of the budget constraint and the multiplier of the loan constraint is equal to the marginal change in utility. If the lending constraint (LC) is binding, then \( \lambda_l > 0 \) and the buyer borrows \( l_b = \bar{l} \). For the case that the lending constraint is not binding, \( \lambda_l = 0 \). In our model, we assume that the borrowing constraint will never be binding. In this case, the consumer chooses \( l_b \) such that in equilibrium the multiplier \( \lambda \) is equal to \( id \). Using this result and again the envelope condition for the first FOC gives the following condition which has to hold if buyers maximize utility:

\[
u'(q_b) = \phi p(1 + id).
\]

A.2 Optimization Problem of Subsistence Producers

Subsistence producers choose the amount \( q_s \) that maximizes profit, thereby taking as given the price \( p \). Producers incur utility costs \( c(q_s) \) for producing the amount \( q_s \).
Subsistence producers’ optimization problem is:

$$\max_{q_s} \{-c(q_s) + W(d, 0, pq_s)\}. \tag{38}$$

The first-order condition is:

$$q_s : c'(q_s) = pW_m = \phi p. \tag{39}$$

### A.3 Optimization Problem of Unbanked Households

The unbanked households can neither hold deposits nor take out loans. Buyers choose how much to demand of good $q$. Their optimization problem is formulated as follows:

$$\max_{q_b} \{u(q_b) + W(0, 0, m - pq_b)\}, \tag{40}$$

s.t. $pq_b \leq m$, \hspace{1cm} (BC)

where the constraint states that households can dispense up to their money holdings $m$. The Lagrangian of the maximization problem of the buyer is

$$L(q_b) = u(q_b) + W(0, 0, m - pq_b) - \lambda_m \phi [pq_b - m]. \tag{41}$$

The first-order conditions are:

$$q_b : u'(q_b) = W_mp + \lambda_m \phi p, \tag{42}$$

$$\lambda_m : m = pq_b.$$ 

The envelope condition for holding money states that $W_m = \phi$. In the optimum, buyers choose $q_b$ such that the following equation is satisfied:

$$u'(q_b) = \phi p(1 + \lambda_m). \tag{43}$$

If the budget restriction of the buyer is binding, then he will consume: $q_b = m/p$. If the buyer’s money balance is sufficiently large, he will consume the amount of the production good that sets marginal utility equal to the real price. In this case, $\lambda_m$ is equal to zero.

Subsistence producers choose the amount $q_s$ that maximizes profit, thereby taking as given the price $p$. Households incur utility costs $c(q_s)$ for producing the amount $q_s$. 

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Their optimization problem is formulated as follows:

$$\max_{q_s} \{ -c(q_s) + W(0, 0, pq_s) \}. \quad (44)$$

The Lagrangian of the maximization problem of the subsistence producer is

$$L(q_s) = -c(q_s) + W(0, 0, m + pq_s). \quad (45)$$

The first-order condition is:

$$c'(q_s) = W_m p = \phi p. \quad (46)$$

### A.4 Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>parameter of the utility function</td>
</tr>
<tr>
<td>a</td>
<td>0.25</td>
<td>entrepreneurs’ cost factor</td>
</tr>
<tr>
<td>B</td>
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<td>entrepreneurs’ private benefit</td>
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<td>b</td>
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<td>discount factor</td>
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<td>monitoring costs</td>
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<td>$\eta$</td>
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<td>parameter of the utility function</td>
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<td>setup costs of entrepreneurs</td>
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<td>success rate, high</td>
</tr>
<tr>
<td>$\mu_l$</td>
<td>0.3</td>
<td>success rate, low</td>
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<tr>
<td>n</td>
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<td>rate of producers and entrepreneurs</td>
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<tr>
<td>$\rho$</td>
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<td>parameter of the production function</td>
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</tbody>
</table>

Notes: Standard parameter values are similar to Lagos and Wright (2005).